Ashcroft Ch 1

Drude model

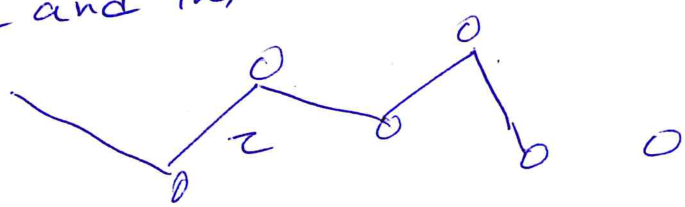
Predates quantum mechanics, atomic model (nucleus was unknown)

Modeled after kinetic theory of gases to a gas of electrons (electron was discovered in 1897 by JJ Thomson, deflection of cathode ~~rays~~ by charged plates)

Drude assumed compensating +ve charge ~~attached to~~ that are immobile heavier particles ⇒ +ve charges do not participate in transport.

1) Like gases, electrons encounter collisions from the ion core (No e-e collisions are assumed)

2) Electrons travel for a certain time τ between collision. Collisions are elastic and instantaneous.



No electric field is implied for electrons to travel, just thermal energy like gases.

3) Probability of collision per unit time is $1/\tau = P$

i.e. electrons are bound to collide after time τ

Probability to collide after time dt ← $\boxed{P\,dt = \left(\frac{1}{\tau}\right) dt}$

$\boxed{P\tau = 1}$

τ → relaxation time, collision time

4) Electrons achieve thermal equilibrium through collisions.

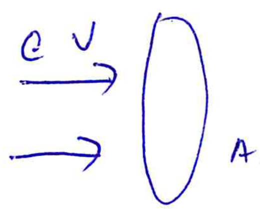
DC conductivity

$V = IR \qquad R = \rho \frac{L}{A}$. $\qquad \rho \rightarrow$ resistivity

$\rightarrow \sigma \rightarrow$ conductivity.

$\vec{E} = \rho \vec{J}$ or $\vec{J} = \sigma \vec{E}$

In general $\vec{E}$ & $\vec{J}$ are not parallel. (resistivity ~~tensor~~ tensor)



Amount of charge crossing A in time $dt = -ne\,v\,dt\,A$

$n \rightarrow$ electrons/vol

~~q = charge/Area~~

current $= \frac{dQ}{dt} = -neVA$

current density $= -ne\bar{V} = \vec{J}$

Due to $\vec{E}$ each electron acquires a velocity

$\vec{V}_{ag} = -\frac{eE\tau}{m} + \langle \vec{V}_0 \rangle$ | $F = qE = -eE = m\frac{dv}{dt}$

$\Rightarrow v = -\frac{eEt}{m}$

$\langle v_0 \rangle$ is the ave velocity just after collision which is zero by assumption since there is no electric field and electrons move in random directions after collision

$j = -nev = \left(\frac{ne^2\tau}{m}\right)\vec{E} = \sigma E$

$\Rightarrow \boxed{\sigma = \frac{ne^2\tau}{m}} \qquad \tau = \frac{m}{\rho n e^2} \qquad \tau = 10^{-14}/10^{-15}$ sec

Mean free path $= |v_0|\tau = l$

$\frac{1}{2}mv_0^2 = \frac{3}{2}kT \Rightarrow V_0 = 10^7$ cm/s $\Rightarrow l = 1\text{-}10\,\text{Å}$

This estimate is mean free path is an order of magnitude low (quantum effects)

At low temp, $\tau \sim 10^3$ Å $\Rightarrow$ electrons travel a very long distance before encountering collisions (Bloch waves)

But the qualitative model is correct. It can be applied to a magnetic field and AC Electric field.

Eqn of motion :- $\vec{J} = -ne\vec{v}$

$$\Rightarrow \vec{J}(t) = -ne\vec{v}(t) = -ne\frac{\vec{p}(t)}{m}$$

Momentum of electron after time ~~t~~ $t+dt$ is $p(t+dt)$

Probability of ~~col~~ collision $= \frac{dt}{\tau}$

Probability of no collision $= 1 - \frac{dt}{\tau}$

External force $\rightarrow \vec{f}(t)$

Addition momentum due to external force $= f(t)dt$

Average momentum/electron taken at random

~~total moment~~ $= \vec{p}(t) + \vec{f}(t)\,dt$

$p(t) \rightarrow$ instantaneous momentum of electron.

$$p(t+dt) = \left(1 - \frac{dt}{\tau}\right)\left[p(t) + f(t)\,dt\right]$$

$$\Rightarrow p(t+dt) - p(t) = -\frac{dt}{\tau}p(t) + f(t)dt$$

$$\Rightarrow \boxed{\frac{dp(t)}{dt} = -\underbrace{\frac{p(t)}{\tau}}_{\text{damping term.}} + f(t)}$$

$$\boxed{\begin{gathered}\text{If } f(t) = 0 \\ p(t) = Ae^{-t/\tau}\end{gathered}}$$