Chapter 12

Gravitation

- Objects fall to earth because of gravity; however, nothing has been said about why they fall or why the rate of descent is 9.8 m/s^2 .
- In addition to his three laws of motion, Newton also provided a coherent understanding of the gravitational force.

The Universal Law of Gravitation

- Every particle in the universe exerts an attractive force on every other particle.
- For two particles, which have masses m₁ and m₂ and are separated by a distance r, the force that each exerts on the other is directed along the line joining the particles

• Newton expressed his law of gravitation by the following equation.

• The symbol G denotes the universal gravitational constant.

 $G = 6.67259 \times 10^{-11} N \cdot m^2 / kg^2$

Example

- What is the magnitude of the gravitational force that acts on each particle?
- Particle m₁ has a mass of 12 kg, while particle m₂ has a mass of 25 kg and the distance separated by a distance of 1.2 meters.

Solution

• For comparison, the force needed to push a doorbell is approximately 1 N.

Definition of Weight

- The weight of an object on the earth is the gravitational force that the earth exerts on an object.
- The magnitude of the weight for a mass, m, can be obtained from the following equation:

Example

- The mass of the Hubble Space Telescope is 11 600 kg.
- Determine the weight of the telescope when it is resting on the surface of the earth and when it is in its orbit 596 km above the earth's surface.

Solution

• On the earth's surface,

• When it is in orbit,

From where does $g = 9.8 \text{ m/s}^2$ come?

• If we use Newton's universal law of gravitation to describe the attraction between the earth and an object on the surface of the earth then we are describing the weight of the object on the earth.

• The equation for gravitation then becomes:

 Since the mass and radius of the earth is nearly constant then we can combine G, M_E, and R into a single number and write the following:

- We now see what the relationship between mass and weight is.
 - What is the value for the Earth's mass?

Gravitational Potential Energy

- Earlier, we derived the gravitation potential energy of the earth assume the force of attraction between a mass and the earth was constant.
- However, we now see that the force of attraction varies with the square of the distance.
- Therefore, our previous equation for potential energy is only an approximation that is useful near the surface of the earth.

- From the work energy theorem we know that the change in potential energy of a mass that is in free fall is equal to negative the work done.
- Therefore,

• The force is given by Newton's law of gravity.

- If we take the origin to be at the center of the earth, then the angle between the force and dr is π radians.
- Therefore,

• The change in gravitational energy of a falling body is:

• Therefore, the gravitational potential energy of a body falling to the earth is defined as:

Example

- Suppose a very strong volleyball player gives a volleyball just enough energy for it to travel upward and never return.
- How fast must the volleyball be struck for this to occur?

Solution

- If the ball has just enough energy to leave the earth, then its final kinetic energy will be zero.
- Furthermore, the final distance of the ball from the earth will be infinite.

Circular Orbits

- Suppose you build a giant cannon to launch pumpkins.
- If the pumpkins are launched horizontally, the greater the charge of the cannon, the farther the pumpkins will travel.
- Eventually, the parabolic trajectory of the pumpkin will match the curvature of the earth.
- At this point the pumpkin curvature will be circular and the pumpkin will be in orbit around the earth.

- If the orbit is circular, then the centripetal force of the satellite will be provided by the gravitational force of the earth.
- Therefore,

Kepler to the Rescue

- With the use of Tyco's data, Kepler disproved the theories of Ptolemy and Copernicus.
- He then developed a new theory of the solar system.



Kepler's First law of Planetary Motion

• Kepler's First law says that planets orbit the sun in ellipses, with the sun at one of the foci.



Kepler's Second Law

• Kepler's second law states that planets sweep out equal areas in equal times.



Kepler's Third Law

- Kepler's third law states that the cube of a planet's semi-major axis is proportional to the square of its period of orbit.
- If we measure a planet's period in earth years and its distance in astronomical units then we can write the following:

Kepler's Second Law

• Kepler's second law can be expressed as a differential equation.

 Here, A is the area swept out by the planet, r is the radius and θ is the angle of displacement. • If we express Kepler's second law in terms of the velocity we have the following:

Since rv sin φ is the magnitude of the cross product of r and v, we can write this equation in terms of the angular momentum.

- Kepler's second law states that the angular momentum of a planet must be constant.
- This can be seen by the definition of torque in terms of angular momentum.

Kepler's Third Law

- Consider the special case of circular orbit.
- We determined that the velocity of an object orbiting the earth was given by:

• We could get a similar result for a planet orbit a star like our sun.

• The period of orbit is the distance once around, divide by the tangential speed.

• If we substitute in for the velocity we get the following:

• Rearranging, we have Kepler's third law.

• The square of the period is equal to the cube of the semi-major axis.