# Chapter 21

**Electric Fields** 

## The Origin of Electricity

- The electrical nature of matter is inherent in the atoms of all substances.
- An atom consists of a small relatively massive nucleus that contains particles called protons and neutrons.

## The Origin of Electricity cont.

- A proton has a mass of 1.673x10<sup>-27</sup> kg, while a neutron has a slightly greater mass of 1.675x10<sup>-27</sup> kg.
- Surrounding the nucleus is a diffuse cloud of orbiting particles called electrons. An electron has a mass of 9.11x10<sup>-31</sup> kg.

## The Origin of Electricity cont.

- Like mass, electric charge is an intrinsic property of protons and electrons, and only two types of charge have been discovered, positive and negative.
- The proton's charge is exactly equal to the electron's.

## The Fundamental Unit of Charge

• The unit for measuring the magnitude of an electric charge is the coulomb. The charge of the electron is equal to:

# $e = 1.60 \times 10^{-19} C$

## Example

• How many electrons are there in one coulomb of negative charge?

## Solution

• The number of electrons is just equal to the total charge divided by the charge of one electron.

$$N = \frac{q}{e} = \frac{1.00 C}{1.60 \times 10^{-19} C} = 6.25 \times 10^{18}$$

## The Conservation of Charge:

- There are several quantities that are conserved in nature. One such quantity is electric charge.
- The conservation of charge states that during any process, the net electric charge of an isolated system remains constant.

# Electric Conductors and Insulators

- Electrical conductors are substances, such as metals, which allow electrons to move through them very easily. Examples of good conductors are: copper, silver, and gold.
- Materials that do not allow the movement of electrons easily through them are called insulators. Examples of insulators are: rubber, plastic, and glass.

## The Differences

- The difference between electrical conductors and insulators is related to their atomic structure.
- In a good conductor, valence electrons become detached from a parent atom and wander more or less freely throughout the material, belonging to no one particular atom.
- In insulators there are very few electrons that are free to move throughout the material.

## Coulomb's Law

- The electrostatic force that stationary charged objects exert on each other depends on the amount of charge on the objects and the distance between them.
- The physicist Charles Augustin Coulomb formulate the law for amount of force between two charges.

# Coulomb's Law

- Charles Coulomb measured the magnitudes of electric forces between two small charged spheres
- He found the force depended on the charges and the distance between them



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# Coulomb's Law, 2

- The electrical force between two stationary charged particles is given by Coulomb's Law
- The force is inversely proportional to the square of the separation *r* between the particles and directed along the line joining them
- The force is proportional to the product of the charges,  $q_1$  and  $q_2$ , on the two particles

# Coulomb's Law, 3

- The force is attractive if the charges are of opposite sign
- The force is repulsive if the charges are of like sign
- The force is a conservative force

# **Point Charge**

- The term **point charge** refers to a particle of zero size that carries an electric charge
  - The electrical behavior of electrons and protons is well described by modeling them as point charges

#### Coulomb's Law cont.

• This law is known as Coulomb's law and is stated as:

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$$

#### Coulomb's Law cont.

- Here the q's represent two charges, r is the distance separating them, and k is a constant equal to  $8.99 \times 10^9$  N m<sup>2</sup> / C<sup>2</sup>.
- The electrostatic force is directed along the line joining the charges, and it is attractive if the charges have unlike signs and repulsive if the charges have like signs.

# A Final Note about Directions

- The sign of the product of q<sub>1</sub>q<sub>2</sub> gives the *relative* direction of the force between q<sub>1</sub> and q<sub>2</sub>
- The *absolute* direction is determined by the actual location of the charges

# Example

- Two objects, whose charges are +1.0 and 1.0 C, are separated by 1.0 km.
- Compared to 1.0 km the sizes of the objects are insignificant.
- Find the magnitude of the attractive force that either charge exerts on the other.

## Solution

• Using coulomb's law and substituting in for the charges and the distance separating them, we can determine the magnitude of the force between them.

# Example

- In the Bohr model of the hydrogen atom, the electron (-e) is in orbit about the nuclear proton (+e) at a radius of  $r = 5.29 \times 10^{-11}$  m.
- Determine the speed of the electron, assuming the orbit is circular.

## Solution

• The electron experiences an electrostatic force of attraction because of the proton, and the magnitude of this force is:

#### Solution cont.

- This force must be equal to the centripetal force that holds the electron in its orbit.
- Therefore, we can use the equation for centripetal force to determine the speed of the electron.

# Example

- Three point charges are on a line that runs along the x axis in a vacuum.
- The charge on particle A is  $-4 \mu$ C, the charge on particle B is  $3 \mu$ C, and the charge on particle C is  $-7 \mu$ C.



#### Example cont.

• Determine the magnitude and direction of the net electrostatic force on particle B.

## Solution

• The magnitudes of the forces are:

#### Solution cont.

• We can find the magnitude of the force between particles B and C in a similar manner.

#### Solution cont.

- Since  $F_{BA}$  points in the negative x direction, and  $F_{BC}$  points in the positive x direction, the net force, F, is given below.
- The direction of the force is along the positive x direction.

## The Electric Field

- Michael Faraday developed the concept of the electric field.
- According to Faraday, a charge creates an electric field about it in all directions.
- In general, electric field lines are directed away from the positive charge and toward the negative charge.

## Electric Field Defined

The electric field that exists at a point in space is the electrostatic force experienced by a small positive test charge placed at that point divided by the charge itself.



# Units of an Electric Field

- The electric field is a vector quantity, and its direction is the same as the direction of the force on the positive test charge.
- The unit of electric field is Newtons per coulomb (N/C).
- It is the surrounding charges that create an electric field at a given point.
- Any positive or negative charge placed at the point interacts with the field, and as a result experiences a force.

# Example

- There is an isolated point charge of +15 micro-coulombs in a vacuum.
- Using a test charge of +0.80 microcoulomb, determine the electric field at a point which is 0.20 m away in the positive x direction.

## Solution

• Following the definition of the electric field, we place the test charge at the point, determine the magnitude of the force acting on the test charge, and then divide the force by the test charge.

## Solution cont.

- The magnitude of the electric field can now be obtained.
- The electric field points in the same direction as the force that the test charge experienced.

# Example

- Can there be places where the magnitude of the electric field is zero?
- Two positive charges,  $q_1 = 16 \ \mu C$  and  $q_2 = 4.0 \ \mu C$ , are separated in a vacuum by a distance of 3.0 m.
- Find the spot on the line between the charges where the net electric field is zero.

# Reasoning

 Between the charges the two field contributions have opposite directions, and the net electric field is zero at the place where the magnitude of E<sub>1</sub> equals that of E<sub>2</sub>.
### Solution

• At the point P the magnitude of the electric fields created by each charge must be equal.

• If we let the distance to the point of zero electric field be d, from the first charge then the distance of that point to the second charge is 3.0m – d.

- There are two possible values for d. The value of 6.0 m corresponds to a location off to the right of both charges, where the magnitudes are equal but the directions are the same so they do not cancel.
- The value of 2.0 m corresponds to the place where the electric field is zero.

## Example

- Two point-charges, one is 25  $\mu$ C and the other is 50  $\mu$ C, are separated by a distance of 10.0 cm.
- Determine the net electric field at a point that is 2.0 cm from the negative charge.
- If an electron is placed at this point what will be its initial acceleration?

### Solution

- The net electric field is equal to the sum of the two electric fields.
- The direction of the electric field generated by the negative charge at the point shown is to the left.



- The direction of the electric field at this point due to the positive charge is also to the left.
- Therefore, the net electric field is given by the following:

• The acceleration of the electron can be determined from Newton's second law.

## The Electric Field Inside a Conductor

- In conducting materials such as copper or iron, electric charges move readily in response to the forces that electric fields exert.
- This characteristic property of conductors has a major effect on the electric field that can exist within and around them.

### E-Field Inside a Conductor cont.

- Suppose that a piece of copper carries a number of excess electrons somewhere within it.
- Each electron would experience a force on it due to the other electrons and they would then move in response to the force.
- Once static equilibrium is reached all the excess charges would be on the surface of the copper.

### E-Field Inside a Conductor cont.

- 1. At equilibrium, any excess charge resides on the surface of a conductor.
- 2. At equilibrium, the electric field at any point within a conductor is zero.
- 3. The conductor shields any charge within it from electric fields created outside the conductor.

# A Charged Particle in an Electric Field

- Consider a set of two parallel conducting plates with a constant electric field between them.
- The electric field is in the positive y-direction and an electron enters the region with an initial velocity of  $v_0$  in the x-direction.
- Ignore gravity, determine the trajectory of the electron.



### Solution

• Newton's equations of motion yield the following:

• We can use Coulomb's law with Newton's second law to calculate the acceleration in the y-direction.

• If we substitute this result back into our previous relation for y we get the following:



## The Electric Field of a Ring

- Suppose we have a ring-shaped conductor that is centered on the x-axis.
- If the ring has a total charge of Q that is uniformly distributed about its circumference, find the electric field at a point that lies on the x-axis a distance x from the origin.



## Solution

- First we divide the ring up into infinitesimal pieces and then we can consider each piece as a point charge.
- The electric field created by each piece is the following:

- First we notice that the y-components sum to zero.
- The magnitude of the component along the x-axis is then:

• To find the total x-component of the electric field we integrate the previous equation.

- Since the distance *x* does not vary as we integrate around the loop then the only variable in the integral is *dQ*.
- Therefore, the electric field becomes:

- Suppose that we have a disk of radius R with a uniform positive surface charge density of  $\sigma$  on its surface.
- What is the electric field at a distance x from the origin along the x-axis?



- Our differential charge is the charge density multiplied by the area of our ring.
- The area of the ring is the differential width dr times the circumference.

- The ring in this problem is similar to the previous problem.
- Therefore, only the xcomponent of the electric field is present.

• To find the total electric field we integrate over r from zero to R.

• If we substitute  $z = x^2 + r^2$  into our integral it becomes:

• If we simplify we get the following:

## An Infinite Sheet Charge

- Suppose we now let the radius of the disk go to infinity, while the surface charge density decreases.
- The second term in parentheses then goes to zero.
- The electric field then becomes:

## Amount of Charge in or on a Small Volume, Surface, or length

- For the volume:  $dq = \rho \, dV$
- For the surface:  $dq = \sigma dA$
- For the length element:  $dq = \lambda d\ell$

# **Problem Solving Hints**

- Units: when using the Coulomb constant, k<sub>e</sub>, the charges must be in C and the distances in m
- Calculating the electric field of point charges: use the superposition principle, find the fields due to the individual charges at the point of interest and then add them as vectors to find the resultant field

## Problem Solving Hints, cont.

- Continuous charge distributions: the vector sums for evaluating the total electric field at some point must be replaced with vector integrals
  - Divide the charge distribution into infinitesimal pieces, calculate the vector sum by integrating over the entire charge distribution
- **Symmetry:** take advantage of any symmetry to simplify calculations

# Motion of Particles, cont

- $\mathbf{F}_e = q\mathbf{E} = m\mathbf{a}$
- If E is uniform, then a is constant
- If the particle has a positive charge, its acceleration is in the direction of the field
- If the particle has a negative charge, its acceleration is in the direction opposite the electric field
- Since the acceleration is constant, the kinematic equations can be used

# Electron in a Uniform Field, Example

- The electron is projected horizontally into a uniform electric field
- The electron undergoes a downward acceleration
  - It is negative, so the acceleration is opposite  ${\bf E}$
- Its motion is parabolic while between the plates



# The Cathode Ray Tube (CRT)

- A CRT is commonly used to obtain a visual display of electronic information in oscilloscopes, radar systems, televisions, etc.
- The CRT is a vacuum tube in which a beam of electrons is accelerated and deflected under the influence of electric or magnetic fields
# CRT, cont

- The electrons are deflected in various directions by two sets of plates
- The placing of charge on the plates creates the electric field between the plates and allows the beam to be steered



## Electric Dipole

- Consider the situation where two charges of equal but opposite signs are held a fixed distance apart.
- If q is placed at d/2 along the y-axis and -q is at -d/2 from the origin, then determine the electric field at a distance y along the y-axis.



## Electric Dipole

• The electric field is:

• Rearranging with a common denominator yields:

# Electric Dipole

• If the distance between the charges is small compared to the distance along the *y*axis then we get:

# Chapter 22

Gauss's Law

## Gauss's Law

- The electric flux is represented by the number of electric field lines penetrating some surface.
- When the surface being penetrated encloses some net charge, the number of lines that go through the surface is proportional to the net charge within the surface.

• The product of the electric field **E**, and a vectorsurface area **A** is called the electric flux. The units for electric flux are Nm<sup>2</sup>/C. The flux can be expressed as:

- The vector **A** has a magnitude equal to the area and a direction that is perpendicular to that area.
- By the definition of the dot product we can write the flux in terms of the angle between **E** and **A**.

• If we wrap the surface completely around the charges that are responsible for the electric field, then we will create a volume which encloses all our charges.

• If we now look at a small infinitesimal area of this volume the amount of flux that passes through this area is given by the following:

• If we want to know the total flux through the surface enclosing our charges we need to integrate the previous equation:

• The previous equation is known as Gauss's Law and the surface that encloses the charges is known as a Gaussian surface.

- If we know the number and magnitude of the enclosed charges then Gauss's Law becomes:
- The constant  $\epsilon_0$  is the permittivity of free space and has a value of 8.854 x 10<sup>-12</sup> C<sup>2</sup> / N m<sup>2</sup>.

# Example

- Consider a uniform electric field E oriented in the x direction.
- Find the net electric flux though the surface of a cube of edges L oriented with its faces perpendicular to the x, y, and z axis.

# Solution

- The net flux can be evaluated by summing up the fluxes through each face of the cube.
- The faces at the top and bottom of the cube, as well as the two that have a normal vector perpendicular to the x axis, have zero flux through them since:

• The net flux through the remaining two faces is

• After integrating we obtain the following for each of the remaining faces.

• If we now sum up all the contributions to the total flux through the cube we get the following:

# Example

- Consider a thin spherical shell of radius R. A positive charge Q is spread uniformly over the shell.
- Find the magnitude of the electric field at any point a) outside the shell and b) inside the shell.

# Solution

 From our definition of Gauss's law we know that the amount of charge enclosed by our Gaussian surface is proportional to the integral over the Gaussian surface of the electric field.

- We choose a sphere as our Gaussian surface. Thus, the electric field is everywhere perpendicular to the surface.
- If we let r represent the radius of our Gaussian surface then the flux for r > R is:

• Then the electric field outside the sphere is the same as that for a point charge.

• To find the magnitude of the electric field inside the charged shell, we select a spherical Gaussian surface that lies inside the shell. According to Gauss's Law:

• But the charge enclosed by the Gaussian surface is zero; therefore, the electric field is zero inside the charged shell.

# The Parallel Plate Capacitor

- A parallel plate capacitor is a device that consists of two parallel metal plates. In our example each circular plate has an area "A".
- A charge + q is spread uniformly over one plate, while a charge -q is spread uniformly over the other plate.
- In the region between the plates and away from the edges, the electric field points from the positive plate to the negative plate and is perpendicular to both.

- Using Gauss's Law we can determine the electric field between the plates.
- For our Gaussian surface we choose a cylinder with its length perpendicular to the plates of the capacitor.
- One end of the cylinder is in the plate and the other end is positioned between the plates.



- The sides of the cylinder do not contribute since no E-field passes through them.
- The only contribution is due to the end of the cylinder.

- We now let our Gaussian surface enclose the entire surface of the plate.
- Our flux is then just the electric field times the area (A) of the plate.

- The charge enclosed by our Gaussian surface can be defined in terms of a surface charge density  $\sigma$ .
- $\sigma$  is the charge per unit area on the plates of the capacitor. Therefore,  $\sigma$  is:

 Substitution of our previous equation into our Gauss law equation for a parallel plate capacitor yields:

# Equivalence of Gauss's and Coulomb's Laws

- Suppose we have a point charge q and we wish to use Gauss' law to determine the electric field produced by this charge.
- We could construct a Gaussian sphere, with radius r, that encloses q with the charge resting at the center of the sphere.

## Equivalence of Laws cont.

- The electric field on the surface of the sphere is independent of position and can therefore be pulled out of the integral.
- Then according to Gauss' law:

## Equivalence of Laws cont.

• Since the electric field was independent of direction the integral is only the area of a sphere.
#### Equivalence of Laws cont.

- If we solve for the electric field we see that we obtain the same result as if we had used Coulomb.
- Therefore, we state their equivalence.

# The Electric Field of an Infinite Line Charge

 Consider an infinite line charge with a uniform charge distribution of λ along its length.



# The Electric Field of an Infinite Line Charge

- We choose for our Gaussian surface a cylinder of radius r.
- The electric field is perpendicular to the sides of the cylinder and parallel to the ends.



### An Infinite Line Charge

- We now integrate to get the flux.
- We note that the ends of the cylinder do not contribute since the electric field through them is zero.

### An Infinite Line Charge

- Solving the integral gives the flux.
- However, if we wish solve for the electric field in terms of the charge density then we make note of the following:

#### An Infinite Line Charge

• Then applying Gauss's law we get that the electric field a distance *r* from an infinite line charge is: