

Chapter 5

Applications of Newton's Laws

Particles in Equilibrium

- When a particle is in equilibrium, the sum of the forces on the particle equals zero.
- Newton's second law then becomes:

$$\sum \vec{F} = 0$$

Steps for solving problems in equilibrium

- 1. Draw a sketch of the problem.
- 2. For each object draw a free-body diagram.
- Draw vectors that represents forces on each object. Include only the forces on each object, not the forces that they exert on other objects.

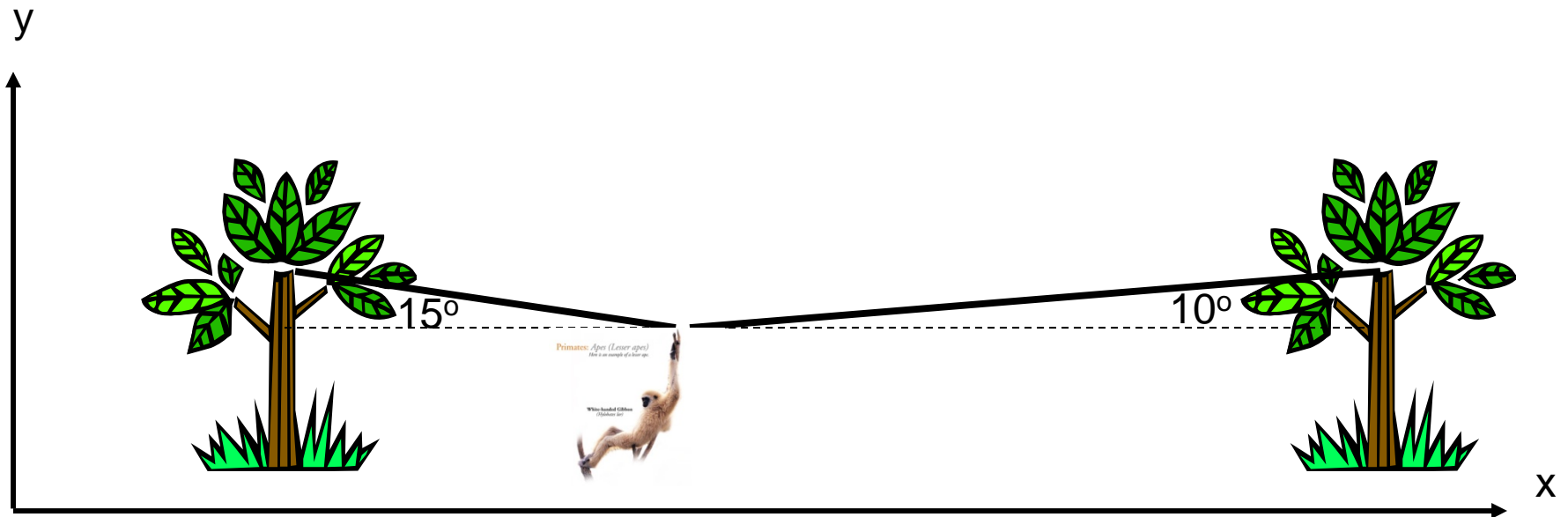
- 3. Set up a coordinate system appropriate to the problem.
- 4. Find the components for each force along each coordinate axis.
- 5. Sum up the components of the forces along each direction and set the sum equal to zero.
Note: These components remain independent.
- 6. Make certain you have n -equations and n -unknown and then solve for the desired variables.

Example

- An ape hangs from a rope that is suspended between two trees.
- The angle between the left side of the rope and the horizontal is 15° and the right side makes an angle of 10° .
- If the ape has a mass of 100.0 kg determine the tension in each section of rope.

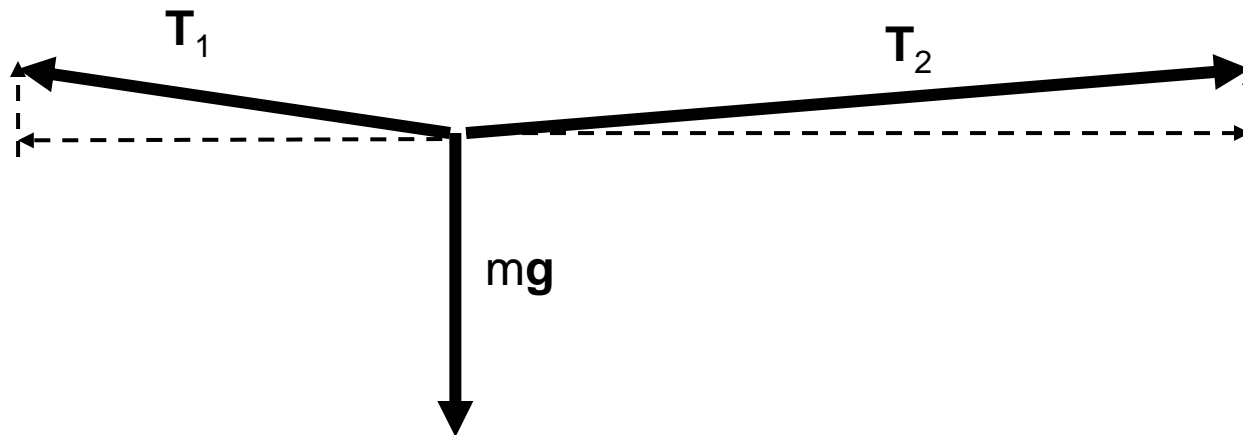
Solution

- First sketch the problem.



Solution cont.

- Next make a free body diagram and label the forces.



Solution cont.

- Now sum up the forces in each direction and set them equal to zero.

Solution cont.

- Now we solve for the appropriate variables.

Solution cont.

- The tensions are:

The Normal Force

- The normal force F_N is the component of the force that a surface exerts on an object with which it is in contact, and is perpendicular to the surface.

Example

- A brick with a weight of 15 Newtons rest on a table.
- Someone pushes downward on the top of the brick with a force of 11 Newtons the resulting normal force is the vector sum to the two forces but in the opposite direction.

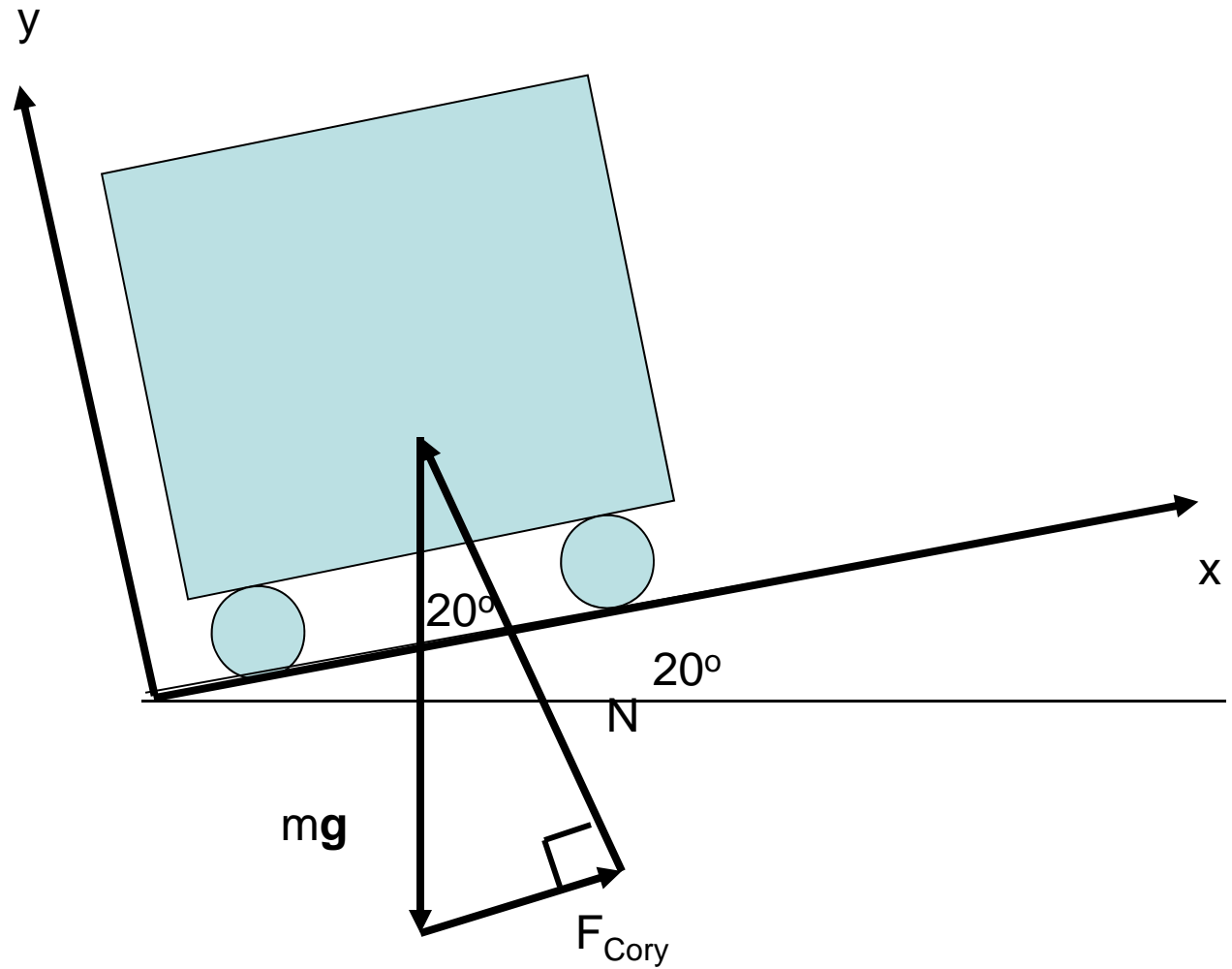
Solution

- If we call the direction of down, negative then the normal force has the following solution:

Example

- Cory Everson is pushing a crate on wheels up an incline.
- The crate moves up the incline with a constant speed and the friction is negligible.
- If she applies a force of 500N on the crate and the angle of the incline is 20° , how much does the crate weigh?
- Determine the normal force between the crate and the plane.

Free-Body Diagram



Solution

- We now sum up the forces in each direction and set them equal to zero.

Solution cont.

- Now solve for the weight of the crate and the normal force.

Particle Dynamics

- For cases where particles are not in equilibrium Newton's second law is:

Component Form

- We can write Newton's second law in component form.
- In Cartesian coordinates it becomes:

Problem solving tips

- 1. Draw a sketch of the problem.
- 2. Draw a free body diagram for each body and show all the forces acting on each body.
- 3. Choose a coordinate system. If you have more than one object that is accelerating, you can choose a different coordinate system for each object.

Problem solving tips

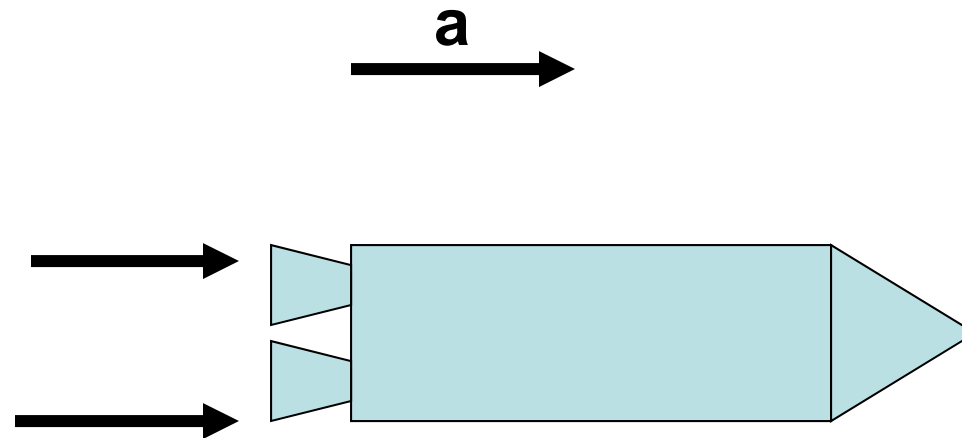
- 4. Identify the particulars about the problem. For instance, you may need to use an equation of motion in addition to Newton's second law to solve the problem.
- 5. Determine the components of all the forces that you are interested in the problem.
- 6. Apply Newton's second law and make certain that you have n -equations and n -unknowns.

Example

- The thrusters on a spaceship provide a constant forward force.
- The spaceship is far away from any significant gravitational influences.
- The spaceship's velocity relative to the earth increases from 500 m/s to 1500 m/s in 10 seconds.
- If the weight of the spaceship on earth is 100,000 N, determine the force of the thrusters.

Sketch

- We first make a sketch.



Free-Body Diagram



Solution

- We now apply Newton's second law.

- Note: We do not know the mass of the spaceship or its acceleration.

Solution cont.

- We can get its mass by the following:

Solution cont.

- The acceleration can be obtained from the equations of motion.

Solution cont.

- The force of the thrusters is then:

Example

- A water-skier is being pulled behind a boat.
- Her position as a function of time is given by the following:

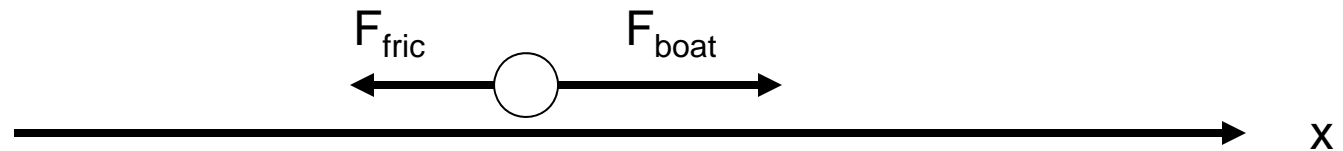
Example cont.

- If the skier's mass is 55 kg and the force of friction on her skies due to the water is 200 N, determine the force applied to her by the boat after 2 seconds.

Picture



Free-Body Diagram



- Now we apply Newton's second law.

Frictional Forces

- When two objects are in contact there is a normal force that is in a direction perpendicular to the surface.
- When the object moves or attempts to move along the surface then there is a force parallel to the surface called friction.

Frictional Forces

- The exact details of how frictional forces arise is not very well understood; however, some empirical relations have been developed that allow for the effects of friction to be taken into account.

- When the surface of one object slides over the surface of another each object exerts a frictional force on the other.
- This frictional force is called the **kinetic frictional force**, or sliding frictional force.
- The magnitude of the kinetic frictional force, f_K , is given by the following:

Coefficient of Kinetic Friction

- The coefficient of kinetic friction, μ_k , is a unitless number that depends on the two surfaces in question.

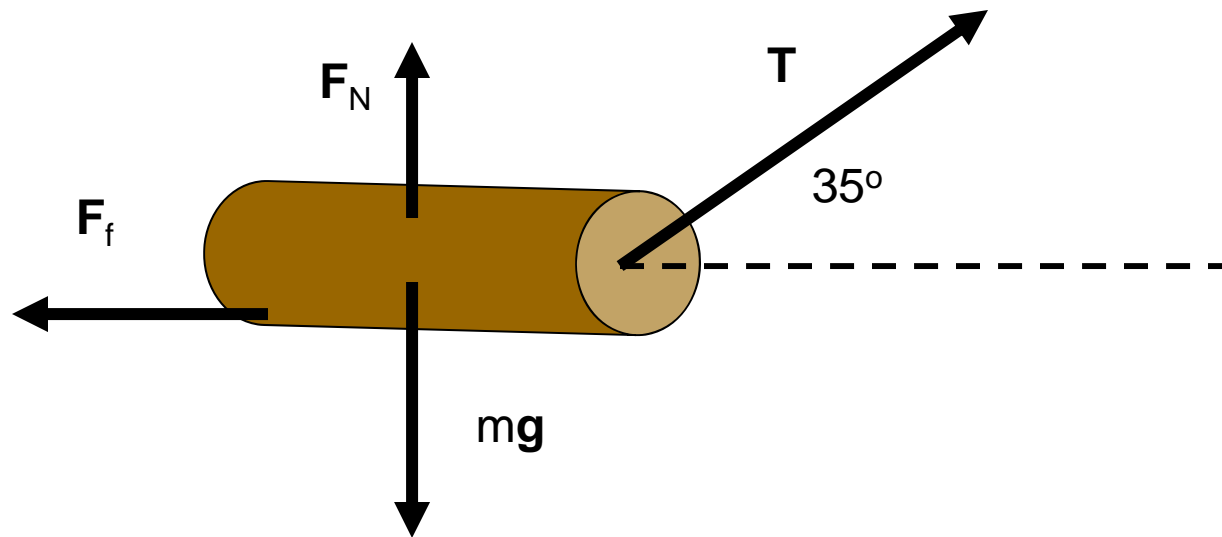
Static Frictional Force

- Another frictional force, called the **static frictional force**, acts even when there is no motion between the surfaces of two objects.
- The magnitude of the static frictional force, f_s , can have any value from zero up to a maximum value that depends on the applied force.
- Therefore,

Example

- An elephant is used to pull a large log across the ground.
- The chain attached to the log makes an angle of 35° with respect to the horizontal.
- The log has a mass of 500-kg and the coefficient of kinetic friction is 0.5.
- Determine the force supplied by the elephant to keep the log moving with a constant velocity.

Free-Body Diagram



Solution

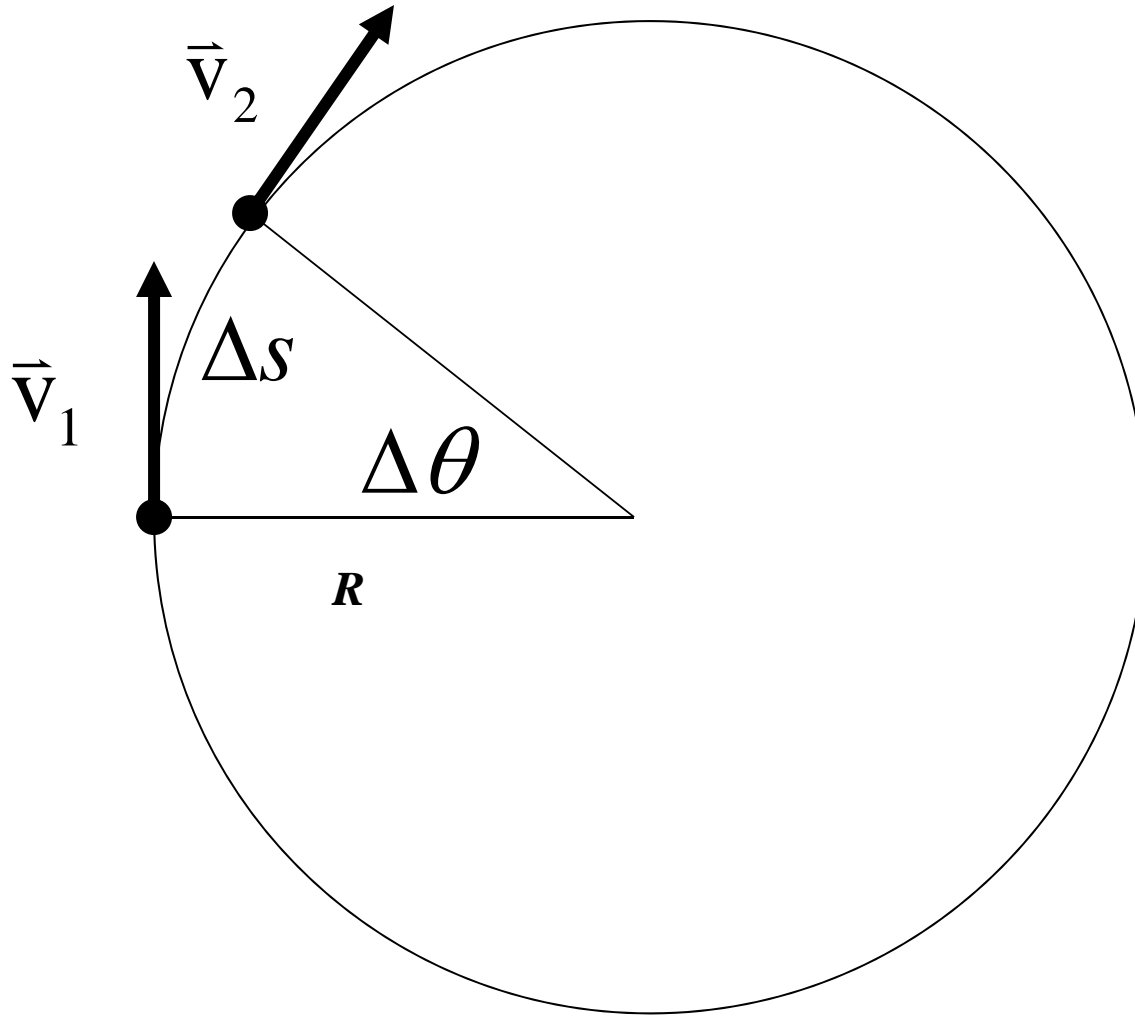
- We apply Newton's second law for each component of the problem.
- The force of friction is equal to the normal force times the coefficient of kinetic friction.
- However, the normal force is not equal to the weight because of the vertical component of the chain.

- We solve for our variables.

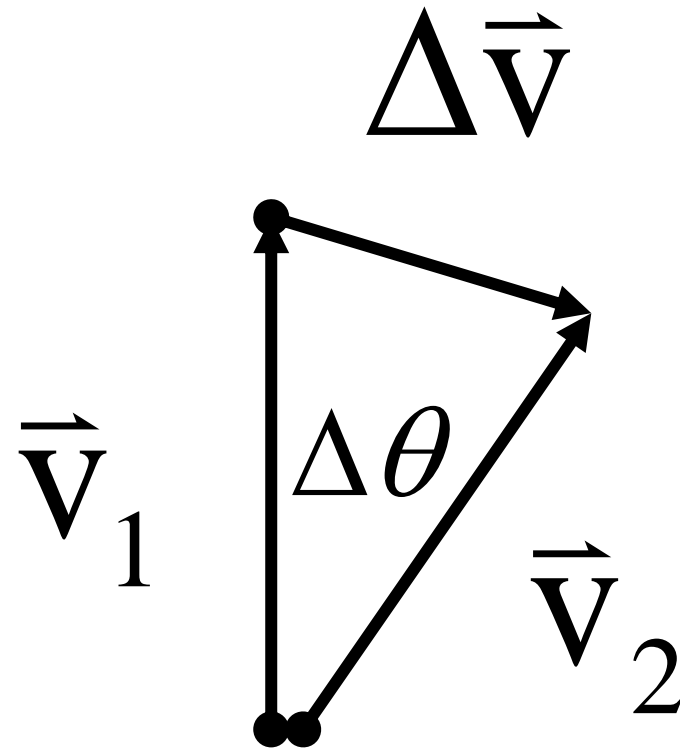
Uniform Circular Motion

- Uniform circular motion occurs when an object moves in a circle with no tangential acceleration.
- The only acceleration that is present is due to the continuously changing direction.
- Such acceleration is directed toward the center of the circle and is called **centripetal acceleration**.

Consider the velocity of circular motion at two different instances.



- The difference between the two velocities can be obtained by the following:



- The magnitude of the change in velocity can be written in terms of the angle and the radius of the circle by the use of similar triangles.

- The magnitude of the acceleration is the change in velocity over the change in time. Therefore, the centripetal acceleration becomes:

- If we drop the subscript on the velocity and note that the definition of the magnitude of the velocity is the change over position with time, then the centripetal acceleration becomes:

Period

- We can also define the centripetal acceleration in terms of the period of revolution.
- The period of revolution, or time to go around once, is:

- The magnitude of the centripetal acceleration in terms of the period becomes:

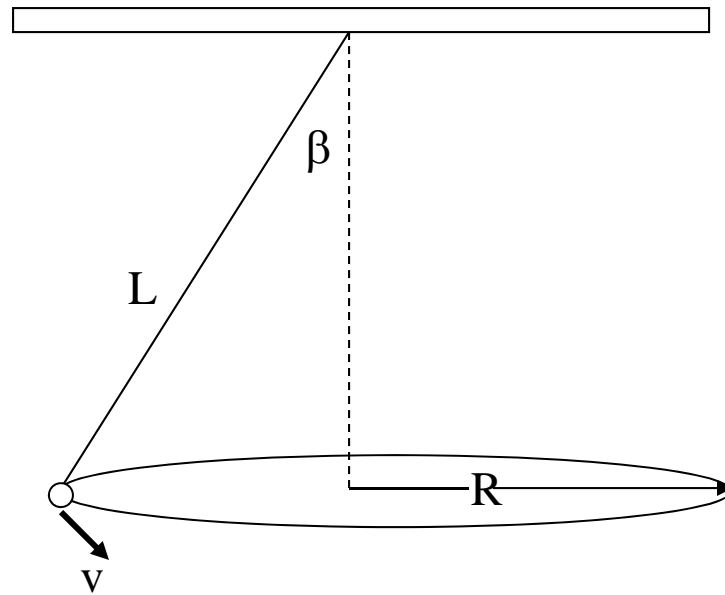
Centripetal Force

- If we apply Newton's second law to uniform circular motion we get the following:

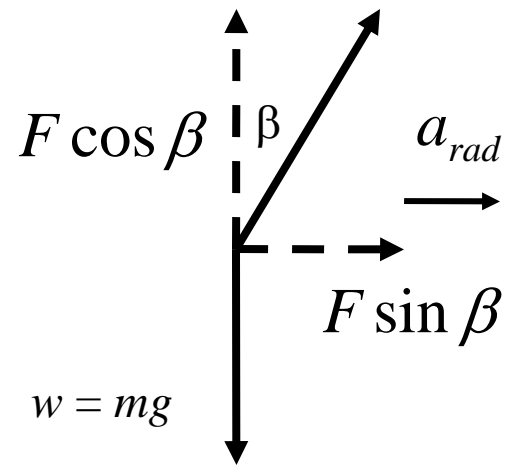
Example

- A mass m is suspended from the ceiling with a wire of length L .
- The mass moves with a speed v in a horizontal circle of radius R .
- The angle that the wire makes with the vertical is β .
- Given the angle β find the tension, F , in the wire and the period T .

Picture



Free-Body Diagram



- We now sum the forces and set them equal to the mass multiplied by the acceleration.

- If we divide the first equation by the second we get the following:

- Therefore the acceleration is:

- We know that the equation relating the radial acceleration to the period is:

- From the geometry we see that the radius R is:

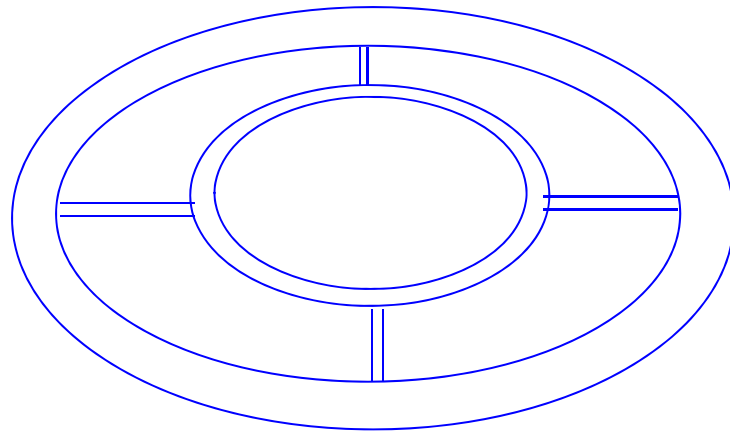
- Therefore, the acceleration is:

- If we now solve for the tension we get the following:

Example

- A space laboratory orbiting Mars is rotating to create artificial gravity.
- Its period of rotation is chosen so that the outer ring, $r_0 = 2150 \text{ m}$, simulates the acceleration due to gravity on the earth.
- What should be the radius of the inner ring such that it simulates gravity on Mars? (3.72 m/s^2)

Picture



Solution

- Since both rings are rigidly connected the period of both rings will be equal.
- Furthermore, the centripetal acceleration must be equal to g for the outer ring.

- Therefore;
- Earth

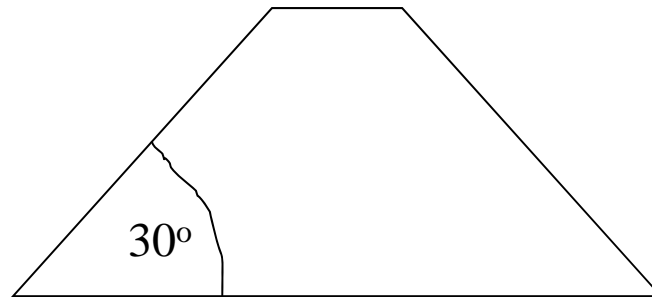
- Mars

- If we divide this equation by the first equation and solve for r_1 we get the following:

Example

- James Bond is lying on a spinning dome atop a missile silo belonging to Goldfinger.
- Static friction holds James in place without slipping.
- The coefficient of static friction between James and the surface of the dome is 0.760.
- The surface of the dome makes an angle of 30° with respect to the horizontal and has a radius of 11.0 m.
- How much time is required for James to go around once?
- Second how does James get off of the dome?

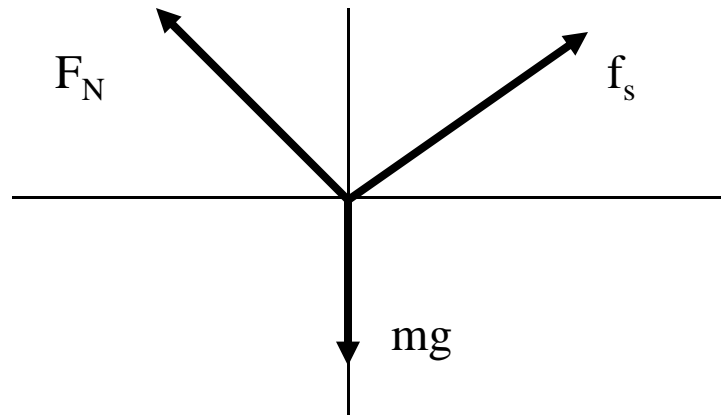
Picture



Solution

- The centripetal force holding James on the dome is:

- We can solve for T once we know the value of the centripetal force.
- A free-body diagram shows the following:



- For the maximum static frictional force we can write:

- If we apply Newton's second law in the y-direction we see that:

- If we solve for the normal force we get:

- The magnitude of the centripetal force is then:

- We now have:

- Solving for T and plugging in our values we get: