Problem 1 Suppose that the force acting on a particle is factorable in to one of the following forms.
(a) F(x<sub>i</sub>,t) = f(x<sub>i</sub>)g(t)
(b)F(x<sub>i</sub>,t) = f(x<sub>i</sub>)g(t)
(c) F(x<sub>i</sub>,x<sub>i</sub>) = f(x<sub>i</sub>)g(x<sub>i</sub>)

For which cases are the equations of motion are integrable?

# 20 points

• **Problem 2:** If a projectile is fired from the origin of the coordinate system with an initial velocity  $v_0$ , and in a direction making an angle  $\alpha$  with the horizontal, calculate the time required for the projectile to cross a line passing through the origin and making an angle  $\beta \leq \alpha$  with the horizontal.

## 20 points

#### • Problem 3

A projectile is fired with a velocity  $v_0$  such that it passes through two points, both a distance above the horizontal. Show that if the gun is adjusted for maximum range, the separation of the points is

$$d = \frac{v_0}{g}\sqrt{v_0^2 - 4gh}$$

#### 20 points

• **Problem 4** Consider a projectile fired vertically in a constant gravitational field. For the same initial velocities, compare the times required for the projectile to reach its maximum height. (a) for zero resisting force

(b) for a resisting force proportional to the instantaneous velocity of the projectile

### 20 points

• **Problem 5** A particle is projected vertically upward in a constant gravitational field with an initial speed  $v_0$ . Show that if there is a retarding force proportional to the square of the instantaneous speed, the speed of the particle when it return to the original position is:

$$\frac{v_0 v_t}{\sqrt{v_0^2 + v_t^2}}$$
(1)

where  $v_t$  is the terminal speed.

20 points