Classical Mechanics - PHYS 310 - Fall 2013 HW # 9
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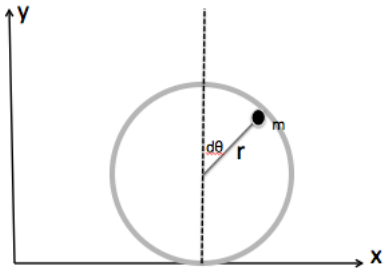
Please return it by the 13th November 2013

- **Problem 1** A point particle of mass m is constrained to move frictionlessly on the inside of a circular wire hoop of radius r, uniform density and mass M. The hoop is in the x-y plane, can roll on a fixed line (the x-axis), but does not slide, nor can it lose contact with the x-axis. The point particle is acted on by gravity exerting force along the negative y-axis. At $t = 0$, suppose the hoop is at rest. At this time, the particle is at the top of the hoop, and is given a velocity v_0 along the x-axis. What is the velocity v_f, with respect to the fixed axis, when the particle comes to the bottom of the hoop? Simplify your answer in the limits $m/M \to 0$, and $M/m \to 0$.



15 points

- **Problem 2** Consider a particle of mass m which is constrained to move on the surface of a sphere of radius R. There are no external forces of any ind on the particle.
(a) What are the generalized coordinates necessary ti describe this problem?
(b) What is the Hamiltonian of the system? Is the Hamiltonian conserved?
(c) Set up the Hamilton's equation of motion.

15 points

- **Problem 3** A particle under the action of gravity slides on the inside of a paraboloid of revolution whose axis is vertical. (Suppose the paraboloid of revolution is generated by parabola which is defined by $z = Ar^2$ in cylindrical coordinates (r, ϕ, z). Using the distance from the axis r, and the azimuthal angle ϕ as generalized coordinates, Find
(a) The Lagrangean of the system
(b) The generalized momenta and the corresponding Hamiltonian
(c) The equation of motion
(d) If $\dot{\phi} = 0$, show that the particle can execute small oscillations about the lowest point of the paraboloid, and find the frequency of these oscillations.

15 points

- **Problem 4** Describe, why the motion of a particle in a central force field can be explained by plane lola coordinates,

10 points

- **Problem 5** Find the force law of a central force field that allows a particle to move in a spiral orbit given by $r = k\theta^2$, where k is a constant.

10 points