Recap from last meeting

Ch12 - A&M

Drude eqn of motion (Chap 1 A&M)

$$\frac{d\vec{p}}{dt} = -\frac{\vec{p}}{\tau} - e\left(\vec{E} + \frac{\vec{p}}{mc} \times B\right)$$

$\vec{v} = \frac{\vec{p}}{m}$; $\mathcal{E} = \frac{p^2}{2m}$; $\vec{p} = \hbar \vec{k}$ $\boxed{\text{A\&M Ch 1 Eqn 1.16-1.21}}$

$\Rightarrow R_H = -\frac{1}{nec}$ and $\rho(H) = \rho(0)$ $\Rightarrow \Delta\rho = 0$

$\Rightarrow \vec{v} = \frac{\hbar \vec{k}}{m}$ always since electrons are free

Semi-classical eqn of motion

$\vec{p} = \hbar \vec{k}$, called crystal momentum

But $\vec{v} \neq \frac{\vec{P}}{m}$. $\vec{v} \rightarrow$ velocity of electron inside a crystal lattice $\rightarrow$ not free

$$\vec{v}_n(k) = \frac{1}{\hbar} \nabla_k \mathcal{E}_n(k)$$

$\mathcal{E}_n(k)$ are the energy bands $\neq \frac{\hbar^2 k^2}{2m}$

$$\frac{d\vec{p}}{dt} = \hbar \dot{\vec{k}} = -e\left(\vec{E} + \frac{\vec{v}_n(k)}{c} \times \vec{B}\right)$$

Consequences of semi-classical eqns

1) Filled bands do not contribute to current

$$\vec{J} = -nev = -e(nv) \qquad n \rightarrow \frac{1}{(4\pi)^3}\int d^3\vec{k} \qquad v = \frac{1}{\hbar}\frac{\partial \mathcal{E}_n(k)}{\partial k}$$

$$= -\frac{e}{(4\pi)^3}\int d^3\vec{k}\, \frac{1}{\hbar}\frac{\partial \mathcal{E}_n(k)}{\partial k}$$

$\vec{J}_{\text{filled band}} = 0 \rightarrow$ (Eqn 12.16 A&M)

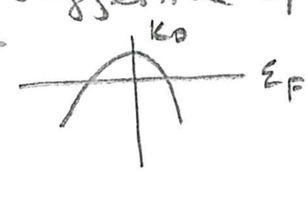
$\Rightarrow$ Current (conduction) is due partially filled bands.

$$\vec{J} = -e \int_{\text{occupied}} \frac{dk}{4\pi^3} v_n(k)$$

---

2) Holes $\rightarrow$ in some neighborhood of energy maximum, we can can expect the electron to behave in a manner suggestive of a positive charge.

$$\mathcal{E}_k \simeq \mathcal{E}(k_0) - A(\vec{k} - \vec{k}_0)^2$$



$$A = \hbar^2 / 2m^*$$

$$v_n(k) = \frac{1}{\hbar}\frac{\partial \mathcal{E}(k)}{\partial k} = -\frac{\hbar(k - k_0)}{m^*}$$

(10pts.)

8. Perform the indicated operation and express in standard form: $\frac{8+2i}{6+7i}$.

Compared to free electron case where $\vec{v} = +\frac{\vec{P}}{m}$ always

we see that for electrons near a band maximum

$$\left.\begin{aligned} \vec{v} &= -\frac{\vec{P}}{m^*} + \text{constant} \\ \text{or } \frac{d\vec{v}}{dt} &= \vec{a} = -\frac{\hbar\dot{\vec{k}}}{m^*} \end{aligned}\right\} \text{Check eqn 12.22 - 12.26 A\&M}$$

This behavior is consistent for a positive charge with a mass m^* that we call holes (or electrons with a -ve mass)

Just like we adopted a definition for $v_n(k) = \frac{1}{\hbar}\frac{\partial \mathcal{E}(k)}{\partial k}$ we can adopt a defintion for the effective mass m^*

$$(m^*)^{-1} = \pm\frac{1}{\hbar^2}\frac{\partial^2 \mathcal{E}_n(k)}{\partial k^2} \quad \text{or as a tensor} \quad (m^*)^{-1}_{ij} = \pm\frac{1}{\hbar^2}\frac{\partial^2 \mathcal{E}(\vec{k})}{\partial k_i \partial k_j}$$

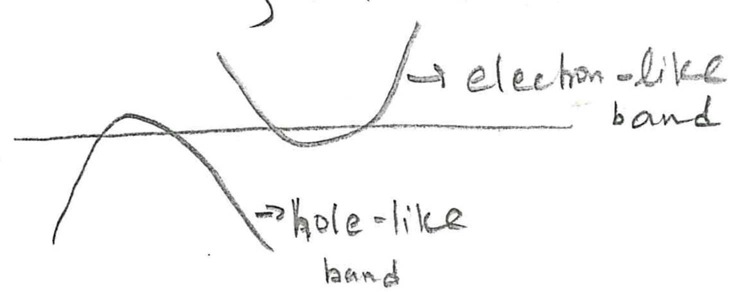
$+ \rightarrow$ electrons (band minimum)

$- \rightarrow$ holes (bands maximum)

$$\Rightarrow \vec{a} = \frac{\pm\hbar\dot{\vec{k}}}{m^*} \;\&\; m^*\vec{a} = \mp e\left(\vec{E} + \frac{\vec{V}\times\vec{B}}{c}\right)$$

So, in a 3D system, there can be electron-like and hole-like behavior coexisting in the material depending on the band structure.

e.g.



Two-band model

$$\sigma = \sigma_{electron} + \sigma_{hole}$$

$$\left.\begin{aligned} \sigma_e &= \frac{n_e e^2 \tau_e}{m_e} \\ \sigma_h &= \frac{n_h e^2 \tau_h}{m_h} \end{aligned}\right\} \text{Gives } B^2 \text{ magneto resistivity}$$

→ heavy holes → Flat bands have high m^* → less mobile

→ light holes → dispersive bands have low m^* → more mobile