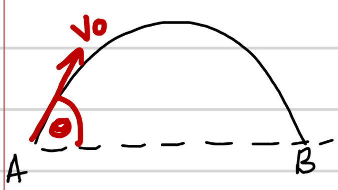
# 310 Homework #1

Question 1:

A long jumper leaves the ground at an angle of $20^\circ$ above the horizontal at a speed of $11.0\ ms^{-1}$.

(a) How far does the jump in the horizontal direction?

(b) What is the maximum height reached.




Initial conditions: $t=0$, $x=0$, $u = V_0 \cos\theta$ $a=-g$

We can use the Kinematic Equations (Because the through out the motion the acceleration is constant)

The motion in the $\hat{x}$ direction:

$$x = u_0 \cos\theta\, t + \frac{1}{2}(0)$$

$$x = u_0 \cos\theta\, t \quad \text{——} (1)$$

In the $\hat{y}$ direction: $y = u_0 \sin\theta\, t - \frac{1}{2} g t^2$ —— (2)

To find the AB distance let's consider the equation (2) at point (B) $y=0$;

$$0 = u_0 \sin\theta\, t - \frac{1}{2} g t^2$$

$$\frac{1}{2} g t = u_0 \sin\theta$$

$$t_B = \frac{2 u_0 \sin\theta}{g}$$

Now let's use $t = t_B$ in equation (1)

$$x_{AB} = u_0 \cos\theta \frac{2 u_0 \sin\theta}{g}$$

$$x_{AB} = u_0^2 \frac{2 \sin\theta \cos\theta}{g}$$

Now for the given problem $\theta = 20^\circ$; $u_0 = 11\ ms^{-1}$

$g = 9.8$

$$x_{AB} = 7.9\ m$$

Maximum Height

Let's use $v = u + at$ in $\hat{y}$ direction, and use $v=0$ at the maximum height.

$$V = V_0 \sin\theta - g t$$

$$t = \frac{V_0 \sin\theta}{g}$$

$$y = V_0 \sin\theta\, t - \frac{1}{2} g t^2$$

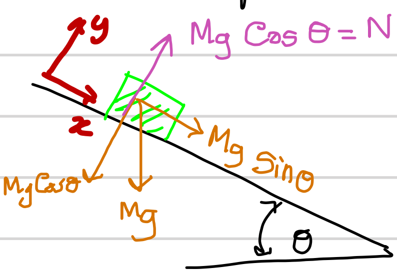
$$= \frac{V_0^2 \sin^2\theta}{g} - \frac{1}{2} g \frac{V_0^2 \sin^2\theta}{g^2}$$

$$y = \frac{v_0^2 \sin^2\theta}{2g}$$

$$y = \frac{(11)^2 (\sin 20)^2}{2g} = 0.72\ m$$

Q-2 In class we have discussed the problem of a box sliding down on an inclined plane by integrating Newton's second law. Can you solve this problem using Kinematics equation. If yes, use Kinematics equations to solve the example problem for both with friction and without friction.
In particular, solve the following 2 problems using Kinematics equations.

(a) A block sliding down a frictionless plane, that is inclined by an angle $\theta$. What is the accelleration of the block? What is the velocity of the block after it moves from rest a distance $x_0$ down the plane.




The only forces acting on the block are the gravitational force and the Normal force.
By considering that no accelleration in $\hat{y}$ direction $N = Mg\cos\theta$

The force in $\hat{x}$ direction $= F_x = Mg\sin\theta$

$$Ma = Mg\sin\theta$$

$$a = g\sin\theta$$

We want to relate the velocity and the distane. By using the Kinematics equation:

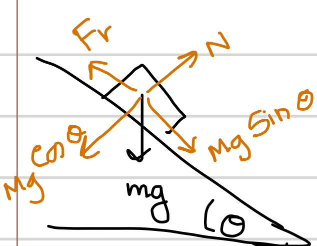
$$V^2 = u^2 + 2aS$$

$$V^2 = 2g\sin\theta\, x_0$$

$$V = \sqrt{2gx_0\sin\theta}$$

(b) If the coefficient of static friction between the block and the plane in the previous example is $\mu_s = 0.4$, at what angle will the block start sliding, if it initially is at rest.

Let's now look at the forces acting on the block.




$\hat{y}$ directon: $F = ma$

$$N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

in $\hat{x}$ direction

$$Mg\sin\theta - F_r = ma$$

$$Mg\sin\theta - \mu N = ma$$

$$Mg\sin\theta - \mu Mg\cos\theta = ma$$

Just before it slides

$$Mg\sin\theta - \mu Mg\cos\theta = 0$$

$$\sin\theta = \mu\cos\theta$$

$$\tan\theta = \mu$$

$$\theta = \tan^{-1}[0.4]$$

$$\theta = 21.8^\circ$$

(c) If the coefficient of Kinetic friction is 0.3, find the acceleration of the block when the inclined angle of the block is 30 degrees.

$\hat{y}$ direction $F = ma \Rightarrow N - Mg\cos\theta = 0 \quad N = Mg\cos\theta$

$\hat{x}$ direction: $F = ma$

$$Mg\sin\theta_0 - \mu_k Mg\cos\theta_0 = Ma$$

$$a = g\sin\theta_0 - \mu_k g\cos\theta_0$$

Q-4 An object is at rest on a smooth horizontal surface at time $t=0$. A time dependent force $F = \frac{F_0 t}{k}$ is applied in the $\hat{x}$ direction.

(a) What is the accelleration of the object?
(b) What is the velocity of the object at time $t = t_0$?
(c) How long the object has travelled within this time $t_0$?

(a) $F = ma$

$$\frac{F_0 t}{k} = ma \qquad a = \frac{F_0 t}{mk} \;//$$

(d) $F = ma$

$$\frac{F_0 t}{k} = m\frac{dV}{dt}$$

$$\frac{F_0}{mk} t\,dt = dV \qquad \frac{F_0}{mk}\frac{t^2}{2} = V + C$$

at $t=0$ $V=0$ $\Rightarrow C=0$

$$V = \frac{F_0}{2mk} t^2 \quad \text{——} \; (1)$$

So at time $t_0$ $V = \frac{F_0}{2mk} t_0^2 \;//$

(c) By integrating the equation one more time:

$$V = \frac{F_0}{2mk} t^2$$

$$\frac{dx}{dt} = \frac{F_0}{2mk} t^2$$

$$dx = \frac{F_0}{2mk} t^2\,dt$$

$$x = \frac{F_0}{2mk}\frac{t^3}{3} + C_2$$

$$x = \frac{F_0 t^3}{6mk}$$

at $t = t_0$, the distance travelled $= x_0 = \frac{F_0 t_0^3}{6mk}$