Classical Mechanics - PHYS 310 - Fall 2013 Lec 12: Supplementary Notes

Department of Physics, Southern Illinois University Instructor: Thushari Jayasekera (thushari@siu.edu)

POISSON'S EQUATION FOR GRAVITATION

In the last two lectures, we basically discussed three ways of calculating the gravitational force distribution. Depending on the type of the problem, we can decide which method is easier.

- Directly calculating the force due to a mass element $dm = \rho(r')dv'$. In this case force due to a mass element is :

$$dF = -G\frac{\rho(r')dv'}{r^2}\hat{r} \tag{1}$$

  The total force is then calculated by integrating over the total distribution of mass:

$$F = -G\int_V \frac{\rho(r')dv'}{r^2}\hat{r} \tag{2}$$

- Calculating the Gravitational Potential and then using $\hat{g} = \vec{\nabla}\phi$, we can calculate the gravitational field vector. In this case, gravitational potential due to a mass element is calculated as:

$$d\phi = -G\frac{\rho(r')dv'}{r} \tag{3}$$

  then in order to get the total gravitational potential, we do an integration:

$$\phi = -G\int_V \frac{\rho(r')dv'}{r} \tag{4}$$

  Notice that, compared to the summation over vector quantity in eq.(**??**), in the second method, we need to sum over a scalar quantity as shown in the eq.(**??**).

- The third method is directly evaluating the gravitational field vector using the Poisson's Equation. This method is more suitable for systems with a high symmetry.

$$\int_S \hat{n}.\hat{g}da = -4\pi GM \tag{5}$$

  If you calculate the gravitational potential using the direct method, we then calculate the gravitational field vector by $g = -\vec{\nabla}\phi$.

  On the other hand, if we calculate the $\hat{g}$ directly using the method II or III above, in the last class, we discussed how to evaluate the potential by considering the fact that the change in potential energy between two points is equal to the work done to move a point mass between these two places.

EXAMPLES

Example I:

A Planet of density ρ_1 (spherical core, readies R_1) with a thick spherical cloud of dust (density ρ_2, radius R_2) is discovered. What is the force on a particle os mass m placed within the dust cloud?

Example II:

Calculate the gravitational potential due to a thin rod of length l and and mass M at a distance R from the center of the rod and in a direction perpendicular to the rod.

Example III:

A particle is dropped in to a hole drilled straight through the center of earth. Neglecting rotational effects, show that the particleÕs motion is simple harmonic if you assume earth has uniform density. Show that the period of oscillation is 84 min.

Example IV:

Assuming that the air resistance is not important, calculate the minimum velocity a particle must have at the surface of Earth to escape from EarthÕs gravitational field. Obtain a numerical value for this result. Do you know what is this velocity called?