PHYS 310 – HW #3 Solutions

Q-1 A child slides a block of mass 2kg along a slick kitchen floor. If the initial speed 4 m/s and the block hits a spring with spring constant $k = 6$ N/m, what is the maximum compression of the spring? What is the result if the Block slides 2m of a rough floor that has $\mu_k = 0.2$?

The initial energy of the Block $T = \frac{1}{2} m v_0^2$

For the maximum compression, all the kinetic energy converts in to the potential energy. (The block comes to a complete stop)

$$\frac{1}{2} m v^2 = \frac{1}{2} k x^2$$

$$x^2 = \frac{m}{k} v^2 = \frac{2\ \text{kg}\ (4)^2 m^2 s^{-2}}{6\ N m^{-1}} = \frac{2 \times 4}{6} \frac{\text{kg}\, m^2 s^{-2}}{\text{kg}\, m s^{-2}\ m^{-1}}$$

$$x^2 = \frac{2 \times 4^2}{6} m^2$$

$$x = \sqrt{\frac{32}{6}}\ m. = 2.30\ m$$

Now if there is a frictional force, part of the energy converts to work against the frictional force.

$$F_f = \mu_k N = \mu_k m g$$

If the Block slides a distance h:

$$\frac{1}{2} m v_0^2 + \mu_k m g h = \frac{1}{2} k x^2 + \mu_k m g x$$

Assuming that the block comes to a complete stop.

$$v_0^2 - 2 \mu_k g h = \frac{k}{m} x^2 + 2 \mu_k g x$$

We are looking for x. This gives an quadratic equation of x.

$$x^2 + \frac{2 \mu_k m g}{k} x + \left(\frac{-m v_0^2}{k} + \frac{2 \mu_k m g h}{k} \right) = 0$$

$$x = \frac{-2 \mu_k m g}{k} \pm \sqrt{\left(\frac{2 \mu_k m g}{k} \right)^2 + 4 \left(\frac{m v_0^2}{k} - \frac{2 \mu_k m g h}{k} \right)}$$

Q-2: A block of mass 1.62 kg slides down a frictionless incline. The block is released from a height $h = 3.91$ m above the bottom of the loop.

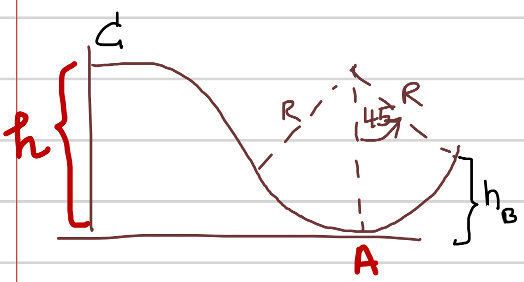
(a) What is the force of the inclined block at the bottom (point A)?

(b) What is the force of the track on the block at point B?

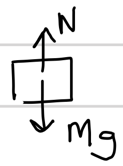
(c) At what speed does the block leave the track?

(d) How far away from point A does the block land on level ground?

(e) Sketch the potential energy $U(x)$ of the block. Indicate the total energy on the sketch.



We did this problem in class. At point A the object is moving with a velocity V in a curved path. The object moves with a constant speed on a curved path means the acceleration on the object is v^2/R



The acceleration is in the radial direction. The total force in the radial direction equals $F = N - mg$

So, by using Newton's Laws

$$F = ma$$

$$N - mg = \frac{mv^2}{R} \quad \text{—— (1)}$$

If we know the V at point A, we know the force.

The energy of the object at point C equals to the energy of the object at point A

$$E_A = E_C$$

$$KE_A + PE_A = KE_C + PE_C$$

$$0 + mgh = \frac{1}{2}mv^2 + 0$$

$$v^2 = 2gh \quad \text{—— (2)}$$

Now by putting this in eq. (1),

$$F = \frac{mv^2}{R} = \frac{m\,2gh}{R}$$

The force on the block from the surface $N = mg + \frac{2mgh}{R} = mg\left(1 + \frac{2h}{R}\right)$

(b) At point B

$$N - mg\cos 45 = \frac{mv_B^2}{R}$$

Notice now I use V_B

$$N = mg\cos 45 + \frac{mV_B^2}{R}$$

Now we can compare the energies at points C & B.

$$E_C = E_B$$

$$KE_C + PE_C = KE_B + PE_B$$

$$0 + mgh = \frac{1}{2}mv_B^2 + mgh_B$$

h_B is marked in the figure in the previous page.

$$h_B = R - R\cos 45 = R - \frac{R}{\sqrt{2}}$$

$$V_B^2 = \left[mgh - mgR\left(1-\frac{1}{\sqrt{2}}\right)\right]\frac{2}{m}$$

$$= 2g\left[h - R\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

$$N = mg\cos 45 + \frac{mV_B^2}{R}$$

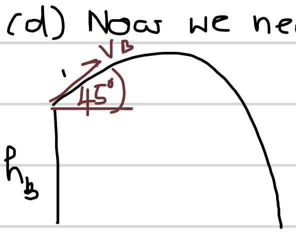
$$= \frac{mg}{\sqrt{2}} + \frac{m}{R}2g\left[h - R\left(1-\frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{2mgh}{R} - 2mg + \frac{2mg}{\sqrt{2}} + \frac{mg}{\sqrt{2}}$$

$$= \frac{2mgh}{R} + \frac{3}{\sqrt{2}}mg - 2mg$$ ⟵ This is the Force on the Block at point B.

(c) The block leaves the track at a speed $v = \left(2g\left[h - R\left(1-\frac{1}{\sqrt{2}}\right)\right]\right)^{1/2}$

(d) Now we need to solve the projectile motion.



Let's use the Kinematics equations

$$y = y_0 + V_0t + \frac{1}{2}at^2$$

When the block reaches the ground $y = 0$

$$0 = h_B + V_0\sin 45\, t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 - V_0\sin 45\, t - h_B = 0$$

$$t^2 - \frac{2V_0}{g}\sin 45\, t - \frac{2h_B}{g} = 0$$

$$t^2 - \frac{2V_0 \sin 45}{g} t - \frac{2h_B}{g} = 0$$

$$t^2 - \frac{2V_0}{\sqrt{2}g} t - \frac{2h_B}{g} = 0$$

$$t^2 - \frac{\sqrt{2}V_0}{g} t - \frac{2h_B}{g} = 0$$

$$t = \left[\frac{\sqrt{2}V_0}{g} \pm \sqrt{\frac{2V_0^2}{g^2} + 4\left(\frac{2h_B}{g}\right)}\right] \Big/ 2$$

$$= \frac{1}{2g}\left[\sqrt{2}V_0 \pm \sqrt{2V_0^2 + 8gh_B}\right]$$

The number in the square is larger than $\sqrt{2}\,V_0$. So out of the two possible solutions for t in the above equation, the negative sign result in a negative time. So we only consider the positive.

$$t = \frac{1}{2g}\left[\sqrt{2}V_0 + \sqrt{2V_0^2 + 8gh_B}\right]$$

Now we will consider the kinematic equations in the $\hat{x}$ direction.

$$x = x_0 + V_0 \cos 45 t + \frac{1}{2}at^2$$

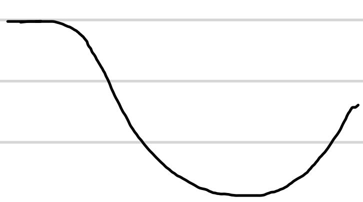
$$x = x_0 + V_0 \cos 45\, t$$

$$= \frac{R}{\sqrt{2}} + \frac{V_0}{\sqrt{2}}\left[\frac{1}{2g}\left\{\sqrt{2}V_0 + \sqrt{2V_0^2 + 8gh_B}\right\}\right]$$

By plugging the h_B, we can simplify it further.

(e) Sketch the potential energy.

it goes as mgh



$\boxed{Q\text{-}3}$ A particle of mass $m = 1\,kg$ is subjected to a one dimensional force $F(t) = kte^{-\alpha t}$, where $k = 1\,N/s$, and $\alpha = 0.5\,s^{-1}$. If the particle is initially at rest. Calculate and plot with the aid of a computer program, the position, speed, and acceleration of the particle as a function of time.

$$F(t) = kte^{-\alpha t}$$
$$m\ddot{x} = kte^{-\alpha t}$$
$$\ddot{x} = \frac{k}{m} te^{-\alpha t}$$

← This is the acceleration of the particle. We know k, m, and α. We can plot it right away.

Let's integrate this once

$$\ddot{x} = \frac{k}{m} te^{-\alpha t}$$

$$\dot{x} = \frac{k}{m}\int te^{-\alpha t}\,dt$$

$$= \frac{k}{m}\int t\left(\frac{-1}{\alpha}\right)\frac{d}{dt}e^{-\alpha t}\,dt$$

$$\int u\,dv = uv - \int v\,du$$

$$= -\frac{k}{m\alpha}\int t\,de^{-\alpha t}$$

$$= \frac{-k}{m\alpha}\left[te^{-\alpha t} - e^{-\alpha t}\,dt\right] + C$$

$$\dot{x} = \frac{-k}{m\alpha}\left[te^{-\alpha t} + \frac{e^{-\alpha t}}{\alpha}\right] + C$$

at $t=0$ $\dot{x}=0$

$$0 = \frac{-k}{m\alpha}\left[0 + \frac{1}{\alpha}\right] + C \qquad C = k/m\alpha^2$$

$$\dot{x} = \frac{-k}{m\alpha}\left[te^{-\alpha t} + \frac{e^{-\alpha t}}{\alpha}\right] + \frac{k}{m\alpha^2}$$

Now, we integrate this equation one more time to get the position.

$$x(t) = \int \frac{k}{m\alpha^2}\,dt - \int \frac{k}{m\alpha^2} e^{-\alpha t}\,dt - \frac{k}{m\alpha}\left[\int te^{-\alpha t}\,dt\right]$$

$$= \frac{k}{m\alpha^2}t + \frac{k}{m\alpha^3}e^{-\alpha t} - \frac{k}{m\alpha}\left(\frac{-1}{\alpha}\right)\left[te^{-\alpha t} + \frac{e^{-\alpha t}}{\alpha}\right] + C$$

$$x(t) = \frac{k}{m\alpha^2}t + \frac{k}{m\alpha^3}e^{-\alpha t} + \frac{k}{m\alpha^2}te^{-\alpha t} + \frac{k}{m\alpha^3}e^{-\alpha t} + C$$

$$x(t) = \frac{k}{m\alpha^2} t + \frac{k}{m\alpha^3} e^{-\alpha t} + \frac{k}{m\alpha^2} t e^{-\alpha t} + \frac{k}{m\alpha^3} e^{-\alpha t} + C$$

Now let's use the initial conditions $x=0$ at $t=0$

$$0 = 0 + \frac{k}{m\alpha^3} + 0 + \frac{k}{m\alpha^3} + C$$

$$C = -\frac{2k}{m\alpha^3}$$

$$x(t) = -\frac{2k}{m\alpha^3} + \frac{k}{m\alpha^2} t + \frac{k}{m\alpha^3} t + \frac{k}{m\alpha^2} t e^{-\alpha t} + \frac{k}{m\alpha^3} e^{-\alpha t}$$

Plots attached

$\boxed{Q-4}$ A particle is released from rest (at $y=0$) and falls under the influence of gravity, and air resistance. Find the relationship between v and the distance of falling y, when the air resistance is equal to (a) αv (b) βv^2

(a) $F = -\alpha V + mg$

$$m\dot{y} = mg - \alpha V$$

$$m\ddot{y} = mg - \alpha V$$

$$\frac{d\dot{y}}{dt} = g - \frac{\alpha}{m} V$$

$$\frac{dV}{dy}\frac{dy}{dt} = -g - \alpha/m V$$

$$V\frac{dV}{dy} = -g - \alpha/m V$$

$$\frac{V\,dV}{g + \frac{\alpha}{m}V} = -dy$$

$$\frac{m}{\alpha} \ln\left[\frac{\alpha}{m}V + g\right] = -y + C$$

at $y=0$ $V=0$ $C = \frac{m}{\alpha}\ln g$

$$\frac{m}{\alpha}\ln\left[\frac{\alpha/m + g}{g}\right] = -y \Rightarrow \ln\left[\frac{\alpha/m V + g}{g}\right] = \frac{-\alpha y}{m}$$

Note to Saeed
Can you double check.
I think I made a mistake
please do this problem

$$\ln\left[\frac{\frac{\alpha}{m}V + g}{g}\right] = -\frac{\alpha y}{m}$$

$$\frac{\frac{\alpha}{m}y + g}{g} = e^{-\alpha y/m}$$

$$\frac{\alpha}{m}V = -g + g e^{-\alpha y/m}$$

(b) $F_f = -\beta v^2$

$$F = ma$$

$$m\ddot{y} = mg - \beta v^2$$

$$m\frac{dv}{dy} = mg - \beta v^2$$

$$\frac{v\,dv}{g - \beta/m\, v^2} = dy$$