Classical Mechanics - PHYS 310 - Fall 2013 HW # 7
Department of Physics, Southern Illinois University
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Please return it by the 25st of October 2013

- **Problem 1** Show that the shortest path between two points on a plane is a straight line

15 points

- **Problem 2** Consider light passing from one medium with index of refraction n_1 in to another medium with index of refraction n_2 . Use the Fermat's principle (lights takes the quickest path) to minimize the time, and derive the law of refraction $n_1 Sin\theta_1 = n_2 Sin\theta_2$

15 points

- **Problem 3** Set up the differential equation to find the shortest path between the (x,y,z) points (0,-1,0) and (0, 1, 0) on the conical surface $z = 1 - \sqrt{x^2 + y^2}$. Note that this is the Shortest Mountain path around a volcano.

15 points

- **Problem 4** Find the curve $y(x)$ that passes through the points $(0, 0)$ and $(1, 1)$ and minimize the functional

$$I[y] = \int_0^1 \left(\left(\frac{dy}{dx} \right)^2 - y^2 \right) dx. \tag{1}$$

(a) What is the minimum value of the integral?
(b) Evaluate I[y] for a straight line $y = x$ between the two point $(0, 0$ and $(1, 1)$.

- **Problem 5** A simple pendulum of length b and the bob with mass m is attached to a massless support moving horizontally with constant acceleration a Determine
(a) the equations of motion and
(b) the period of small oscillations.

15 points

- **Problem 6** A simple pendulum of length b and bob with mass m is attached to a massless support moving vertically upward with a constant acceleration a. Determine the
(a) Equations of motion and
(b) the period of small oscillations.

15 points

- **Problem 7** A pendulum consists of a mass m suspended by a massless spring with unextended length b and spring constant k. Find the Lagrange's equations of motion.

15 points

- **Problem 8** The point of support of a simple pendulum of mass m and length b is driven horizontally by $x = aSin\omega t$. Find the pendulum's equation of motion.

15 points

- **Problem 9** A particle of mass m rests on a smooth plane. The plane is raised to an inclination angle θ at a constant rate α ($\theta = 0$ at $t = 0$), causing the particle to move down the plane . Determine the motion of the particle.

15 points