IDENTIFYING AND USING AUXILIARY CONDITIONS

In the last lecture, we discussed how rouse Auxiliary conditions in Euler's Equations with Lagrange's Undetermined Multipliers.

Let's do some examples:

EXAMPLE

Repeat Example 6.4 finding the shortest path between any two points on the surface of a sphere, but use the method of the Euler Equations with an Auxiliary condition imposed.

Example 6.4 is: A geodesic is a line that represents the shortest path between any two points when the path is restricted to a particular surface. Explain the procedure to find the geodesic on the sphere with the Lagrangean undetermined multiplied method.

EXAMPLE

Consider a disk rolling without slipping on an inclined plane . Determine he equation of constraint in terms of the coordinates y and θ.

Example:

A disk of readies R rolls without slipping inside the parabola $y = ax^2$. Find the equation of constraint.

EXAMPLE:

Consider alight passing from one medium with index of refraction n_1 in to another medium with index of refraction n_2 . Use the Fermat's Principle to minimize the time and derive the law of refraction $n_1 Sin\theta_1 = n_2 Sin\theta_2$

EXAMPLE

Set up the differential equation to find the shortest path between the (x, y, z) points $(0, -1, 0)$ and $(0, 1, 0)$ on the conical surface $z = 1 - \sqrt{x^2 + y^2}$. Note that this is the Shortest Mountain path around a volcano.