Chapter 11

Equilibrium

Equilibrium

- If an object is in equilibrium then its motion is not changing.
- Therefore, according to Newton's second law, the net force must be zero.
- Likewise, if its motion is constant then the rotation must be constant and Newton's second law says that the net torque must also be zero.

- We can express the equilibrium condition as follows:

- Since both the force on an object and the torque are vectors then the components of each must also be zero.

- The two equations imply that both the translational momentum and the rotational momentum of a system remains constant.
- We therefore, say that they are conserved.

- If we put one further restriction on the system and demand that the motion itself is zero then we have the case of static equilibrium.

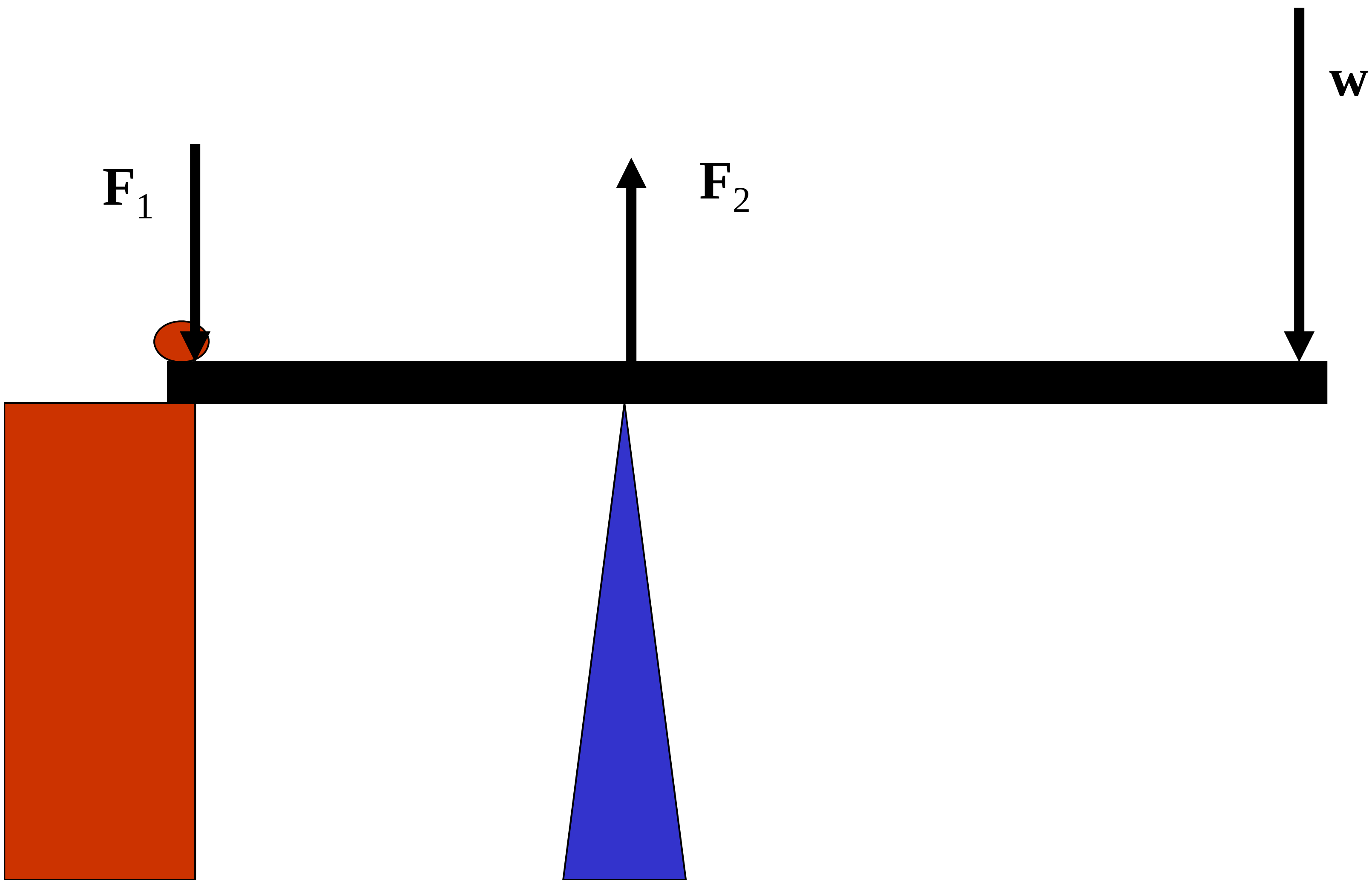
Static Equilibrium

- The conditions for static equilibrium are:

Example

- A woman whose weight is 530 N is poised at the end of a diving board, whose length is 3.90 m.
- The board has negligible weight and is bolted down at the opposite end, while being supported 1.40 m away by a fulcrum.
- Find the forces F_1 and F_2 that the bolt and the fulcrum, respectively, exert on the board.

Sketch



Solution

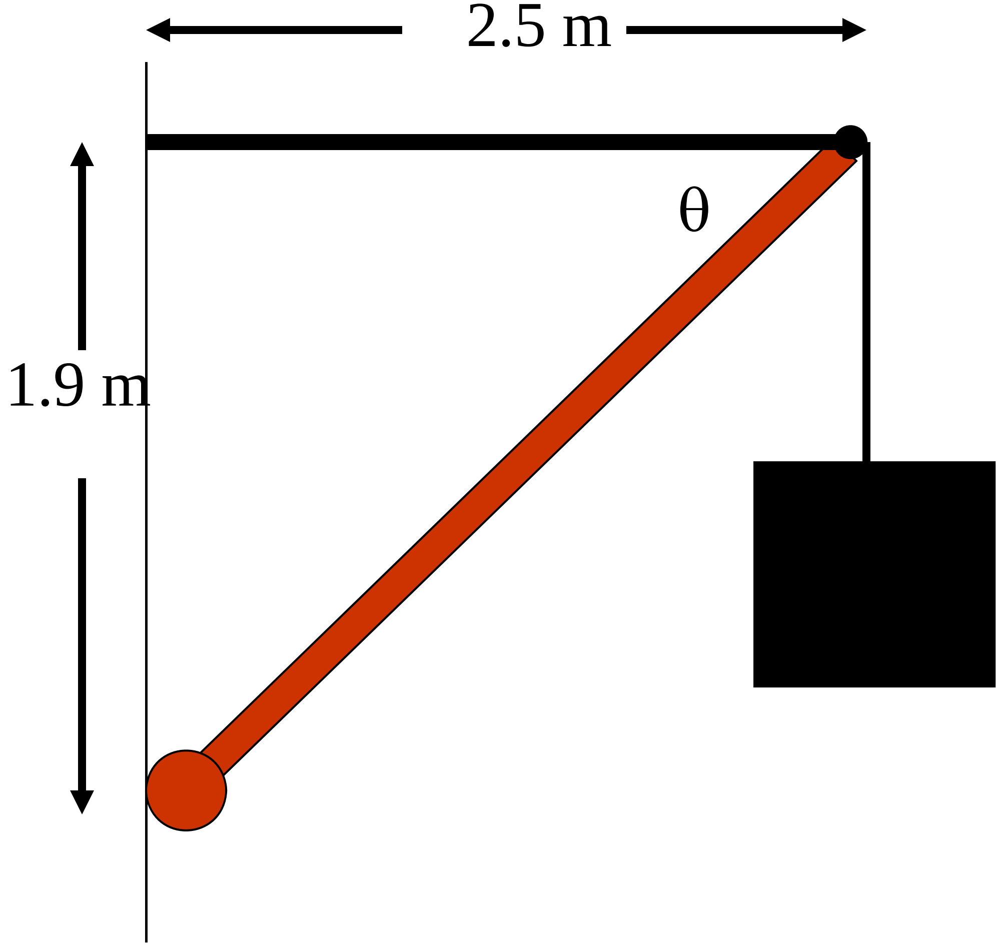
- Since the board is in equilibrium, the sum of the vertical forces must be zero:

- Similarly, the sum of the torques must be zero.

Example

- A crate of mass 430-kg is hanging by a rope from a boom.
- The boom consists of a hinged beam and a horizontal cable.
- The uniform beam has a mass of 85-kg and the masses of the cable and rope are negligible.
- Determine the tension in the cable.
- Find the magnitude of the net force on the beam from the hinge.

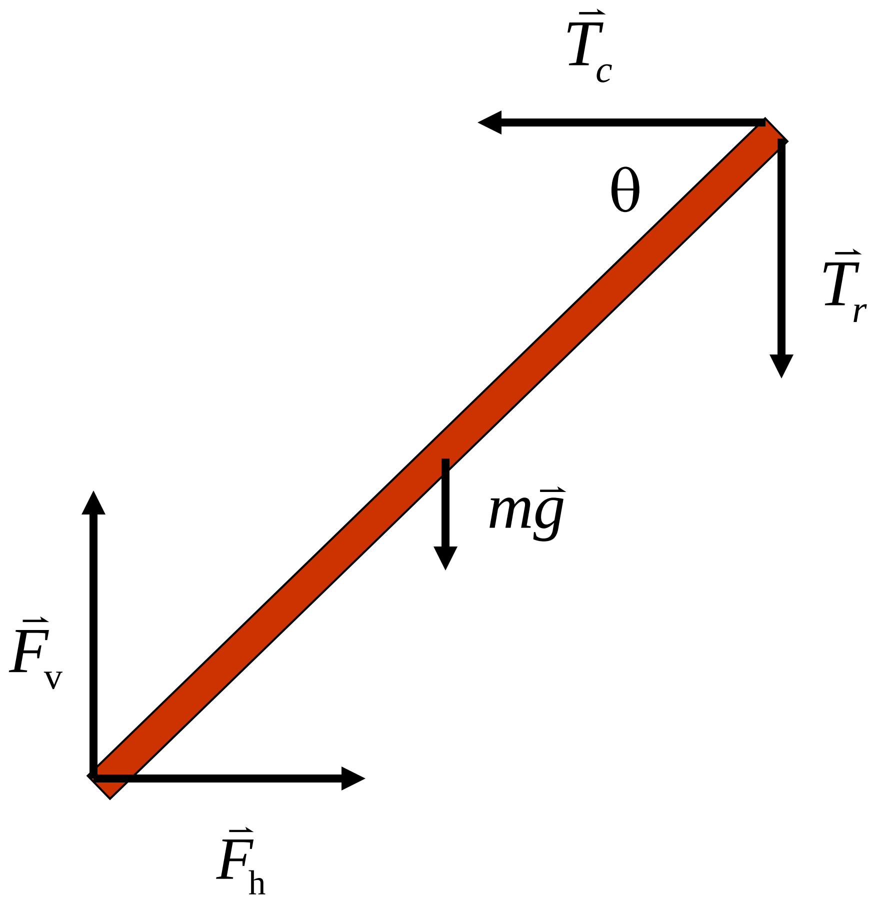
Picture



Solution

- We first draw a free body diagram of the forces on the beam.
- We may choose to place the origin at the hinge.

Free-Body Diagram



- If we set the forces in the x and y directions equal to zero we get the following:

- We can calculate the torques about the hinge and set them equal to zero.

- We note that T_r is just the weight of the crate; therefore,

- From our force equations we get:

- The magnitude of the force is then:

Rigid-Bodies in Equilibrium

- A rigid body is said to be in equilibrium if the sum of the forces, as well as the sum of the torques about any point on the body are equal to zero.
- Hence:

- When solving problems of rigid-body equilibrium it is important to do the following:
- a) Draw a sketch of the problem.

  b) Draw a free body diagram, labeling all the forces that act on the body.

- c) Draw a coordinate system and represent all vector quantities in terms of their components.

- d) Choose points at which to compute torques that simplify the problem.
- e) Write down the equations that express the equilibrium conditions.
- f) Make certain that you have the same number of equations as unknowns and then use the appropriate method to solve for the unknowns.

Example:

Sir Lancelot is trying to rescue the Lady Elayne from Castle Von Doom by climbing a uniform ladder that is 5.0 m long and weighs 180 N.

Lancelot, who weighs 800 N, stops a third of the way up the ladder.

The bottom of the ladder rests on a horizontal stone ledge and leans across the moat in equilibrium against a vertical wall that is frictionless because of a thick layer of moss.

The ladder makes an angle of 53.1^{o} with the horizontal, conveniently forming a 3-4-5 right triangle.

a) Find the normal and frictional forces on the ladder at its base.

b) Find the minimum coefficient of static friction needed to prevent slipping at the base.

c) Find the contact force on the ladder at the base.

Solution: a

- The forces at the bottom of the ladder are the upward normal force N_2 and the static friction force f_s, which points to the right to prevent slipping.
- The frictionless wall exerts a normal force N_1 at the top of the ladder that points away from the wall.

Solution: a cont.

- The sums of the forces in the x and y directions are:

- We can solve for N_2 in the second equation but the first equation has two unknowns therefore we need an additional equation to finish the problem.
- The magnitude of N_2 can be solved for and yields:

- To solve the rest of the problem we need to take a torque about some point. A good choice is at the base of the ladder.
- The two forces N_2 and f_s have no torque about that point.
- The lever arm for the ladder's weight is 1.5 m, the lever arm for Lancelot's weight is 1.0 m, and the lever arm for N_1 is 4.0 m.
- The torque is then:

- Solving for N_1 we get:

We can now substitute this result back into our x direction force equation and then solve for the static frictional force:

Solution: b)

- The static friction force f_s cannot exceed $\mu_s N_2$, so the minimum coefficient of static friction to prevent slipping is:

Solution: c)

- The components of the contact force at the base are the static friction force and the normal force, therefore:

Solution: c)

- The magnitude of F_B can now be determined:

Solution: c)

- The angle from horizontal is then: