Chapter 8

Momentum, Impulse,

And

Collisions

Momentum and Impulse

- During a collision of two objects there are forces exerted on the objects and the forces are experienced during a finite amount of time.
- If we wish to know something about the forces involved and the amount of time in which they act then we need to define two new quantities called the momentum and impulse.

Momentum

- We first define a vector quantity called momentum.
- The momentum of an object is the product of its mass and its velocity.

Newton’s Second Law

- Isaac Newton originally wrote his second law in terms of momentum.

- We can derive the more familiar form of Newton’s second law if we let the mass be a constant.

- We now see that our original version of Newton’s second law was only valid for constant mass.
- If the mass changes then Newton’s second law becomes:

Impulse

- We can derive the impulse of a particle by using our definition of momentum according to Newton's second law.

- We integrate to obtain the impulse.

- The quantity on the left is called the impulse.
- It is equivalent to the change in the momentum.

Impulse-Momentum theorem

Example

- A golf club exerts an average force of 500 N on a 0.1-kg golf ball.
- The contact time with the ball is 0.01s.
- What is the magnitude of the average impulse delivered by the club?
- What is the change in the velocity of the golf ball?

Solution

- The average impulse is the product of the average force and the time of impact.

Solution cont.

- We can determine the change in velocity by dividing the impulse by the mass of the golf ball.

The Total Momentum of a System

- Suppose we have a system of N particles.
- The total momentum of is then:

Types of Collisions

- **Elastic:** An elastic collision is one in which two or more objects collide and bounce away from one another, while conserving kinetic energy.
- **Inelastic:** An inelastic collision is one in which two or more objects collide and completely stick together. Kinetic energy is <u>not</u> conserved in the process.
- **Partially inelastic:** A partially inelastic collision is one in which two or more objects collide and do not stick together; however, the kinetic energy of the system is <u>not</u> conserved.

Elastic Collisions

- An elastic collision is one in which the kinetic energy of each colliding body can change but the total kinetic energy of the system must remain the same.
- Therefore, in addition to the conservation of momentum, we can apply the conservation of energy when solving problems.

Example:

- Two billiard balls colliding on a frictionless surface.

- The kinetic energy of the system before and after collision is the same.
- Note in an isolated system we can always apply the conservation of momentum.

Example cont.

- Suppose the first ball has a velocity of 5.0 m/s in the positive y-direction while the second ball is motionless.
- Assuming the masses of both balls are the same, determine the resultant velocity of the balls after collision.

Solution

- Since the system conserves both kinetic energy and momentum, we can write the following:

- From the conservation of energy we get:

- The conservation of momentum yields:

- From the conservation of energy we see that the two balls must have velocities that are perpendicular to one another.
- Therefore, the x-components of the momentum of each ball after collision must be equal and opposite.

- The y-components of the momentum must sum to the initial momentum of the system.
- Furthermore, by the Pythagorean theorem, they must be equal
- Since the masses are equal we get that the y-components of the velocities are:

- Plugging in for the velocity we get:

- Solving for the y-component of one of the velocities yields:

- The y-component of the velocity is:

- Of course v_2 is exactly the same.

- According to the conservation of kinetic energy we must have the following:

- Plugging in for the y-component we get:

- Therefore, the velocity after impact of the two balls is:

Inelastic Collisions

- Two ice-skaters collide on the ice.
- The first skater has a mass of 50-kg while the second skater has a mass of 40-kg and is at rest before the collision.
- The coefficient of kinetic friction between the skates and the ice is 0.020.
- If the two skaters slide together along the surface for a distance of 6.50 m before stopping, determine the initial speed of the first skater.

Solution

- Since there are external forces present, the total momentum of the system is not conserved.
- However, we can apply the conservation of momentum immediately after the collision before friction has had a chance to act on the system.

- Immediately after the collision we have:

- For the second part of the problem we exam the skaters from the time of the collision until they come to rest.
- The velocity of the skaters immediately after the collision from the first part of the problem now becomes the initial velocity of the skaters as they are slowed by friction.
- There final velocity will be zero.
- We can use the equations of motion to write the acceleration in terms of the velocity and displacement.

- The acceleration is:

- If we apply Newton’s second law we get:

- Our velocity is then:

- This is our final velocity from the first part of the problem involving the conservation of momentum.

- Plugging in to the conservation of momentum equation we get:

Example

- Consider a partially inelastic collision between the skull of a soccar player (British football) and the ball.
- If the impact last for 0.20s and the change in velocity of the ball is 20 m/s, determine the force imparted to the player if the mass of the ball is 0.43-kg?

Solution

Center of Mass

- Consider a system composed of N particles of various masses.
- At a particular instant in time the coordinates of all the mass could be expressed by the following:

- If we differentiate the center of mass coordinate with respect to time we get the velocity of the center of mass of the system.

The Big Pee

- If we denote the total mass of the system as M and the total momentum as **P**, then we can write the following:

Example

- Arnold and Monica are having a tug of war on a slippery surface to see who can get to a glass of Guinness that has been placed half way between them.
- Arnold's mass is 125-kg while Monica's mass is 50-kg.
- Initially, they are 20.0-m apart.
- When Arnold has moved 6.0-m towards the Guinness how close will Monica be to the Guinness?

Solution

- If the surface they are standing on is virtually frictionless, then the net external force on the system will be zero.
- Therefore, the total momentum of the system will be constant.
- Since the two are initially at rest, the center of mass of the system must remain fixed.

- The center of mass of the system can be determined by the following:

- Since the center of mass of the system does not move we can write the following:

Rocket Propulsion

- Consider a rocket exhausting fuel out the rear.
- According to Newton’s third law, the rocket experiences a forward force.
- However, since the mass of the rocket is changing we cannot simply write Newton’s second law as:

- To deal with this problem we need to use Newton’s original formulation for the second law of motion.

- If the rocket is in space and there are no other forces acting on it then the total momentum of the system must be conserved.
- Hence,

- Therefore, Newton’s second law becomes:

- Rearranging we get the following:

- The $\mathbf{v}_{ex}$ in the previous equation is the velocity of the exhaust from the rocket.
- Note: the time rate of change of the mass is negative because the rocket is losing mass.
- We can solve this first order ordinary differential equation by the separation of variables method.

- The velocity of the rocket as a function of the relative lost mass is then:

- We see that the larger the mass of fuel to mass of rocket ratio, the greater will be the velocity of the rocket.