Chapter 7

The Conservation of Energy

- Consider an object dropped near the surface of the earth.
- If the distance is small then the gravitational force between the earth and the object will be nearly constant.
- If the object drops from a height y_1 to a height y_2, the work done by its weight will be:

Potential Energy

- The gravitational potential energy is defined as:

- Or

- Here, *h* is the height.

- The initial and final values for the gravitational potential energy of the falling object are:

- The change in the gravitational potential energy is then equal to negative the work done.

Conservation of Energy

- Suppose a body is allowed to fall under the influence of gravity near the surface of the earth.
- If no other forces are present then the total mechanical energy of the system will be conserved.

- As a body falls its kinetic energy increases and its potential energy decreases.
- Therefore, the total energy of the system can be expressed by the following:

Conservation of Mechanical Energy

- Note: The path that a body takes during its descent is independent of the previous equation.
- Therefore, if we wish to apply the conservation of energy to a problem, we need only consider the vertical displacement, through which it falls.

Example

- The highest hill of a particular roller coaster is 27-m.
- If the loaded cars have an initial speed of 5.0 m/s when they crest the largest hill, determine the maximum speed of the roller coaster during the ride.

Solution

- If we assume that air resistance and friction are negligible, then the maximum speed can be obtained from the conservation of energy.

Solution cont.

- Solving for the final speed we get:

- The maximum speed of the roller coaster should be slightly less than our calculated value due to friction and air resistance.

Elastic Potential Energy

- Previously, we determined the work done compressing or stretching a spring.
- Using the work energy theorem we can now write the following:

- The potential energy can now be defined as:

Example

- Wile Coyote, in an attempt to catch the Road Runner, constructs a catapult.
- The catapult has a spring constant of 100,000 N/m and is compressed by 3.5 m.
- A large rock with a mass of 1000 kg is placed on the catapult.
- As usual, things go wrong and Wile is launched with the rock into the air.

Example cont.

- If Wile's mass is 40-kg, determine the maximum speed of Wile as he is launched with the rock.
- Determine the maximum height of the rock and Wile.

Solution

- If we apply the conservation of energy to the problem we get:

- Solving for the speed we get:

Solution part 2

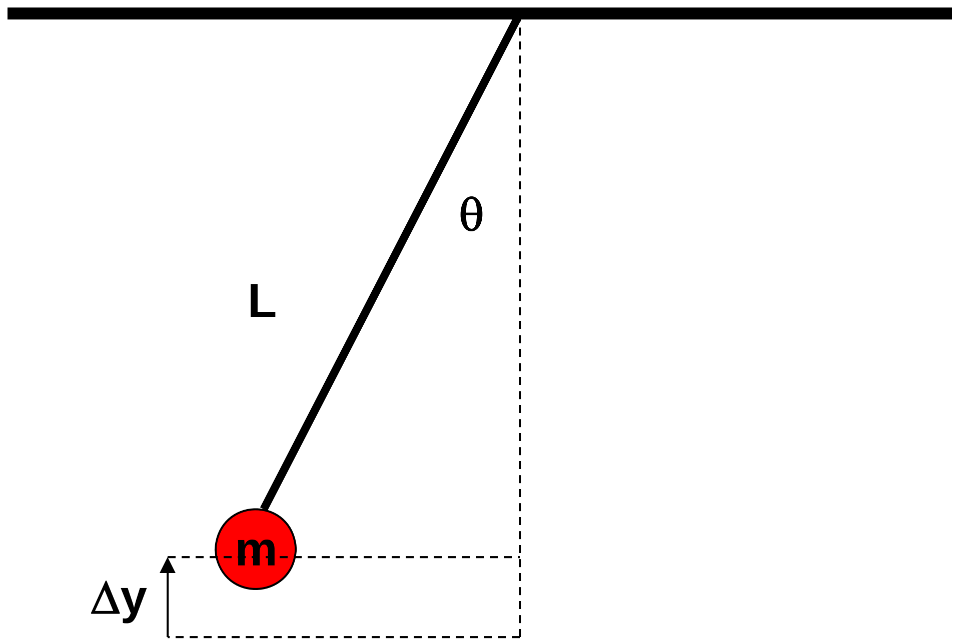
- To determine the maximum height we assume that the rock and Wile will travel straight upward.
- The maximum height occurs when the kinetic energy is zero.
- Therefore,

Solution part 2 cont.

Example

- Consider a pendulum.
- If there is no friction or air resistance present, then the total mechanical energy of the pendulum must remain constant.
- At the most upward portion of the swing the energy is all potential.
- Meanwhile, at the lowest portion of the swing the energy is all kinetic.

Pendulum



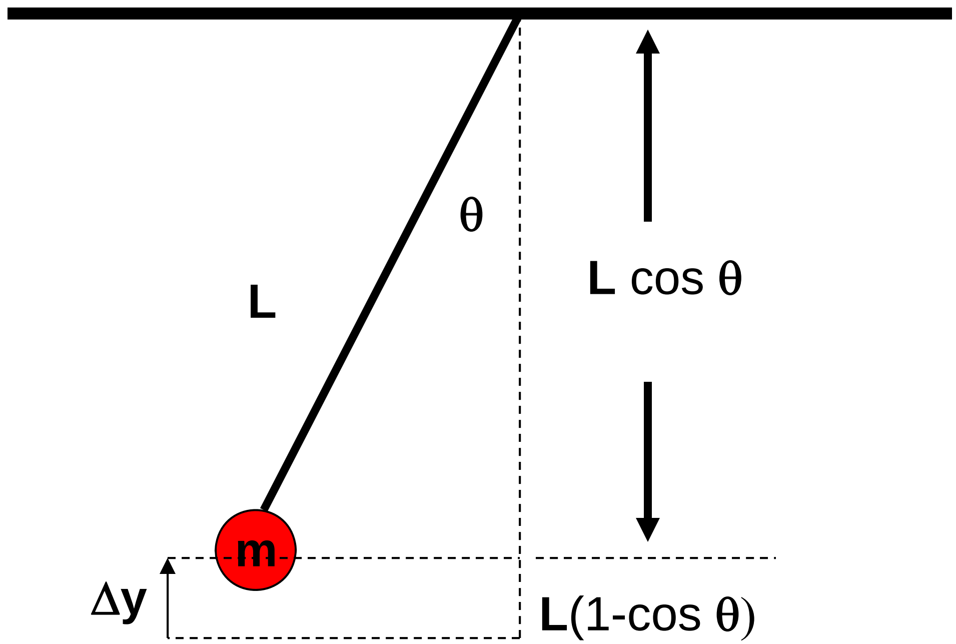
Example Continued

- Determine the relationship for the maximum kinetic energy of the pendulum.

Solution

- We can write the vertical displacement in terms of the length of the pendulum.

Pendulum



- The energy of the system is then given by the following:

- If the pendulum starts from rest then the maximum kinetic energy is:

Non-conservative Forces

- A non-conservative force is one in which the total mechanical energy is not conserved.
- For such cases we must subtract the work done by these non-conservative force from the total energy of the system.
- The work done by non-conservative forces, unlike conservative forces, is path **dependent**.

Example

- Consider the naked roller coaster once again with a total mass of 2000-kg.
- Suppose that the drag force of the air and the frictional force of the track gave a net resistive force of 750-N.
- Furthermore, assume that the length of track from the top of the hill to the bottom is 100-m.
- Determine the speed of the passengers at the bottom of the hill.

Solution

Solution cont.

On Force and Potential Energy

- Consider the equation of work in one dimensional.
- If the force is parallel to the displacement then we can write the following:

- By the work energy theorem this is also equal to negative the change in the potential energy.

- Now consider the work done by a force applied over a tiny displacement.

- Rearranging and letting Δx go to zero we get:

Example

- As a check lets see what result we obtain if we apply the previous equation to the potential of a spring.

Force and Potential Energy in 3-D

- We wish to derive a force from a potential energy.
- However, force is a vector quantity, while potential energy is a scalar.
- Therefore, we need a vector operator that can transform a scalar into a vector.

The Gradient

- Consider the following vector operator in Cartesian coordinates.

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

- This operator differentiates a scalar function and converts the result into a vector.

- If we operate on the negative of our potential with the gradient we get the force.

$$\vec{F} = -\nabla U = \frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}$$