12 Magnetoresistance

In this chapter we consider an important transport problem—the electrical conductivity of metals in a magnetic field. A large effort of theoretical physicists in recent years has gone into the derivation of improved solutions to transport problems, in gases, plasmas, and metals. Pioneer papers dealing with the quantum theory of charge transport in metals include those by J. M. Luttinger and W. Kohn, *Phys. Rev.* **109,** 1892 (1958) and by I. M. Lifshitz, *Soviet Phys.—JETP* **5,** 1227 (1957). The classical theory of magnetoresistance is developed rather fully in the books by Wilson and by Ziman. In Chapters 16 and 17 we treat several interesting, but somewhat complicated, problems by classical methods. But the startling highlights of the observed magnetoresistive phenomena in solids can be elucidated qualitatively by relatively elementary methods. The analysis of the experimental results bears directly on the shape and connectivity of the fermi surface.

By magnetoresistance we mean the increase in the electrical resistance of a metal or semiconductor when placed in a magnetic field. The effect of greatest interest is the transverse magnetoresistance, which is usually studied in the following geometrical arrangement: a long thin wire is directed along the x axis, and a d-c electric field E_x is established in the wire by means of an external power supply. A uniform magnetic field H_z is applied along the z axis, thus normal to the axis of the wire. The most interesting experiments are those carried out at low temperatures on very pure specimens in strong magnetic fields, as here the product $|\omega_c|\tau$ of the cyclotron frequency and the relaxation time may be $\gg 1$. In these conditions the details of the collision processes are suppressed and the details of the fermi surface enhanced.

In the geometry described, which we shall refer to as the *standard geometry*, the effect of a weak magnetic field ($|\omega_c|\tau \ll 1$) is to increase

the resistance by an additive term proportional to H^2. The additive term may be of the order of magnitude of $(\omega_c\tau)^2$:

$$\frac{R(H) - R(0)}{R(0)} \approx (\omega_c\tau)^2. \tag{1}$$

On dimensional grounds we could not expect much else, bearing in mind that a term linear in H is inconsistent with the obvious symmetry of the problem with respect to the sign of the magnetic field. We note (*ISSP*, p. 238) that in copper at room temperature the relaxation time is $\approx 2 \times 10^{-14}$ sec; for $m^* = m$ and $H = 30$ koe we have $|\omega_c| \approx 8 \times 10^{11}$ sec^{-1}, so that $|\omega_c|\tau \approx 0.02$. At 4°K the conductivity of a fairly pure crystal of copper may be higher than at room temperature by 10^3 or more; thus τ is lengthened by 10^3 and in the same magnetic field $|\omega_c|\tau \approx 20$.

In very strong fields, that is, for $|\omega_c|\tau \gg 1$, the transverse magnetoresistance of a crystal may generally do one of three quite different things.

(*a*) The resistance may saturate, that is, may become independent of H, perhaps at a resistance of several times the zero field value. Saturation occurs for all orientations of the crystal axes relative to the measurement axes.

(*b*) The resistance may continue to increase up to the highest fields studied for all crystal orientations.

(*c*) The resistance may saturate in some crystal directions, but not saturate in other, often very nearby, crystal directions. This behavior is exhibited as an extraordinary anisotropy of the resistance in a magnetic field, as illustrated by Fig. 1.

Crystals are known in all three categories. We shall see that the first category comprises crystals with *closed* fermi surfaces, such as In, Al, Na, and Li. The second category comprises crystals with equal numbers of electrons and holes, such as Bi, Sb, W, and Mo. The third category comprises crystals with fermi surfaces having *open orbits* for some directions of the magnetic field; this category is known to include Cu, Ag, Au, Mg, Zn, Cd, Ga, Tl, Sn, Pb, and Pt. The value of magnetoresistance as a tool is that it tells us whether the fermi surface is closed or contains open orbits, and in which directions the open orbits lie. There are geometrical situations possible for which open fermi surfaces do not contain open orbits.

Many interesting features can be explained by an elementary drift velocity treatment or by simple extensions thereof. We give this

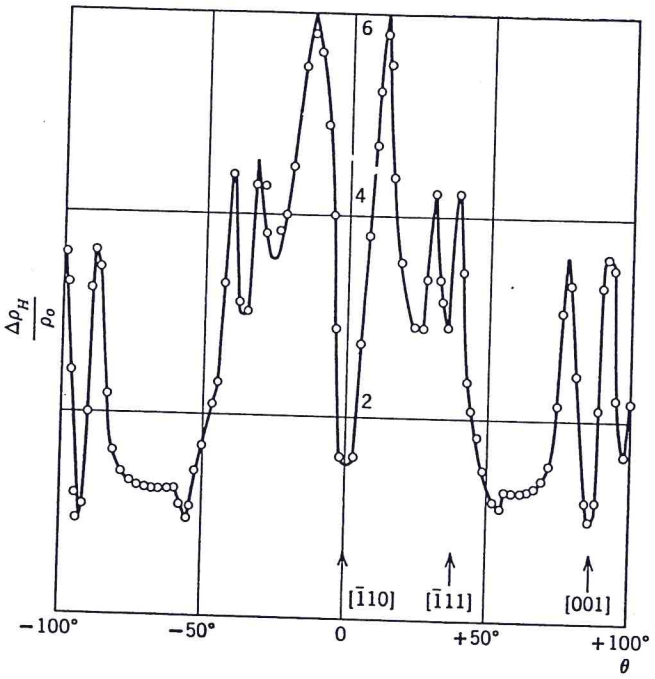

FIG. 1. Variation of transverse magnetoresistance with field direction in a field of 23.5 kG, for a single crystal Au specimen with current‖ [110]. (From Gaidukov, 1959.)

treatment first as a preliminary to the application of more detailed transport theory. The drift velocity $\mathbf{v}$ is defined as the average carrier velocity:

$$\mathbf{v} = \frac{1}{N}\sum_i \mathbf{v}_i. \tag{2}$$

Single Carrier-Type Isotropic Effective Mass and Constant Relaxation Time. The equation of motion of the drift velocity of a gas of carriers having isotropic mass m^* is, according to *ISSP*, Chapter 10,

$$m^*\left(\dot{\mathbf{v}} + \frac{1}{\tau}\mathbf{v}\right) = e\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right), \tag{3}$$

where τ is the relaxation time of the charge carriers. The relaxation time is approximately related to the mean free path Λ by $\Lambda \cong \overline{|\mathbf{v}_i|}\tau$, where $\overline{|\mathbf{v}_i|}$ is the mean magnitude of the particle velocity. We take $\mathbf{H}$

in the z direction. In the steady state $\dot{\mathbf{v}} = 0$, so that

$$\mathbf{v} = \frac{e\tau}{m^*}\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H}\right). \tag{4}$$

If we set

$$\mu \equiv e\tau/m^*; \qquad \xi \equiv \mu H = eH\tau/m^*c = -\omega_c\tau, \tag{5}$$

then (4) becomes

$$v_x = \mu E_x + \xi v_y; \qquad v_y = \mu E_y - \xi v_x; \qquad v_z = \mu E_z. \tag{6}$$

Thus, on solving for v_x and v_y,

$$v_x = \mu E_x + \mu\xi E_y - \xi^2 v_x; \qquad v_y = \mu E_y - \mu\xi E_x - \xi^2 v_y, \tag{7}$$

or

$$v_x = \frac{\mu}{1+\xi^2}(E_x + \xi E_y); \qquad v_y = \frac{\mu}{1+\xi^2}(E_y - \xi E_x). \tag{8}$$

The current density component j_λ is obtained from the velocity component v_λ by multiplying by ne, where n is the carrier concentration. The conductivity tensor component $\sigma_{\lambda\nu}$ is defined by

$$j_\lambda = \sigma_{\lambda\nu}E_\nu. \tag{9}$$

From (8) we have, for $H \parallel \hat{z}$,

$$\bar{\sigma} = \frac{ne\mu}{1+\xi^2}\begin{pmatrix} 1 & \xi & 0 \\ -\xi & 1 & 0 \\ 0 & 0 & 1+\xi^2 \end{pmatrix}. \tag{10}$$

The components satisfy the condition

$$\sigma_{\lambda\nu}(H) = \sigma_{\nu\lambda}(-H), \tag{11}$$

as a general consequence of the theory of the thermodynamics of irreversible processes.

In our standard geometry the boundary conditions permit current flow only in the x direction, thus

$$j_y = j_z = 0. \tag{12}$$

From (8) we see that the boundary conditions can be satisfied only if

$$E_y = \xi E_x; \qquad E_z = 0. \tag{13}$$

The field E_y is known as the hall field. From (10) and (13),

$$j_x = \frac{ne\mu}{1+\xi^2}(E_x + \xi E_y) = ne\mu E_x; \tag{14}$$

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thus in this geometry the *effective* conductivity in the x direction as calculated on our model is independent of the magnetic field in the z direction, even though the conductivity tensor components (10) do involve the magnetic field. That is, our model gives zero for the transverse magnetoresistance.

The resistivity tensor $\bar{\rho}$ is the inverse of the conductivity tensor, so that $E_\lambda = \rho_{\lambda\nu} j_\nu$. The components are given by

$$\rho_{\lambda\nu} = \Delta_{\lambda\nu}/\Delta, \tag{15}$$

where Δ is the determinant of $\bar{\sigma}$; $\Delta_{\lambda\nu}$ is the $\lambda\nu$-th minor; and

$$\Delta = \frac{(ne\mu)^3}{1+\xi^2}. \tag{16}$$

Thus for $\bar{\sigma}$ given by (10) the resistivity tensor is

$$\bar{\rho} = \frac{1}{ne\mu}\begin{pmatrix} 1 & -\xi & 0 \\ \xi & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

This is consistent with (6), from which $\bar{\rho}$ is most easily found. For the standard geometry with $j_y = 0$ we have from (17) that

$$E_x = \frac{1}{ne\mu} j_x; \qquad E_y = \frac{\xi}{ne\mu} j_x = \frac{H_z}{ne} j_x = \xi E_x, \tag{18}$$

in agreement with (13) and (14).

The absence of magnetoresistance on this model in the standard geometry is the result of the hall electric field E_y, which just balances the lorentz force of the magnetic field. The balance can be maintained only for the one kinematical quantity v included in the equations of motion. But usually the relaxation time depends on the speed v_i of an individual carrier, so that we cannot expect to describe the motion of the carriers in terms of a single drift velocity. Then the cancellation will not take place. The experimental situation is that a transverse magnetoresistance is always, or nearly always, observed. A simple and important alteration of the drift velocity model is to introduce a second carrier type. With two carrier types the identical hall electric field cannot rectify the orbits of both carrier types at once. This is an important practical situation—the two carrier types may be electrons and holes; s electrons and d electrons; open orbits and closed orbits; etc.

Two Carrier Types—High Field Limit. There is a special value in treating the problem of two carrier types in the high field limit. In any field the steady state drift velocity equations are, by analogy

with (4),

$$\mathbf{v}_1 = (e\tau_1/m_1^*)\mathbf{E} + (e\tau_1/m_1^* c)\mathbf{v}_1 \times \mathbf{H}; \tag{19}$$

$$\mathbf{v}_2 = -(e\tau_2/m_2^*)\mathbf{E} - (e\tau_2/m_2^* c)\mathbf{v}_2 \times \mathbf{H}; \tag{20}$$

where the carriers of type 1 are taken to be electrons of effective mass m_1^*, relaxation time τ_1, and concentration n_1. The carriers of type 2 are taken to be holes. We now consider fields such that $|\omega_{c1}|\tau_1 \gg 1$ and $|\omega_{c2}|\tau_2 \gg 1$. Then we can neglect $\mathbf{v}_1$, $\mathbf{v}_2$ when they appear on the left-hand side of (19) and (20), whence for the x components of these equations we have

$$E_x + \frac{H}{c} v_{1y} = 0; \qquad E_x + \frac{H}{c} v_{2y} = 0. \tag{21}$$

Thus

$$j_y \equiv n_1 e v_{1y} - n_2 e v_{2y} = \frac{(n_2 - n_1)ec}{H} E_x, \tag{22}$$

whence

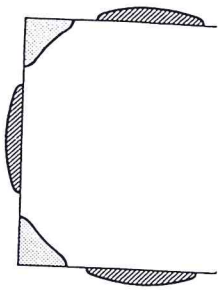
$$\boxed{\sigma_{yx} = (n_2 - n_1)ec/H.} \tag{23}$$

This is a crucial result, because it shows that for equal numbers of holes and electrons $\sigma_{yx} = 0$. But if $\sigma_{yx} = 0$, there is no hall voltage E_y, as $j_y = 0$ without benefit of an E_y. Without E_y the effective resistivity becomes simply $1/\sigma_{xx}$, where σ_{xx} is given by (10) and in this limit

$$\sigma_{xx} \cong \frac{n|e|}{H^2}\left(\frac{1}{|\mu_1|} + \frac{1}{|\mu_2|}\right), \tag{24}$$

where $n = n_1 = n_2$. Thus the *transverse magnetoresistance does not saturate if there are equal numbers of holes and electrons.*

Divalent metals having one atom (and two valence electrons) per primitive cell will necessarily have equal numbers of holes and electrons ($n_- = n_+$), provided there are no open orbits. There is one point in $\mathbf{k}$ space in each Brillouin zone for each primitive cell in the crystal. Equality of electron and hole concentrations can also occur in metals of odd valence if the primitive cell contains an even number of atoms. Under these conditions it is observed that the transverse magnetoresistance does not saturate. The topology of the equality of electrons and holes is easily understood in two dimensions; see, for example, Fig. 2, where the fermi surface has been constructed with parts in two zones, but with the total filled area just equal to the area of one zone.



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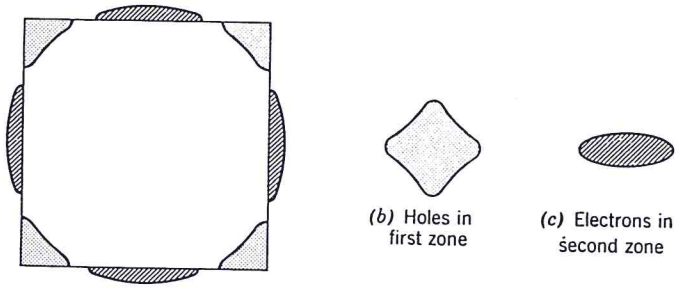

FIG. 2. (*a*) Fermi surface in two dimensions enclosing an area equal to the area of one Brillouin zone. (*b*) Hole orbit as connected. (*c*) One electron orbit as connected.

This result (23) holds also for a general fermi surface, at least in the semiclassical approximation developed in the preceding chapter for the dynamics of an electron in a magnetic field. We consider a thin section of an electron fermi surface, bounded by planes normal to H, with α states per unit area of the section. In constant fields H_z and E_x the energy ε of an electron in the section changes according to

$$\dot{\varepsilon} = ev_xE_x = -c\dot{k}_yE_x/H, \tag{25}$$

because $\dot{k}_y = -(e/c)v_xH$, if we may neglect collisions. Thus the shift of the fermi surface from equilibrium is given by

$$\Delta\varepsilon = -ck_yE_x/H, \tag{26}$$

within an additive constant. The resultant current in the y direction is, integrating over the surface of the section,

$$J_y = \alpha e \int dk_x\, dk_y \frac{\partial \epsilon}{\partial k_y} = -\alpha ec(E_x/H_z) \int dk_x\, dk_y. \tag{27}$$

The integral on the right-hand side is just the area of the section, so that J_y is the number of states in the section times ecE_x/H. For the whole fermi surface we integrate over dk_z, recalling that $\alpha \int dk_x\, dk_y\, dk_z$ gives the number of states in the volume. Thus the total current density is

$$j_y = \frac{ec}{H}(n_+ - n_-)E_x, \tag{28}$$

where n_+ is the hole concentration and n_- the electron concentration. For $n_- = n_+$ we have $\sigma_{yx} = 0$, in agreement with (23), and by the above argument the magnetoresistance does not saturate, in agreement with experiment. This result is independent of crystal orientation and explains the second variety of magnetoresistive behavior, as enumerated earlier.

INFLUENCE OF OPEN ORBITS

It is a remarkable experimental fact that in some crystals the magnetoresistance saturates *except* for certain special crystal orientations. The absence of saturation in certain directions may be explained in terms of open orbits. In strong magnetic fields an open orbit carries current essentially only in a single direction in the plane normal to the magnetic field; thus the open orbit cannot be saturated by the field. Suppose that for a given crystal orientation there are open orbits parallel to k_x; in real space these orbits carry current parallel to the y axis. We can associate a conductivity σ_{yy} with the open orbits; let us write the open-orbit conductivity as equal to $sne\mu$; this defines s. The high field limit of the conductivity tensor (10) is

$$\bar{\sigma} \approx ne\mu \begin{pmatrix} \xi^{-2} & \xi^{-1} & 0 \\ -\xi^{-1} & \xi^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{29}$$

not considering the contribution of open orbits. We recall that $\xi \propto H$. With the contribution of the open orbits we have

$$\bar{\sigma} \approx ne\mu \begin{pmatrix} \xi^{-2} & \xi^{-1} & 0 \\ -\xi^{-1} & s & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{30}$$

Here we have dropped ξ^{-2} in comparison with s in the term σ_{yy}. We have assumed for convenience that $\sigma_{xz} = 0 = \sigma_{zx}$; this is not the most general situation. The applicability of (29) and (30) as high field limits is demonstrated by Pippard, *LTP*, pp. 93–95. We note that an open orbit parallel to k_x has an average velocity component only in the y direction and does not contribute to σ_{xy}, σ_{xx}, etc. The strength of the magnetic field does not affect the average carrier velocity v_y on the orbit; the field strength only affects the rate $\dot{k}_x$ at which the open orbit is traversed in **k** space.

With (30) we obtain $j_y = 0$ when

$$-\frac{E_x}{\xi} + sE_y = 0, \qquad \text{or} \qquad E_y = \frac{E_x}{s\xi}; \tag{31}$$

thus

$$j_x \approx ne\mu(\xi^{-2}E_x + \xi^{-1}E_y) = ne\mu\left(1 + \frac{1}{s}\right)\xi^{-2}E_x; \tag{32}$$

whence the effective resistivity is

$$\rho \approx \frac{\xi^2}{ne\mu}\frac{s}{s+1}. \tag{33}$$

The resistivity does not saturate, but increases as H^2. Crystal distributions tend to reduce the exponent towards 1. We have thus accounted for the third variety of magnetoresistive behavior, namely nonsaturation only in special crystal orientations.

Suppose the crystal is oriented so that the open orbit carries current in the x direction. Then

$$\bar{\sigma} \cong ne\mu\begin{pmatrix} s & \xi^{-1} & 0 \\ -\xi^{-1} & \xi^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{34}$$

and $j_y = 0$ if $E_y = \xi E_x$, so that

$$j_x \approx (s+1)ne\mu E_x. \tag{35}$$

For this orientation the magnetoresistance saturates.

If the open orbit runs in a general direction in the xy plane, the conductivity tensor has the form, again in the limit $\xi \gg 1$,

$$\bar{\bar{\sigma}} \approx ne\mu\begin{pmatrix} s\sin^2\theta + \xi^{-2} & -s\sin\theta\cos\theta + \xi^{-1} & 0 \\ s\sin\theta\cos\theta - \xi^{-1} & s\cos^2\theta + \xi^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{36}$$

This gives $j_y = 0$ when $E_y = (\xi^{-1} - s\sin\theta\cos\theta)E_x/(s\cos^2\theta + \xi^{-2})$ for $\theta \neq 0$; we have

$$j_x = ne\mu\left\{s\sin^2\theta + \xi^{-2} + \frac{(s\sin\theta\cos\theta - \xi^{-1})^2}{s\cos^2\theta + \xi^{-2}}\right\}E_x \tag{37}$$

$$\rightarrow 2ne\mu s\sin^2\theta E_x.$$

Thus the *magnetoresistance saturates except when the open orbit carries current almost precisely parallel to the y direction.* By the geometrical rules discussed in Chapter 11 this requirement is that the orbit in **k** space must run in the k_x direction.

The circumstance that the magnetoresistance saturates in sufficiently strong magnetic fields, except when there are open orbits in the k_x direction, explains the extraordinary anisotropy of the transverse magnetoresistance observed in single crystals. The anisotropy is a striking feature of the experimental results, as illustrated for gold in

Fig. 1. Thus high-field studies of the angular dependence of the transverse magnetoresistance in single crystals can provide information on the presence of open orbits and on the connectivity of the fermi surface. In directions in which open orbits exist the resistance does not saturate; in other directions it does, except in special directions in which the metal may simulate a two-band metal with equal numbers of electrons and holes.

TRANSPORT EQUATIONS FOR MAGNETORESISTANCE

The Chambers or kinetic formulation of the transport equation is rather more revealing for the magnetoresistance problem with a general fermi surface than is the usual iterative formulation of the boltzmann method, in which magnetic effects appear only in second order. We first work with a version of the theory linearized in **E**. If the distribution is $f = f_0 + f_1$, where f_0 is the equilibrium distribution, then the electric current density is

$$\mathbf{j} = \frac{2e}{(2\pi)^3}\int d^3k\, \mathbf{v} f_1 = \frac{2e}{(2\pi)^3}\int d^3k\, \mathbf{v}\frac{df_0}{d\varepsilon}\Delta\varepsilon, \tag{38}$$

where $\Delta\varepsilon$ is the mean energy gained by the electron from the electric field **E** in the time between collisions. It is assumed that immediately after a collision the electron is in the equilibrium distribution. Then

$$\Delta\varepsilon = e\int_{-\infty}^{0} dt\, \mathbf{E}\cdot\mathbf{v}(t)e^{t/\tau}, \tag{39}$$

if the relaxation time τ is a constant. Here $e^{-|t|/\tau}$ is the probability that the last collision took place at least a time $|t|$ from the next collision, taken at $t = 0$. It is a simple matter to generalize (39) to problems in which τ is known as a function of **k**.

The general nonlinearized result of Chambers [*Proc. Phys. Soc.* (*London*) **A65,** 458 (1952)] for the distribution function is

$$f = \int_{-\infty}^{t}\frac{dt'}{\tau(\mathbf{k}(t'))} f_0(\varepsilon - \Delta\varepsilon(t'))\exp\left(-\int_{t'}^{t}\frac{ds}{\tau(\mathbf{k}(s))}\right), \tag{40}$$

where

$$\Delta\varepsilon = \int_{t'}^{t} dt''\, \mathbf{F}\cdot\mathbf{v}(t'') \tag{41}$$

is the energy gain from the force **F** between times t' and t in the absence of collisions; τ is the relaxation time; and f_0 is the equilibrium distribution function. A proof is given by H. Budd, *Phys. Rev.* **127,** 4 (1962), that (40) satisfies the boltzmann equation.

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(49) $\sigma_{xy} = \frac{2e^2}{(2\pi)^3}\omega_c\tau$

many times between collisions and the integral (39) will approach zero for the velocity components in the plane normal to the magnetic field $\mathbf{H}$; the component parallel to $\mathbf{H}$ ($= H_z$) may be replaced by the average $\langle v_{\mathrm{H}}(k_{\mathrm{H}})\rangle$ taken over the orbit at constant k_{H}. Thus if the magnetic field is in the z direction,

(42) $$\sigma_{xx} = \sigma_{xy} = \sigma_{yy} = \sigma_{yx} = 0;$$

(43) $$\sigma_{zz} = \frac{2e^2}{(2\pi)^3\tau}\int d^3k\, v_z\langle v_z\rangle \frac{df_0}{d\varepsilon}.$$

This value of σ_{zz} for $H = \infty$ is in general lower than the value

(44) $$\sigma_{zz}(0)\,\frac{2e^2}{(2\pi)^3\tau}\int d^3k\, {v_z}^2 \frac{df_0}{d\varepsilon}$$

for zero magnetic field by an amount depending on the anisotropy of v_z around the orbit. Thus there is a longitudinal magnetoresistance, which always saturates.

If open orbits are present in the k_x direction, then $\langle v_x\rangle = 0$, but $\langle v_y\rangle \neq 0$. Then $\sigma_{yy} \neq 0$ in the high-field limit, just as found in (30).

Let us apply the kinetic method to transverse magnetoresistance in weak magnetic fields such that $|\omega_c|\tau \ll 1$. We are concerned with

$$\int_{-\infty}^{0} dt\, v_\mu(t)e^{t/\tau},$$

for constant relaxation time τ. We expand

(45) $$v_\mu(t) = v_\mu(0) + t\dot{v}_\mu(0) + \tfrac{1}{2}t^2\ddot{v}_\mu(0) + \cdots.$$

The integrals are trivial and we have

(46) $$\int_{-\infty}^{0} dt\, v_\mu(t)e^{t/\tau} = \tau v_\mu(0) + \tau^2\dot{v}_\mu(0) + \tau^3\ddot{v}_\mu(0) + \cdots.$$

As an example, consider

(47) $$\sigma_{xy} = \frac{2e^2}{(2\pi)^3}\int d^3k\, v_x \frac{df_0}{d\varepsilon}(\tau v_y + \tau^2\dot{v}_y + \tau^3\ddot{v}_y + \cdots).$$

The term in v_y vanishes on integration, by symmetry. The term in $\dot{v}_y$ for a free-electron gas is obtained by using

(48) $$m\dot{v}_y = -\frac{e}{c}v_xH; \qquad \dot{v}_y = \omega_c v_x.$$

Thus to $O(H)$

(49) $$\sigma_{xy} = \frac{2e^2}{(2\pi)^3}\omega_c\tau^2\int d^3k\, {v_x}^2\frac{df_0}{d\varepsilon} = \omega_c\tau\sigma_{xx}(0).$$

The term of $O(H^2)$ in σ_{xx} or σ_{yy} is obtained by evaluating $\ddot{v}_y$, for example. We have

$$\ddot{v}_y = \omega_c \dot{v}_x = -\omega_c{}^2 v_y, \tag{50}$$

so that

$$\sigma_{yy} = \sigma_{yy}(0) - (\omega_c\tau)^2\sigma_{yy}(0). \tag{51}$$

These results for the free-electron gas agree to the appropriate order with the drift-velocity treatment given earlier, but the present formulation can be applied to general energy surfaces. For example, to evaluate σ_{xy} we need

$$\dot{v}_y = \dot{k}_\nu \frac{\partial^2\varepsilon}{\partial k_\nu\, \partial k_y} = \frac{eH}{c}\left(\frac{\partial\varepsilon}{\partial k_y}\frac{\partial^2\varepsilon}{\partial k_x\, \partial k_y} - \frac{\partial\varepsilon}{\partial k_x}\frac{\partial^2\varepsilon}{\partial k_y{}^2}\right), \tag{52}$$

whence to $O(H)$ we have a well-known result:

(53)

$$\sigma_{xy} = \frac{2e^2}{(2\pi)^3}\cdot\frac{eH\tau^2}{c}\int d^3k \frac{\partial f_0}{\partial\varepsilon}\left[\frac{\partial\varepsilon}{\partial k_x}\left(\frac{\partial\varepsilon}{\partial k_y}\frac{\partial^2\varepsilon}{\partial k_x \partial k_y} - \frac{\partial\varepsilon}{\partial k_x}\frac{\partial^2\varepsilon}{\partial k_y{}^2}\right)\right].$$

For $\omega_c > k_BT$ and $\omega_c\tau \gg 1$ an oscillatory behavior is observed in the conductivity components. The quantum oscillations in transport properties have the same origin as the susceptibility oscillations in the de Haas-van Alphen effect considered in Chapter 11.

PROBLEMS

1. Show that the transverse magnetoresistance vanishes for a conductor with an isotropic relaxation time and an ellipsoidal mass surface. An elegant derivation of this result is quoted by Chambers in *The fermi surface*, Wiley.

2. Discuss the magnetoresistive properties of a conductor with a fermi surface in the form of an infinite circular cylinder.

3. Consider the magnetoresistance of the two-carrier-type problem for all values of the magnetic field, but assuming $m_1 = m_2$; $\tau_1 = \tau_2$. Show that for the standard geometry

$$j_x = (n_1 + n_2)e\mu\frac{1}{1+\xi^2}\left[1 + \frac{(n_1 - n_2)^2}{(n_1 + n_2)^2}\xi^2\right]E_x. \tag{54}$$

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[1] F. S. Ham, *Solid state*
[2] J. R. Reitz, *Solid state*
[3] T. O. Woodruff, *Solid*
[4] J. Callaway, *Solid state*
[5] H. Brooks, *Nuovo cime*
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