Chapter 10

Dynamics of Rotational Motion

Torque

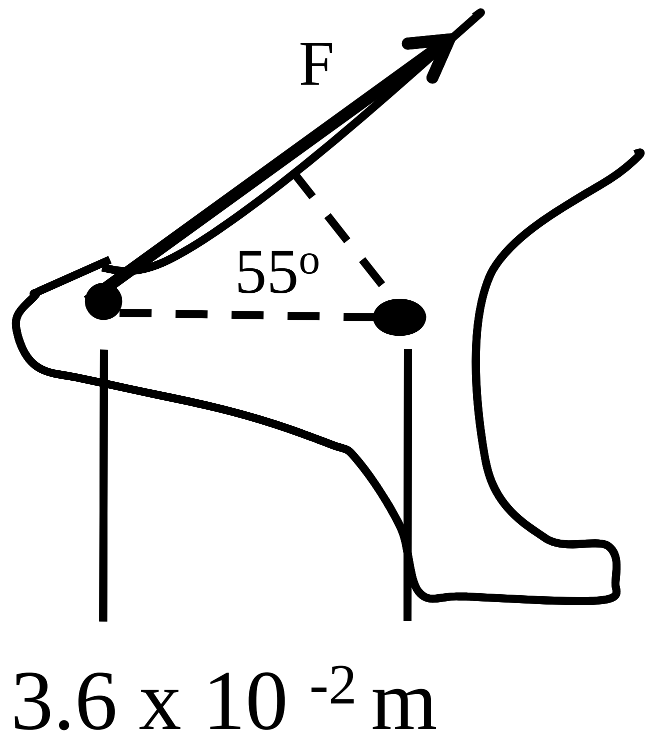
- Definition:
- A force can be defined as something that causes linear motion to change.
- A torque can be similarly defined as something that causes a change in rotational motion.

- The torque applied to a rigid body depends on the magnitude of the applied force, the distance away from the point of rotation (lever arm), and the direction of the applied force.
- We can express the torque as a vector cross product.

Example

- The Achilles tendon of a person is exerting a force of magnitude 720 N on the heel at a point that is located 3.6×10^{-2} m away from the point of rotation.
- Determine the torque about the ankle (point of rotation).
- Assume the force is perpendicular to the radial arm.

Picture of foot and Achilles tendon



Solution

- The magnitude of the torque is:

Torque and Angular Acceleration

- Consider a point on a rigid body.
- If a force is applied we can determine the tangential acceleration by Newton’s second law.

- We can express this in terms of the angular acceleration by the following relation.

- The force on the particle can now be written as:

- If we apply the definition of torque to our previous relation we get the following:

- We can use the vector triple cross product identity to rewrite this expression.

Vector Triple Cross Product

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}\cdot\vec{C}\right)-\vec{C}\left(\vec{A}\cdot\vec{B}\right)$$

- If we apply this identity to our product we get the following:

- However, **r** and α are perpendicular; therefore,

- The torque now becomes:

  - We can rewrite this in terms of the moment of inertia.

Rigid-Body Rotations About a Moving Axis

- Consider the motion of a wheel on an automobile as it moves along the road.
- Since the wheel is in motion it possesses kinetic energy of motion.
- However, since the wheel is both translating along the road and rotating, the kinetic energy of the wheel is shared between these two motions.

- The kinetic energy of a rotating body about a moving axis is given by:

Example

- Determine the kinetic energy of a solid cylinder of radius R and mass M if it rolls without slipping.

Solution

- If an object rolls without slipping the speed of the center of mass is related to the angular speed by the following:

- The kinetic energy is given by:

- The moment of inertia for a solid cylinder is:

- The kinetic energy then becomes:

Work and Power for Rotations

- The work done rotating a rigid object through and infinitesimal angle about some axis is given by the following:

- The total work done rotating the object from some initial angle to some final angle is:

- If we examine the integrand we see that it can be rewritten as:

- We integrate to get the work done.

Power

- The power of a rotating body can be obtained by differentiating the work.

- If we differentiate we get:

Angular Momentum

- We define the linear moment of a system as:

- We saw that Newton’s second law can be written in terms of the linear momentum.

- We can define momentum for rotations as well.
- We define the angular momentum of a system as:

- The value of the angular momentum depends upon the choice of origin.
- The units for angular momentum are:

$$kg \cdot m^2 / s$$

Newton’s Second Law

- Suppose an object is subjected to a net torque.
- How does this affect the angular momentum?

- Suppose the angular momentum changes with time.
- We can write the following:

- Now suppose the mass is constant.
- When we differentiate we get:

- The first term is equal to zero due to the properties of the cross product.
- Therefore, the change in angular momentum is:

Thus, we have Newton’s second law for rotations.

- The angular momentum around a symmetry axis can be expressed in terms of the angular velocity.

- If we take the time derivative of the angular momentum, once again we arrive at Newton’s second law.

$$= I\vec{\alpha} = \vec{\tau}$$

Example

- A woman with mass 50-kg is standing on the rim of a large disk (a carousel) that is rotating at 0.50 rev/s about an axis through it center.
- The disk has mass 110-kg and a radius 4.0m.
- Calculate the magnitude of the total angular momentum of the woman plus disk system.
- Treat the woman as a point particle.

Solution

- The angular momentum of the system is:

- The magnitude of the angular momentum of the system is:

- The magnitude of the linear momentum of the woman is:

- The magnitude of the angular momentum of the woman is:

- The magnitude of the angular momentum of the disk is:

- The magnitude of the total angular momentum of the system is:

The Conservation of Angular Momentum

- Consider a system with no net torque acting upon it.

- The equation above implies that if the net torque on a system is zero, then the angular momentum of the system will be constant.