Magnetoresistance in real metals

Let us now look at some characteristic examples of magnetoresistance in real metals to see how far they depart from the free-electron model for which no change is predicted. In due course we shall be concerned to understand them in detail but for the moment they serve to illustrate the variety of forms which makes the subject interesting. The curves have been redrawn from published data, experimental points being omitted since the shapes are not in dispute. When it is reasonable to do so values of B are quoted directly and also converted by use of (3) into $\omega_c\tau$ (some authors disconcertingly present values of $\omega_c\tau$ for metals which do not approximate to the free-electron model, without making clear how they are calculated). We shall see that different metals may present widely different changes of resistance at the same $\omega_c\tau$.

It is conventional to express the degree of purity of the sample by its residual resistance ratio (RRR in many papers, but r_0 here), being the ratio of its resistance at 0 °C to its residual resistance, for which the resistance at 4 K is usually an adequate measure; and a large fraction of the measurements to be discussed were taken at this temperature in a bath of liquid helium boiling at atmospheric pressure. The magnitude of the magnetoresistance is usually expressed by $\Delta R(B)/R_0$, R_0 being the sample resistance in zero magnetic field, $R(B)$ the resistance in field **B**, and $\Delta R(B) = R(B) - R_0$. In all the examples presented here the field was transverse to the current.

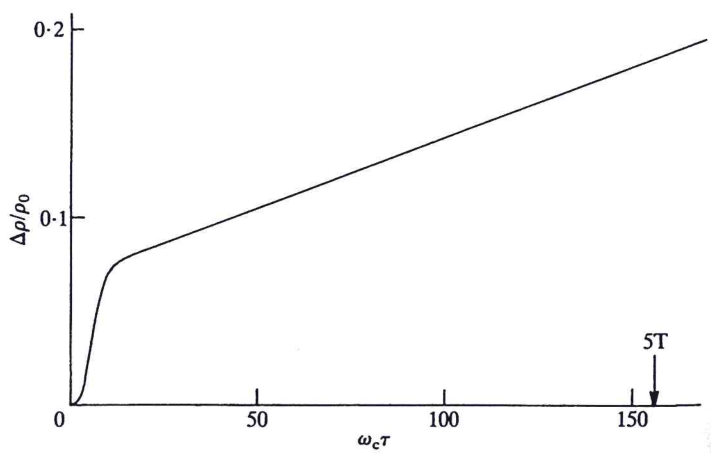

Figure 1.3 Transverse magnetoresistance at 4 K of potassium (Simpson[10]).

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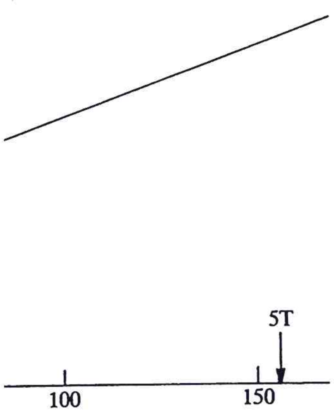

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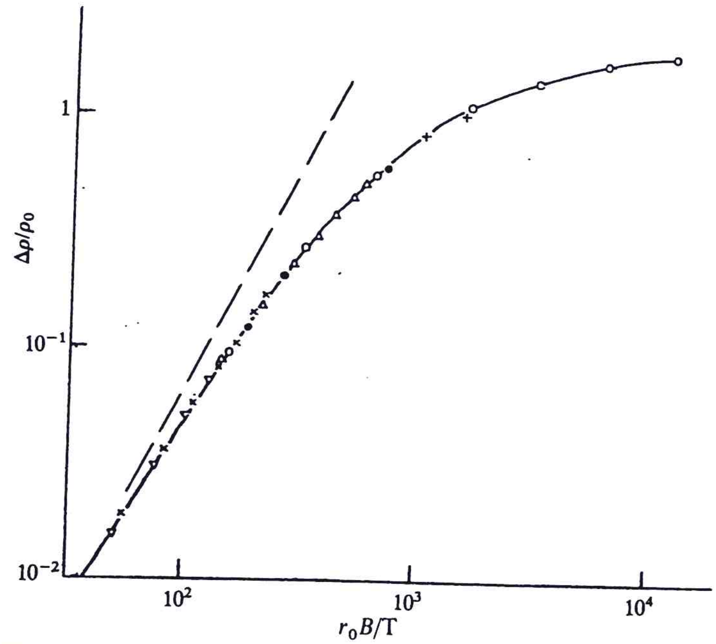


Figure 1.4 Kohler plot for transverse magnetoresistance of polycrystalline indium (Olsen[11]). The broken line shows the slope for quadratic dependence.

The metal that most closely resembles the free-electron model is potassium, and fig. 3 shows how small the magnetoresistance may be in a good sample. The linear rise, extending to the highest values of $\omega_c\tau$ attained, is not explained by any simple theory, but this is only one of the mysteries
presented by potassium, as will be discussed in chapter 5. All the same, the 186
very fact that one can achieve a value of $\Delta R/R_0$ that is 1000 times less than $\omega_c\tau$ should be seen as confirmation of the paradoxical vanishing of magnetoresistance in the ideal metal.

The linear variation never extends back to zero field; since field reversal normally leaves the resistance unchanged one must expect $\Delta R/R_0$ to vary as B^2 in weak fields at least, and this is confirmed fairly well, as fig. 4 shows. The quadratic range may be very short, however, as in curve (*a*) of fig. 5, where it is hardly discernible, or may continue up to high values of $\omega_c\tau$, as in curve (*b*). Both curves relate to single crystals of tin, with **B** in different directions, and it should be noted that while (*a*) saturates with $\Delta R/R_0 \sim 6$, (*b*) is still rising sharply at 750. The fact that $\Delta R/R_0$ in curve (*b*) is not far from $(\omega_c\tau)^2$, itself a rather dubiously defined quantity, is not a coincidence,

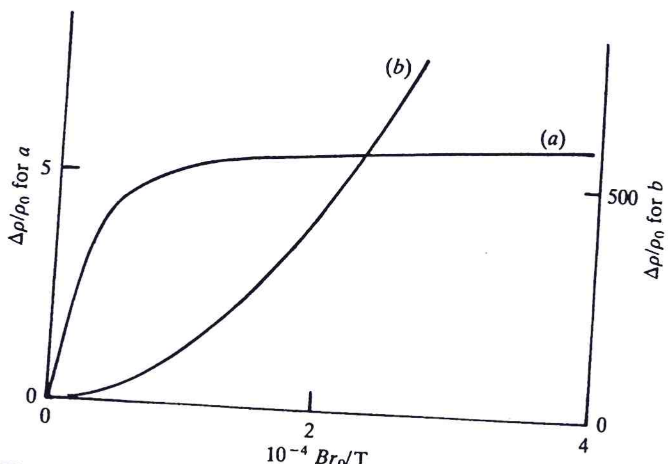
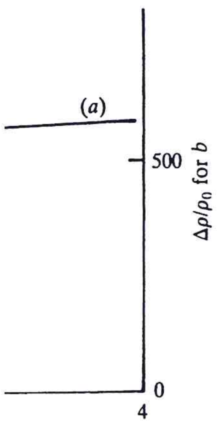


Figure 1.5 Transverse magnetoresistance at 4 K of a single crystal of tin ($r_0 = 11\,000$) with **J** along the *c*-axis and two different orientations of **B** in the basal plane (Alekseevskii and Gaidukov[12]).

30 as we shall see. The same holds in bismuth, in a pure sample of which a field of 1 mT sufficed to bring $\omega_c\tau$ to unity.[6] Values of $\Delta R/R_0$ well in excess of 10^6 are therefore to be expected when B approaches 10 T.

It will be observed in both fig. 4 and fig. 5(*a*) that a slow linear rise, like that in potassium, seems to supervene at high field strengths, marring what might have been complete saturation. This should not be confused with a dominant linear variation such as Kapitza[3] surmised might be the norm. Limited as he was, at the time of his high-field experiments, to temperatures no lower than that of liquid nitrogen, he rarely achieved values of $\omega_c\tau$ above 5. As the initial portion of fig. 5(*a*) exemplifies, there may well be a long, nearly linear stretch before saturation begins, and it is only with a few metals such as polycrystalline copper (fig. 6) that it continues up to high values of $\omega_c\tau$. Later work, in fact, has provided no support for what was sometimes referred to as Kapitza's law, except for certain polycrystalline
184 samples which demand special theoretical treatment.

There are many metals that can be treated adequately by imagining the conduction electrons to behave much like classical particles. The effect of **B** is solely to cause them to be deflected into orbits which are, however, frequently more complicated than the helices described by free electrons. In these, so long as the resistivity is governed by scattering processes which depend on the velocity of an electron and its direction of motion, but not on **B** or on position in the sample, the magnetoresistance is positive as in the examples so far presented. In ferromagnets, especially when they are close



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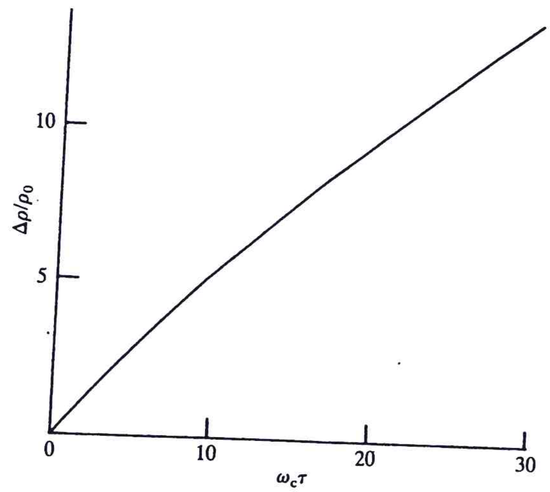

Figure 1.6 Transverse magnetoresistance at 4 K of polycrystalline copper (de Launay *et al.*[4]).

to their Curie point, the polarization of the electrons and hence their scattering can be affected by **B** and lead to negative magnetoresistance, as in fig. 7. Spatial variation of scattering plays a part in samples whose dimensions are comparable to the electronic free path, for the winding up of the orbits may reduce the importance of surface scattering and produce

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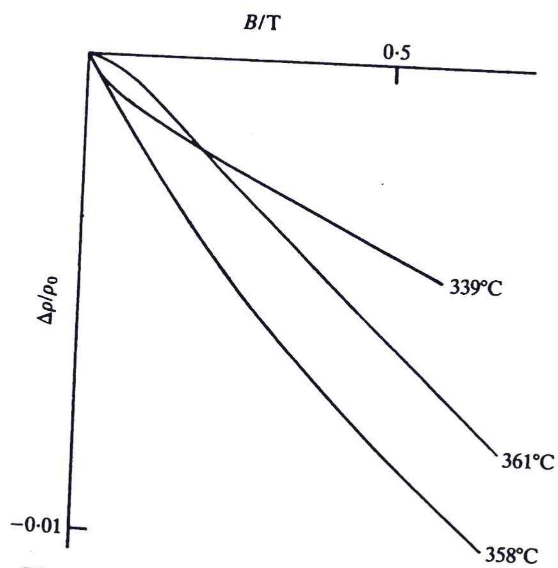

Figure 1.7 Negative transverse magnetoresistance of nickel near its Curie temperature (357 °C) (Potter[13]).

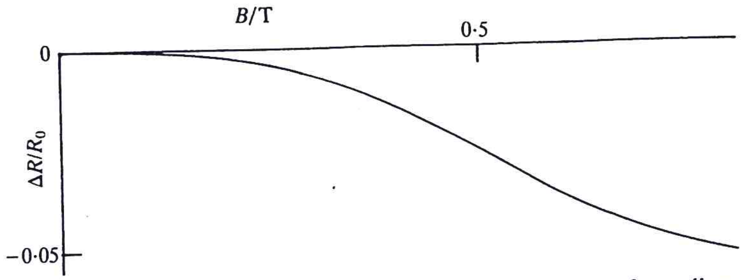

Figure 1.8 Negative longitudinal magnetoresistance at 4 K of a sodium wire, 20 μ in diameter (MacDonald and Sarginson[14]).

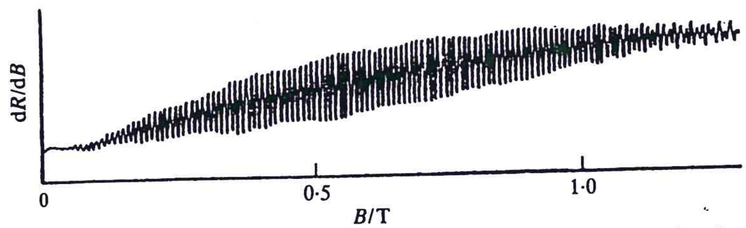

Figure 1.9 Differential transverse magnetoresistance at 1.3 K of a flat single-crystal plate of gallium, showing size-effect oscillations (Munarin and Marcus[15]).

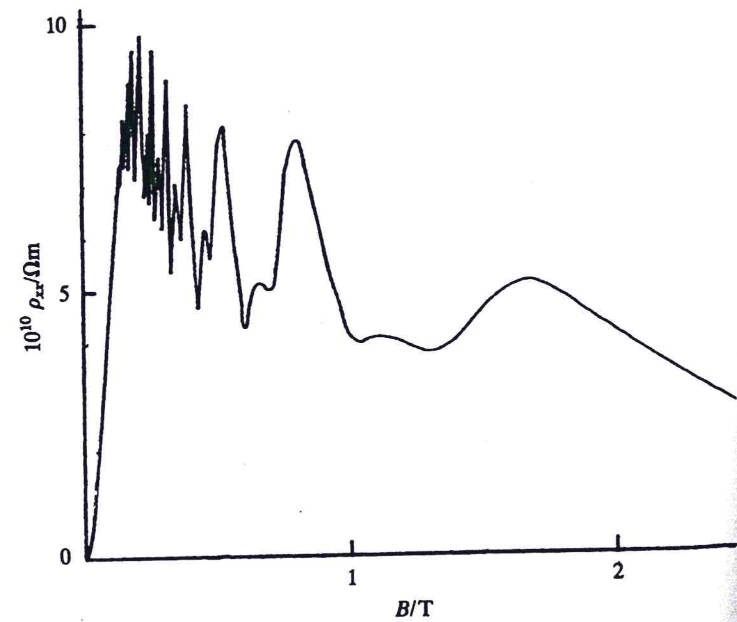

Figure 1.10 Transverse magnetoresistance at 1.6 K of a single crystal of zinc, with **B** along the hexad axis (Stark[16]).

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$$\rho l = (3\pi^2/n^2)^{1/3}\hbar/e^2.$$

In potassium, $n = 13.94 \times 10^{27}$ m

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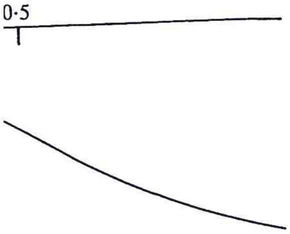

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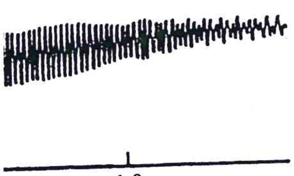

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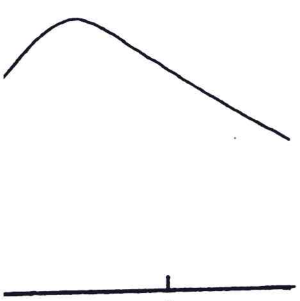

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negative magnetoresistance (fig. 8). Sometimes, however, the resistance change shows an oscillatory component, which may not appear striking when observed directly but which can be distinguished from the smooth variation by recording $\mathrm{d}R/\mathrm{d}B$ instead of R. The oscillatory component thus revealed in a flat plate of gallium is shown in fig. 9, where it can be seen that the period of oscillation is constant and unaffected by the magnitude of $\mathbf{B}$. This is characteristic of size-effect oscillations. 212

On the other hand, oscillations of resistance having their origin in quantization of the electron orbits show a constant period only when plotted against $1/B$. In a direct plot against B, as in fig. 10, the period lengthens markedly towards higher field strengths.

This concludes the preliminary exposure of typical magnetoresistance behaviour which it is the business of this book to explain in as much detail as possible. The rest of the chapter is concerned with the general theoretical concepts which form the basis of the variety of special treatments required.

Dynamics of an electron

In a Fermi gas of electrons at 0 K, all states up to the Fermi energy are filled and those above are empty. On raising the temperature, thermal excitation affects only those within a few $k_B T$ of the Fermi surface, a very small fraction at the temperature of most measurements. Only these few can suffer collisions, the rest having no empty state available into which they could be scattered. We visualise the conduction process as involving the bodily shift, by the action of $\mathscr{E}$, of the k-vectors of all filled states. Most states which were initially filled remain so after the application of $\mathscr{E}$ and therefore play no part in forming a current. The changes that matter are that a few empty states near the Fermi surface become filled and a few on the opposite side, that initially were filled, become empty. It is these that carry the current, and the scattering of these electrons round the Fermi surface that destroys it.

It is clear from the high conductivity of pure metals (when $\mathbf{B}=0$) at low temperatures that the free path, l, of electrons near the Fermi surface must be long; let us make an estimate for a free-electron metal. The relaxation time, τ, in (1) may be replaced by l/v_F, v_F being the Fermi velocity, and $m^* v_F$ is the Fermi momentum, $\hbar k_F$. Also the volume of the Fermi sphere is $\frac{4}{3}\pi k_F^3$ and this volume is packed with $1/4\pi^3$ states per unit volume of metal, so that $n = k_F^3/3\pi^2$; hence,

$$\rho l = (3\pi^2/n^2)^{1/3}\hbar/e^2. \qquad (1.21)$$

In potassium, $n = 13.94 \times 10^{27}\,\mathrm{m}^{-3}$ and $\rho l = 2.19 \times 10^{-15}\,\Omega\mathrm{m}^2$. A very