

Direct Current Circuits

Electromotive Force

- An electromotive force device, or emf device, is a source of constant potential.
- The emf describes the work done per unit charge and has units of volts.



A parallel circuit is one that is connected in such a way that the same voltage is applied across each device.

Resistors in Parallel

Suppose we have two resistors wired together in parallel. *I* Since the voltage across each resistor is the same we can write the following:

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V\left(\frac{1}{R_p}\right)$$

• Here R_p is the equivalent resistance.

Resistors in Parallel cont.

 If we have N resistors wired together in parallel then the reciprocal of the equivalent resistance becomes:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N} = \sum_{i=1}^N \frac{1}{R_i}$$

Example

- Consider the parallel circuit in a kitchen. There are three appliances plugged into the same circuit, each with a different resistance.
- Suppose the first appliance has a resistance of 25 Ω.
- The second has a resistance of 100Ω .
- The third has a resistance of 500Ω .

Example cont.

- Determine the following for the 125 V ac circuit for this house.
- A) the equivalent resistance of the circuit.
- B) the total current of the circuit.
- C) The total average power delivered by the circuit.
- D) The average power dissipated by each appliance.

Solution

• The equivalent resistance is given by the following equation. $\frac{1}{R_p} = \frac{1}{25\Omega} + \frac{1}{100\Omega} + \frac{1}{500\Omega} = \frac{26}{500\Omega}$

$$R_p = \frac{500\,\Omega}{26} = 19.2\,\Omega$$

• The total current can be obtained by using our value for the equivalent resistance.

$$I_{rms} = \frac{V_{rms}}{R_P} = \frac{125V}{19.2\Omega} = 6.5 A$$

The average power in the circuit is obtained by the following:

$$\overline{P} = I_{rms}V_{rms} = (6.5 A)(125V) = 812.5W$$

• The current in each appliance is:

$$I_{rms} = \left(\frac{V_{rms}}{R}\right) = \left(\frac{125V}{25\Omega}\right) = 5A$$
$$I_{rms} = \frac{V_{rms}}{R} = \frac{125V}{100\Omega} = 1.25A$$
$$I_{rms} = \frac{V_{rms}}{R} = \frac{125V}{500\Omega} = 0.25A$$

• We can now determine the average power of each appliance.

 $\overline{P} = (5A)(125V) = 625W$

 $\overline{P} = (1.25 A)(125V) = 156.25W$

 $\overline{P} = (0.25 A)(125 V) = 31.25 W$

And Now for Something Completely Different

Circuits wired together partially in series and partially in parallel.

Consider the following circuit:



Parallel-Series Circuits cont.

- The previous circuit has resistors that are wired together both in series and in parallel with one another.
- In order to reduce this circuit to one equivalent resistance we must look at each part of the circuit separately.

Reducing the Circuit

If we recognize that the most outer resistors are in series we can redraw our circuit as follows.



Reducing the Circuit cont.

• We now see that the circuit has a resistor in parallel with our equivalent resistance.



The Final Reduction

• Finally, we see that our new equivalent resistance is in series with the first resistor.





Treating batteries more realistically.

Internal Resistance cont.

Any emf device has within it some amount of internal resistance. For instance, the wires inside a generator have some small resistance and the chemicals that produce the emf of a battery also provide some resistance as well.

Internal Resistance cont.

- Because of this added resistance the terminal voltage of a battery will be less than the maximum emf produced by the battery.
- To account for this added resistance to the circuit we simply add, in series, the internal resistance of the emf source to the circuit.

Example

• A battery has an emf of 12.0 V and an internal resistance of 0.15 Ω . What is the terminal voltage when the battery is connected to a 1.50 Ω ?

Solution

We can determine the equivalent resistance of the circuit by adding the internal resistance of the battery in series with the resistor of the circuit.

 By Ohm's law the current in the circuit is then:

• We can now find the terminal voltage of the battery.



A method of analyzing complex circuits was developed by Gustav Kirchhoff in the eighteen hundreds.

The Two Rules of Kirchhoff

 1. Junction Rule: The sum of the magnitudes of the currents directed into a junction equals the sum of the magnitudes of the currents directed out of the junction.

 2. Loop Rule: Around any closed circuit loop, the sum of the potential drops equals the sum of the potential rises.

Example

 Determine the magnitude of the current in the circuit and the magnitude of the voltage between the points labeled A and B.



Solution

- The current can be found by using Kirchhoff's loop rule.
- Then the voltage between points A and B can be determined.

 Starting in the upper left corner and going clockwise around the circuit, we set the potential drops equal to the potential rises:



Solving for the current gives:

The voltage between points A and B is then:

Example

- Consider the circuit to the right.
- Determine the current through the resistors.



Solution

• If we apply the junction rule to point A and B we get the following:

 Now we apply the loop rule to loop
BEFA and we get that the potential drops
equal the potential
rises.



 Apply the loop rule to loop CDEB, and once more, the potential drops equal the potential rises.



- We now have three equations and three unknowns.
- These can be solved to yield:

RC Circuits

- Consider a circuit with a resistor, a capacitor, and an emf device all hooked together in series.
- Suppose that the capacitor has no charge initially.
- The emf device is then turned on and the capacitor begins to charge up.

• The equation expressing this circuit is as follows:

If we now solve for the current and substitute into this equation the definition of current in terms of charge we get:

Notational Interlude

- To save space we will now represent the emf with the symbol below.
- Sorry I do not have the same symbol that the book uses.

 Our first order equation is a separable equation; therefore, we can solve it using the following method.

 If we integrate both sides we obtain a solution for q as a function of t.

• If we solve for *q* as a function of *t* we get the following, with the maximum charge being represented by *Q*.

If we want the current as a function of time we just differentiate the previous equation with respect to time.

- If we now wish to discharge the capacitor we simply turn off the emf device.
- The resistor will then dissipate the energy stored in the capacitor.
- The equation for this is:

• If we separate the variables and integrate we get:

 If we now solve for q as a function of time we get the following:

 Once again, to obtain an expression for the current as a function of time we differentiate the previous equation.