Chapter 3

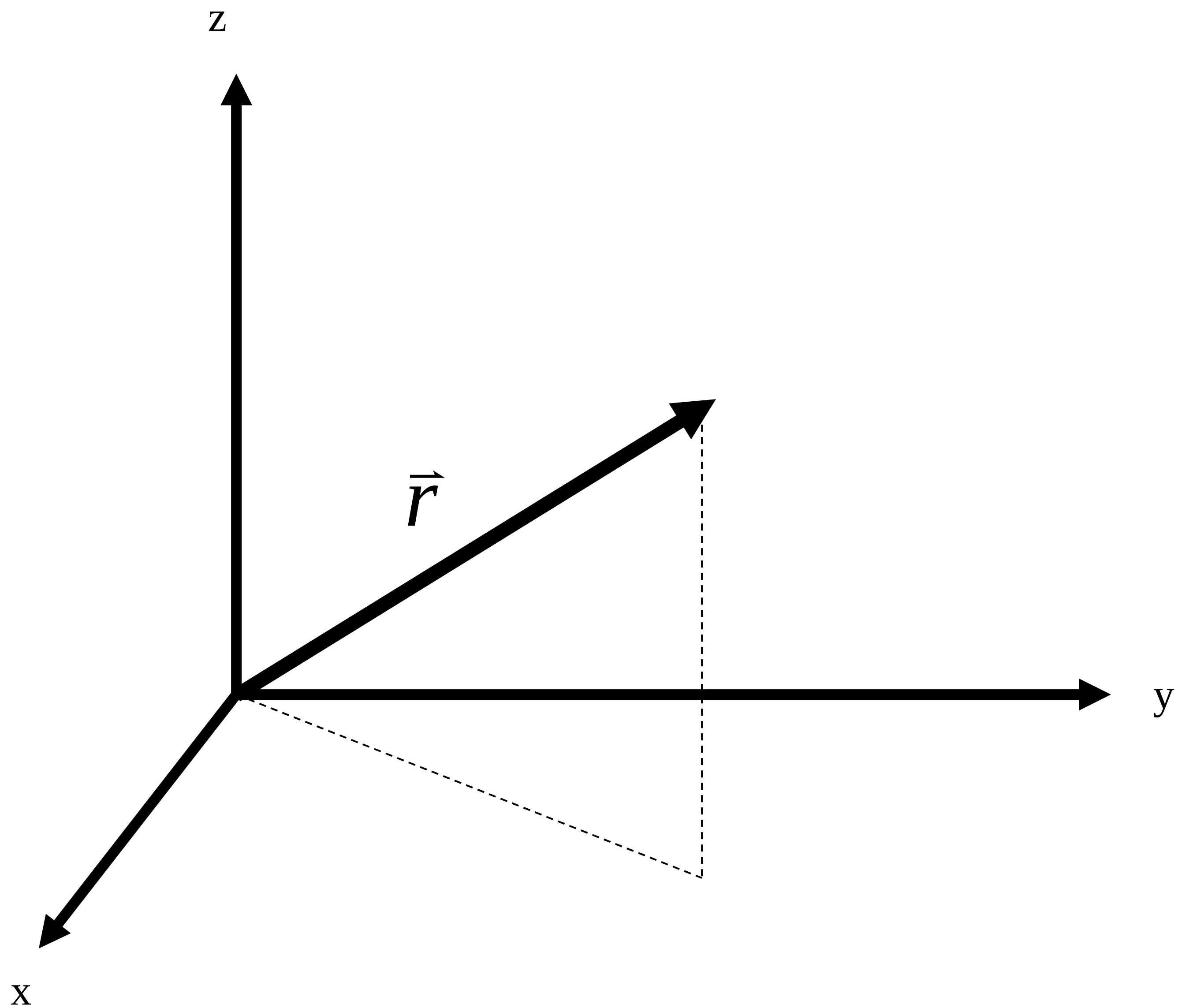
Motion in Two or Three Dimensions

Position

- Consider the following position vector expressed in Cartesian coordinates.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

- This vector defines the position of a particle at some instant in time, relative to our coordinate system.


z
$\vec{r}$
y
x

Average Velocity

- Now suppose our position vector is changing with time.
- In other words, the particle is moving.
- The average velocity of the particle is:

Instantaneous Velocity

- The instantaneous velocity can be obtained by letting the time difference Δt, approach zero.

- In Cartesian coordinates velocity can be written as:

Example

- In pursuit of prey, a moray eel's position vector is given below.
- Determine the velocity of the eel 3 seconds into the hunt.

Solution

- To determine the eel’s velocity as a function of time, we first differentiate our position vector with respect to time.

Solution cont.

- Now we evaluate the velocity at time
- $t = 3s$.

The Acceleration Vector

- When the velocity of an object is not constant then we that it is accelerating.
- The acceleration vector in Cartesian coordinates is:

Example

- Calculate the acceleration of the moray eel in the previous example.

Solution

- Since we have already determined the velocity of the eel as a function of time, we only need to differentiate it once with respect to time to determine its acceleration.

Acceleration Vector

- We can also express acceleration in terms of the position vector.

Acceleration Vector cont.

- In terms of Cartesian coordinates this becomes:

Example

- An intrepid person takes a running jump off of a cliff into the water below.
- If the height of the cliff is 30 meters and the person runs with a speed of 5.0 m/s, how far from the bottom of the cliff will she strike the water?

Solution

- Once she leaves the cliff her only acceleration will be that of gravity.
- Therefore, her velocity in the x-direction will remain constant throughout the fall.
- If we knew her time of flight then, with the equation below, we could determine how far she was from the base of the cliff when she entered the water.

- We can determine her time of flight by looking at her vertical motion.
- Therefore, we need to use an equation of motion that has time in it.
- Furthermore, since we know her initial velocity in the y-direction, her displacement, and her acceleration we use the following equation.

- We note that her initial velocity in the y-direction is zero and we solve for time.

- Plugging in our values we get:

- Now we can plug this time back into our previous equation involving x and get her horizontal displacement.

Example

- Suppose that you lob a tennis ball with an initial speed of 15.0 m/s, at an angle of 50.0 degrees above the horizontal.
- At this instant your opponent is 10.0 m away from the ball. She begins moving away from you 0.30 s later, hoping to reach the ball and hit it back at the moment that it is 2.10 m above its launch point.
- With what minimum average speed must she move?

Solution

- Using the data given in the problem, we can find the maximum flight time t of the ball using

- Once the flight time is known, we can use the definition of average velocity to find the minimum speed required to cover the distance in that time.

- Solving for the time yields the following:

- The first root corresponds to the time required for the ball to reach a vertical displacement of $y = +2.10$ m as it travels upward, and the second root corresponds to the time required for the ball to have a vertical displacement of $y = +2.10$ m as the ball travels upward and then downward.
- The desired flight time t is 2.145 s.
- During the 2.145 s, the horizontal distance traveled by the ball is

- Thus, the opponent must move 20.68 m – 10 m = 10.68m in 2.145s – 0.3 s = 1.845 s.
- The opponent must, therefore, move with a minimum average speed of

Example

How to Sack a Castle.

- The evil lord Percy has barricaded himself in the castle keep.

- The keep is located at the center of a courtyard which is completely surrounded by a 12 meter high castle wall.

- Meanwhile, Baldrick the Brave,

•but not so handsome has positioned a catapult just outside the castle wall.

•Baldrick’s intent is to launch a projectile over the castle wall and smash the keep.

- The keep itself is located 75 meters inside the castle wall, while the catapult is 75 meters outside the castle wall.
- Determine the initial velocity (speed and angle with respect to the horizontal) of the projectile if it is to just clear the castle wall and impact the keep.
- Remember the wall is 12 meters high.

Solution

- Baldrick needs to accomplish two things.
- First the projectile must clear the height of the castle wall.
- Second, once over the castle wall, the projectile must have enough range to reach the castle keep.
- The equations for projectile motion are:

- Suppose the projectile just clears the castle wall at its highest point of flight.
- At this point the y-component of the velocity is zero.
- Therefore, the time (t_1) required for the projectile to reach the top of the wall is:

- We can now eliminate the time in the second equation of motion.

- The second criterion that must be met is the range.
- The range is given by the last equation of motion.

- We can get the time of flight (t_2) from the second equation by noting that the height at impact is zero.

- Plugging back into our equation for the range and we get:

- We now have two equations and two unknowns:

Almost There

- We solve the initial velocity in the second equation and substitute it back into the first equation.

- We can now solve for the angle of elevation.

- We can now solve for the initial speed of the projectile.

Finally

- Therefore, the initial velocity needed to clear the wall and hit the castle keep is: