

MAGNETORESISTANCE OF FERROMAGNETIC METALS AND ALLOYS AT LOW TEMPERATURES

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Communication No. 286a from the Kamerlingh Onnes Laboratory, Leiden, Nederland

Synopsis

The magnetoresistance of pure Ni and Fe, of Ni-Fe-, Ni-Co-, and Ni-Cu-alloys, and of Heusler's alloy has been measured at room temperature and at temperatures of liquid nitrogen and liquid hydrogen. The behaviour of the pure metals is essentially different from that of the alloys. At low temperatures a maximum in the magnetoresistance is observed for alloys having about one Bohr magneton per atom. The positive difference between the longitudinal and the transversal resistance can be explained by means of the spin-orbit interaction.

At low temperatures the pure metals show an increase in resistance with increasing field just as the non-ferromagnetic metals. From this the value of the internal field, acting on the conduction electrons, could be determined, and was found to be approximately equal to the induction B .

1. *Introduction.* In a recent note *Snoek*¹⁾ has emphasized that from the *Heitler-London* approximation for the $3d$ -electrons in ferromagnetic metals and alloys one would expect singularities in alloys containing an integral number of Bohr magnetons, n_B , per atom. In this respect it was pointed out that the mean saturation-magnetostriction of Ni-Fe- and Ni-Co-alloys vanishes for $n_B = 1$. Another coincidence noted was the occurrence of a maximum in the magnetoresistance at about the same compositions.

These considerations were the starting point for some measurements of the second effect mentioned, the results of which will be reported in this paper.

It is wellknown that in strong magnetic fields the resistance of ferromagnetic metals varies about linearly with the field, the small

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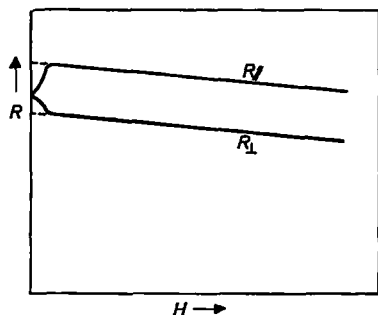


Fig. 1. Schematic plot of the longitudinal and transversal resistances for ferromagnetic metals.

negative slope being the same for all orientations (fig. 1). At present we are interested only in the effects at strong fields, and these straight lines are extrapolated to $H = 0$.

Under these conditions the resistance tensor, referred to the cubic axes of the crystal, depends on the orientation and can be expressed in terms of the direction cosines of the magnetization as follows:

$$\begin{aligned} r_{11} &= r_0 + C_1 a_1^2 + C_3 (a_2^2 a_3^2 + a_3^2 a_1^2 + a_1^2 a_2^2) + C_5 a_2^2 a_3^2 \\ r_{12} &= C_2 a_1 a_2 + C_4 a_1 a_2 a_3^2, \end{aligned} \quad (1)$$

terms higher than those of the fourth degree being omitted.

Most measurements have been carried out on polycrystalline samples. In that case only one single constant is found, being the difference between the resistance in a longitudinal and in a transversal field. Experiments show that always $R_{\parallel} > R_{\perp}$.

In addition to the two effects mentioned above, which are of pure ferromagnetic origin, namely the small negative value of dR/dH and the orientation effect described by (1), at low temperatures the normal increase in resistance becomes important, and in the case of the pure metals, *e.g.* Ni and Fe, predominates.

After a brief outline of the experimental technique, the three aspects of the problem will be treated separately, and an attempt to arrive at a qualitative explanation will be given.

2. Experimental technique. The measurements have been performed in the large magnet of the Kamerlingh Onnes Laboratory, the maximum field strength used being about 22000 oersteds. The resistances have been measured in the usual way on a potentiometer. The samples were furnished by the Philips Laboratories. The value of the resistances varied from 10^{-5} to $10^{-2} \Omega$. For most measurements wires have been used, with a length of about 25 mm and diameters of some tenths of a mm. In this case the transversal external field has been corrected for the demagnetizing field. In

some other cases foils of some tenths of a mm thickness were available. All specimens have been annealed. As only some qualitative features are important, no tables will be given.

3. *The normal increase in resistance.* At low temperatures a similar increase in the resistance has been found as for the non-ferromagnetic metals. This effect is due to the Lorentz force acting on the conduction electrons, which gives them curved paths. The effect is appreciable only if the mean free path is comparable with the radius of curvature, which is inversely proportional to the field strength. Accordingly for the alloys the effect was small (at most about 1% at low temperatures). For the pure metals Ni and Fe at liquid hydrogen temperatures the effects are large and of the same order of magnitude as for the noble metals. Here R_{\perp} always exceeds R_{\parallel} .

The main difference between the curves for the ferromagnetic metals and those for the normal ones was that for Ni and Fe at weak fields the difference between longitudinal and transversal resistance did not vanish, except of course for $H = 0$, since the remanence of the material is negligible. The only possible conclusion is that the field acting on the conduction electrons is not equal to the external field, but a field caused by the internal dipoles has to be added. In the case of iron we were able to evaluate this extra field by means of the Kohler diagram. Kohler²⁾ has stated that $\Delta R/R_{H=0}$ can be plotted as a function of $H/R_{H=0}$ only, for all temperatures and purities. The diagram in which $\log(\Delta R/R_{H=0})$ is plotted *versus* $\log(H/R_{H=0})$ is called the "Kohler diagram".

The values of the additive internal field and those of $R_{H=0}$ (which define an additive constant in ΔR_{\parallel} and ΔR_{\perp}) can now be chosen in such a way as to fit all the points on a Kohler diagram. The actual procedure for doing this was as follows.

Firstly we have to correct for the orientation effect described by (1). For iron this amounted to 0.5% at room temperature, and we assumed this to be independent of the temperature, as was found approximately for nickel (see section 5). Because of the smallness of this effect for iron no large error can be introduced. The values of the resistance at zero total field could be extrapolated with sufficient accuracy. By plotting the Kohler diagram with different values for the additive internal field, and taking for ΔR the difference between R_{\perp} and R_{\parallel} , we found that all measurements (at $T = 14, 20, 64, 69$

and 77°K) lie on one straight line for an extra field of 22000 ± 1000 oersteds. This is approximately equal to $4\pi I$. Thus the field acting on the conduction electrons is equal to the induction B , which is the mean magnetic field within the metal. Three specimens have been used of different purity (labelled from I to III in fig. 2, I being the

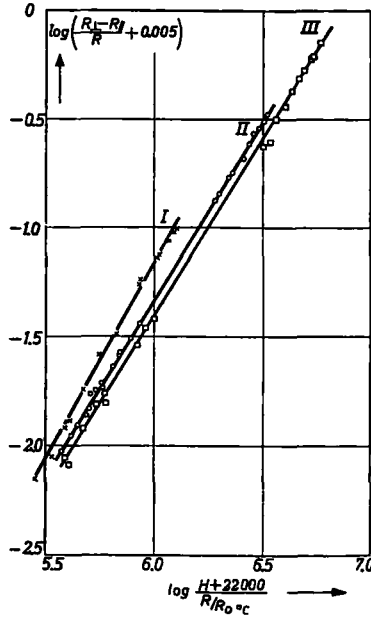


Fig. 2. Reduced Kohler diagram for iron. For ΔR the difference between $R_{//}$ and R_{\perp} , corrected for the orientation effect (1), has been taken. The three specimens have different purity (I is the impurest, and III the purest one). The lower points refer to measurements at temperatures of liquid nitrogen, and the higher ones to temperatures of liquid hydrogen.

most impure one and III the purest). It is remarkable that the three curves do not coincide, and that the curve for the most impure metal is highest. This is the side of the diagram where the metals with highest and most anisotropic interaction between the conduction electrons and the lattice are situated (Cd, Zn, etc.). It is possible that the impurities not only shorten the mean free path but also change the electronic and lattice vibration structure in a manner favourable for a greater magnetoresistance. Now the curves for $\Delta R_{\perp}/R$ and $\Delta R_{//}/R$ could be derived separately (fig. 3).

For nickel the extra internal field could not be estimated with the same accuracy as for iron, and may range from 6000 to 12000 oersteds. The reasons may be firstly that in the Kohler diagram no straight lines were obtained and secondly that the ferromagnetic orientation effect is much greater ($\pm 3\%$). Therefore we took for the additive field also $4\pi I$ (6000 gauss). The results are given in fig. 3. From this figure we see that the transversal effect in iron shows no saturation, whereas nickel behaves similarly to the odd-valent

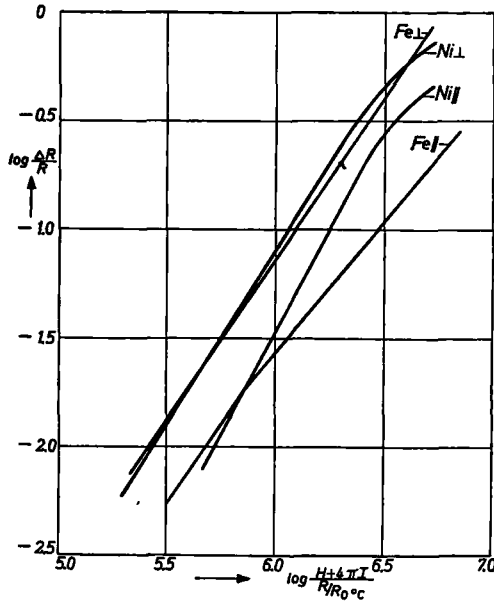


Fig. 3. Reduced Kohler diagrams for iron and nickel, both for the field longitudinal and perpendicular to the current.

metals, with the exception that the ratio between the longitudinal and the transversal effect increases with increasing field or decreasing temperature. It may be assumed that the same effective field is responsible for the Hall effect in ferromagnetic metals. Recently P u g h ³⁾ a.o. have shown that for nickel this field can be described by the formula $H_{\text{eff}} = H + a \cdot 4\pi I$. At room temperature a is about 10, but decreases rapidly with decreasing temperature. Our experiments seem to show that a converges to unity at absolute zero. It is not correct to use the value $a = 1$ at liquid nitrogen temperatures, and indeed for nickel Kohler's rule is better fulfilled by taking $a \approx 2$

at that temperature. For iron the Curie temperature is much higher and α is much closer to unity at that temperature.

The explanation for the field ($\alpha - 1$). $4\pi I$ has not yet been given. This field is undoubtedly due to the incomplete alignment of spins at finite temperatures. Samojlowski and Konjokof⁴) have tried to explain the effect by means of spin-orbit interaction, but they did not take into account explicitly the decrease of the magnetization for $T \neq 0$.

4. *Dependence of R on H.* We now proceed with the effects of purely ferromagnetic origin. If we compare the change in resistance

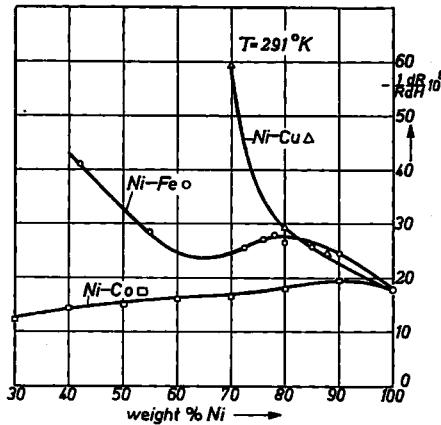


Fig. 4. Relative slope of the R, H -curves in strong fields at room temperature. The value for pure iron is -4×10^{-8} 1/oersted.

with the magnetostriction, then the linear saturation magnetostriction corresponds to the orientation effect (1), both being of a tensor dependent on the orientation of the magnetization, whereas the volume magnetostriction corresponds to the small negative value of dR/dH at strong fields, independently of the orientation. It is clear that these volume effects are due to the increase of the intrinsic magnetization with increasing field. The direct influence of the volume magnetostriction on the resistance of the sample is negligible ($1/R \cdot dR/dH < 2 \cdot 10^{-8}$ 1/oersted) so the effects observed are due to a change of the specific resistance.

At low temperatures both the volume magnetostriction and dR/dH must vanish. Fig. 4 shows $-1/R \cdot dR/dH$ at room temper-

ature for the alloys Ni-Fe, Ni-Co and Ni-Cu. The value for iron is 4×10^{-8} 1/oersted. The results for Ni-Fe agree with those obtained by B o z o r t h ⁵). In the neighbourhood of 76% Ni-Fe a superlattice occurs, whilst the rise of the Cu-curve is without doubt due to the lowering of the Curie point.

It may be assumed that R depends only on the magnetization I , or

$$dR/dH = dR/dI \cdot dI/dH. \tag{2}$$

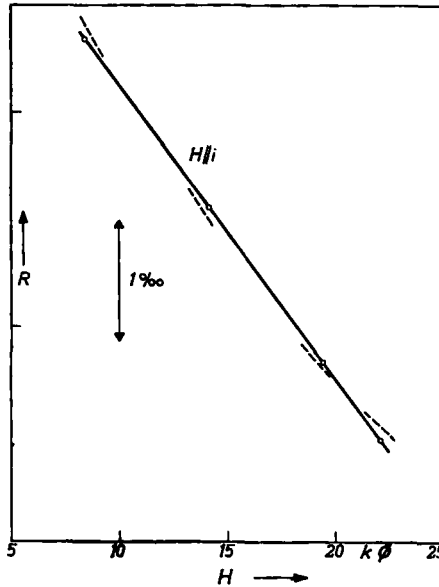


Fig. 5. R, H -curve in a longitudinal field for Ni-Fe 90—10, showing that a linear relationship exists. The dashed curve is the closest fitting parabola $R = a - b\sqrt{H}$.

A further assumption is that dR/dI does not vary appreciably with I for normal fields, so the dependence of dR/dH on H is determined by that of dI/dH , the intrinsic susceptibility. There are two theories concerning this susceptibility. A k u l o v' theory ⁶), based on the phenomenological Weiss theory, predicts a field-independent dI/dH , whereas Holstein and Primakoff ⁷) using the Bloch spin waves, including dipole-dipole interaction derive for strong fields $dI/dH \sim H^{-1/2}$. In the first case the R, H -curve should be a straight line, and in the latter a parabola. Of course all these

considerations hold only when all Weiss domains are orientated ($H > 5000$ oersteds).

It is rather difficult to decide experimentally between the two possibilities. In one case (Ni-Fe, 90—10) we succeeded in determining the field dependence of R with a sufficient accuracy (10^{-5}). The result has been plotted in fig. 5. The four points are situated exactly on a straight line, which is drawn together with the closest fitting parabola. At low temperatures the slope must vanish because saturation is reached and dI/dH consequently is zero. This was verified for alloys containing besides nickel 10—30% iron, 10—70% cobalt and 10—20% copper. At $T = 77^\circ\text{K}$, $-1/R \cdot dR/dH$ never exceeded 8×10^{-8} 1/oersted.

The alloy Ni-Fe 42—58 behaves in quite an anomalous way: $-1/R \cdot dR/dH$ at low temperatures (44×10^{-8} 1/oersted) was found to be even greater than at room temperature (41×10^{-8} 1/oersted). This might be explained by assuming that even at $T = 0^\circ\text{K}$ no complete alignment of spins exists, and consequently $dI/dH \neq 0$.

It has been shown by Néel⁸⁾ that in such a case dI/dT will not vanish at low temperatures. It is interesting to note that early measurements of Hegg⁹⁾ indicate that for this alloy dI/dT does not vanish down to liquid nitrogen temperatures.

These singularities may perhaps be accounted for by the negative exchange interaction between the spins of the iron atoms in the γ -phase. From Néel's paper it can be derived that in such a case there is a critical concentration of nickel in the Ni-Fe-range below which the orientation of the iron spins ceases to be complete, even at absolute zero. For the Ni-Fe 55—45 alloy the slope had not yet disappeared completely ($-1/R \cdot dR/dH = 13 \times 10^{-8}$ 1/oersted at 77°K and 7×10^{-8} at 20°K), so we may assume that this critical composition is about Ni-Fe 60—40. For the Ni-Cu 70—30 alloy there was also still a small slope at $T = 20^\circ\text{K}$ (11×10^{-8} 1/oersted), but this must have a different cause.

5. *The orientation effect.* For the polycrystalline samples we measured the difference between the longitudinal and the transversal resistance. This difference was always positive, in contradistinction to the magnetostriction. In fig. 6 the relative change in resistance, $(R_{//} - R_{\perp})/R$ extrapolated to $H = 0$, at $T = 20^\circ\text{K}$, has been plotted

as a function of the mean number of Bohr magnetons n_B . For R has been taken $R = \frac{1}{3}R_{//} + \frac{2}{3}R_{\perp}$.

Snoek's maximum is rather pronounced at $n_B = 1$, and only one exception occurs for Ni-Co 80—20, possibly due to the polycrystalline state of the sample.

Measurements with non-annealed samples showed always smaller values for $\Delta R/R$.

We see that the values for all alloys are nearly a function of the

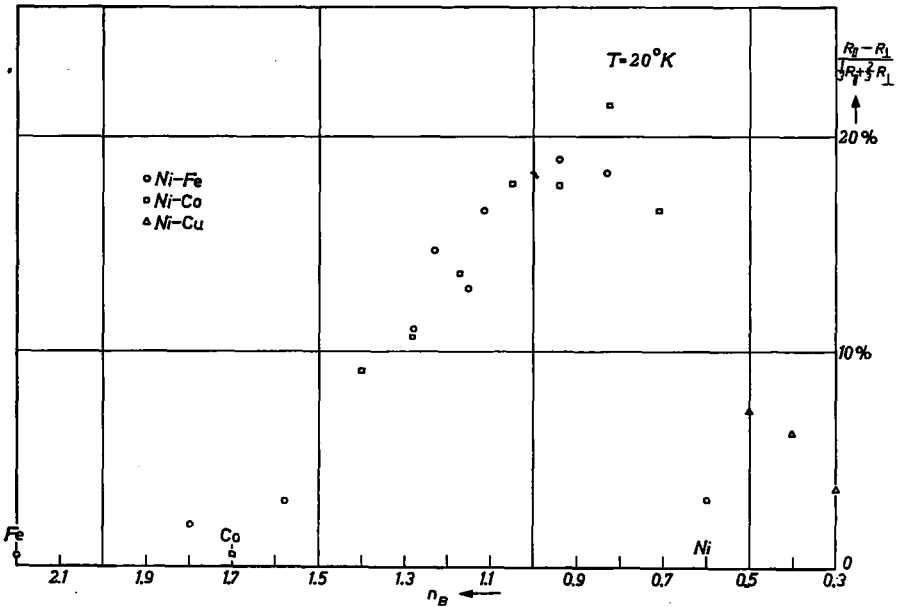


Fig. 6. The relative difference between $R_{//}$ and R_{\perp} at low temperatures as a function of the mean number of Bohr magnetons. Note the small values for pure nickel and for Ni_3Fe ($n_B = 1.2$).

Bohr magneton number only. Exceptions occur for the alloy Ni-Fe 76—24 and for the pure metals Ni and Co. For Co the value given by Bates¹⁰⁾ has been taken, assuming that $\Delta R/R$ does not vary appreciably with temperature. This was verified in the case of Ni, where the effect at 20°K was only 12% greater than at room temperature.

The behaviour of the alloys as a function of the temperature is quite different. There ΔR itself does not vary appreciably with temperature. In fig. 7 the ratios between ΔR at room temperature

and at low temperature and also those of R are given. The curves approach the values for pure nickel, which are 144 and 163 respectively.

Some measurements have been made on samples (Ni-Fe 80—20 and 90—10) having a marked texture, so it may be expected that they behave as single-crystals. We were able to measure C_3 of formula (1) at different temperatures (see fig. 8). The most interesting result is the strong decrease of $C_3/R_{T=0}$ with temperature.

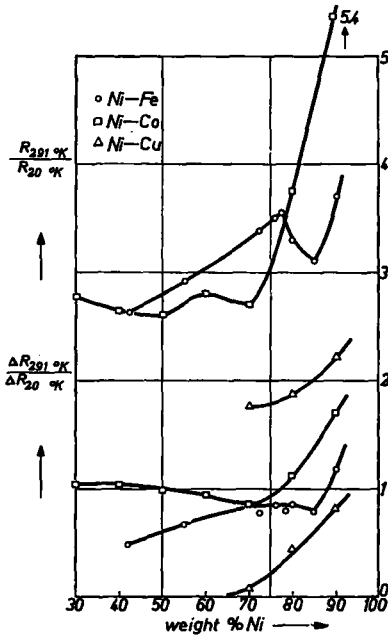


Fig. 7. The three lower curves give the ratios between the values of $R_{\parallel} - R_{\perp}$ at room temperature and at low temperature. The three upper curves the same for R itself.

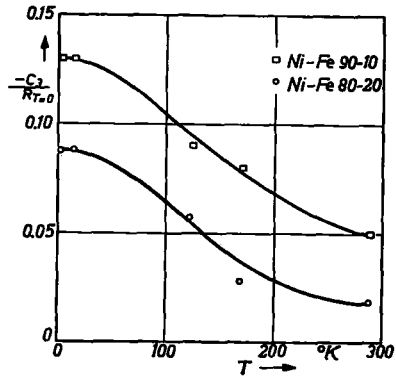


Fig. 8. The anisotropy of the orientation effect as a function of temperature.

6. *Discussion of the orientation effect.* In this section we shall outline some qualitative ideas and results of calculations which may perhaps serve to explain some of the properties of the orientation effect. Some more detailed calculations follow in section 7.

All these considerations will be based on the theory of Mott¹¹⁾ which explains the resistance-temperature curve for nickel very well.

Mott assumes that the 4s-electrons are mainly responsible for the current, and thus for the resistance, and that during a transition from an s to an s- or d-state the spin direction does not change and the spin exchange between s-electrons can be ignored. These assumptions are equivalent to dividing the current into two independent parts, one current of the s-electrons with parallel spin direction, and the other with the opposite spin. As the density of the 3d-states is very large, transitions to these states account for a great part of the resistance in the paramagnetic state. At low temperatures, however, the 3d-states of parallel spin are almost completely occupied, so the parallel 4s-electrons can jump only to other 4s-states. Hence their resistance is much smaller (Mott estimates by a factor 5) than for the antiparallel 4s-electrons, where the possibility for transitions to 3d-states is still present. Therefore at low temperatures we may assume that the resistance is determined mainly by the s-s-transitions of the electrons with parallel spin. This state of affairs may be

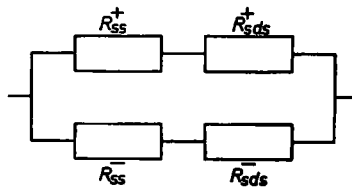


Fig. 9. Schematic circuit for the resistance of a ferromagnetic metal.

demonstrated by the scheme of fig. 9. In the paramagnetic state the two branches of the circuit are equal ($R_{sds} \approx 4R_{ss}$, R_{ss} is due to direct s-s-transitions, and R_{sds} via a d-state). At low temperatures R_{sds}^+ drops out. The influence of the magnetic field strength as discussed in section 4 can be explained as a decrease of the density at the Fermi surface of the parallel 3d-states with increasing magnetization, resulting in a decrease of the resistance (R_{sds}^+).

So far no cause for an anisotropy in the resistance is present, at a given orientation of the magnetization. In our opinion this can only be introduced by the interaction between the spin system and the lattice *via* the spin-orbit coupling. This interaction is also taken to explain the deviation of the g -factor from 2, the crystal anisotropy, and the magnetostriction. If the spin-orbit interaction operator is treated as a small perturbation, then it is shown in section 7 that it

will give some mixing of parallel and antiparallel states. As a consequence even at $T = 0^\circ\text{K}$ unoccupied parallel d -states will be present, and on general grounds it can be proved that these holes are not equally distributed over the five possible d -orbits, but that there is a deficit of hole-orbits perpendicular to the magnetization. In our scheme R_{sds}^+ does not vanish, even at absolute zero, and it is this resistance which we think to be responsible for the anisotropy.

We have now considered in section 7 the transition probability for an s -electron to a d -state, under the influence of a perturbing potential, caused by lattice imperfections or lattice vibrations. It is shown that this transition probability is smallest if the s -electron moves perpendicular to the plane of the orbit, just as would be expected from a purely classical picture. But, as we have seen that there are relatively few holes in orbits perpendicular to the magnetization, it follows that s -electrons moving in the direction of the magnetization are more easily trapped than in a transversal direction. As the resistance is mainly determined by the scattering of the electrons moving in the direction of the current, we may conclude that $R_{\parallel} > R_{\perp}$. An estimation of the order of magnitude is given in section 7, which agrees with the experimental values. Further it is shown that the anisotropy in the transition probability is largest for a disordered, rather diluted alloy, where the perturbing potential is due to the foreign ferromagnetic ions, and smaller if it is due to irregular stresses, non magnetic ions or lattice vibrations. This may explain three experimental facts:

- a. that the value for the pure metal Ni does not fit in the curve (fig. 6) for the alloys at low temperatures, but is lower;
- b. that the ordered alloys Ni_3Fe has also a lower value, for the same reason as nickel. If the alloy were completely ordered one might expect a value of the same order of magnitude as nickel;
- c. that for non-annealed samples the effect is less than for annealed ones.

It is now clear that for the alloys the ratio $\Delta R/R$ will decrease with increasing temperature, since the lattice vibrations give a smaller effect. Moreover it may be expected that the anisotropies of the two kinds of transition probabilities cancel out to some extent, as they are uncorrelated, in agreement with the experimental fact that in some cases ΔR itself decreases with increasing temperature. The relation between the magnetoresistance and the number of Bohr

magneton remains as obscure as before. In order to test these ideas we have carried out some preliminary measurements on a Heusler alloy *) (13 Al-61 Cu-26 Mn †, 1 hour at 750°C in H₂ and 48 hours at 200°C, $4\pi I_s = 8000$ gauss).

Measurements of the Barnett effect and also of the ferromagnetic resonance ¹²⁾ show that this alloy has a g -factor almost exactly equal to 2. Thus one is led to the conclusion that spin-orbit interaction is absent in this alloy. According to our view the anisotropy in the resistance should then vanish. Measurements at $T = 20^\circ\text{K}$ showed that $|(R_{//} - R_{\perp})/R|$ does not exceed a few times 10^{-4} which is much smaller than the effects for the other alloys investigated.

7. *Calculations concerning the orientation effect.* Assume that the spin-orbit interaction operator is of the form

$$A(\mathbf{L} \cdot \mathbf{S}) = A\{L_x S_x + \frac{1}{2}(L_x - iL_y)(S_x + iS_y) + \frac{1}{2}(L_x + iL_y)(S_x - iS_y)\}. \quad (3)$$

$\hbar\mathbf{L}$ is the orbital momentum operator, and S_x , S_y and S_z are the Pauli matrices. The magnetization is in the z -direction. The term $L_x S_x$ causes the g -factor to be different from 2. We are only interested in the last term with $\frac{1}{2}(S_x - iS_y)$, which gives some mixing of parallel states with an originally pure antiparallel state. If this last state were not occupied, then the parallel states, which are added, are not occupied either, as the second order energy perturbation is very small. From the fact that the operator $(L_x + iL_y)$ is not symmetrical with respect to x , y and z , it may be expected that not all orbits are mixed to the same amount. To illustrate this we shall take as zero order approximation for the $3d$ -states in the metal the atomic d -functions in a cubic field.

Owing to the degeneracy still existing in this field, this approximation is not sufficient to calculate the g -factor. The spin-orbit interaction $(L_x S_x)$ would give a zero-order perturbation, with values for g largely different from 2. Instead it is assumed that the orbits of neighbouring atoms are quenched by some antiferromagnetic coupling. For the other terms of the perturbing-energy operator we do not encounter these difficulties, as they combine only states of

*) A fuller account of these measurements will be published by Mr H. C. v a n E l s t.

†) The measurement of the magnetization has been carried out by Dr J. V o l g e r of the Philips Laboratories.

antiparallel spin orientation, which have an energy difference 2δ , δ being the exchange energy in the molecular field approximation. Thus we may expect that for our problem these atomic wave functions still give the essential features, and we omit the term $L_x S_x$.

The cubic field splits up the five-fold degeneracy into a doublet and a triplet, with an energy difference ΔE . The doublet is magnetically inactive, and if it were highest, and ΔE large enough, the difficulties mentioned above should vanish. As it may not be assumed, however, that ΔE is appreciable, we shall omit it here. The unperturbed wave functions are

$$\varphi_1 = yz f(r) \quad \varphi_2 = xz f(r) \quad \varphi_3 = xy f(r) \quad \varphi_4 = \frac{1}{2}(x^2 - y^2) f(r), \quad \varphi_5 = \frac{1}{2}3^{-\frac{1}{2}}(r^2 - 3z^2) f(r). \tag{4}$$

To a first approximation we then get for the perturbed wave functions:

$$\left. \begin{aligned} \varphi_1^- &= \varphi_1^- + (A/2\delta) (\varphi_3^+ - i\varphi_4^+ + i\sqrt{3} \varphi_5^+) \\ \varphi_2^- &= \varphi_2^- + (A/2\delta) (i\varphi_3^+ - \varphi_4^+ - \sqrt{3} \varphi_5^+) \\ \varphi_3^- &= \varphi_3^- + (A/2\delta) (\varphi_1^+ - i\varphi_2^+) \\ \varphi_4^- &= \varphi_4^- + (A/2\delta) (i\varphi_1^+ + \varphi_2^+) \\ \varphi_5^- &= \varphi_5^- + (A/2\delta) (i\sqrt{3} \varphi_1^+ + \sqrt{3} \varphi_2^+) \end{aligned} \right\} \tag{5}$$

We see that the mixing of φ_1^+ -, φ_2^+ - and φ_5^+ -states predominates over that of φ_3^+ - and φ_4^+ -states. In the mean there is a deficit of $\frac{7}{5} (A/2\delta)^2 \varphi_3^+$ -state per state.

We now proceed to calculate the transition probability from an s-electron to a φ_3 -state in a very crude manner. Taking for the s-function the free electron value, we find

$$P_{sd} \sim \left| \iiint e^{-ikr} \cdot V \cdot xy f(r) dx dy dz \right|^2, \tag{6}$$

V being the perturbing potential. Assuming that V is spherically symmetrical, (6) is proportional to

$$P_{sd} \sim k_x^2 k_y^2 g(k). \tag{7}$$

The wave functions in (6) are not orthogonal, but the ignored terms are proportional to the overlap integral and do not change the orientation properties of (7). As it may be expected that in the mean V does not possess extremely large orientation properties, (7) is the most anisotropic function which can be obtained. We see that electrons moving in the z -direction, thus parallel to the magnetization, cannot be trapped. This leads to the consequences already

reported qualitatively in section 6. In what cases can we expect a result like (7)? We think that the conditions are most favourable in an alloy with a low concentration of the second ferromagnetic element at low temperatures in the annealed state. Then the perturbing potential is only caused by the foreign ions and is practically confined to their regions. Thus only the foreign ferromagnetic ions can trap conduction electrons. The perturbing potential is then a screened Coulomb-field of the form e^{-qr}/r , which is spherically symmetrical. In the other cases of lattice distortions both thermal and static, only on the average is V spherically symmetrical and a constant term has to be added in (7) which lowers the relative anisotropy.

The evaluation of the anisotropy in the resistance is now straightforward. Assume that in the unperturbed state there is an isotropic time of relaxation τ_0 . This will be slightly changed by an anisotropic term

$$\tau(\mathbf{k}) = \tau_0(E) - \tau_1(\mathbf{k}) \quad \tau_1 \ll \tau_0. \quad (8)$$

This holds for the parallel electrons. Assuming as already done tacitly, that the energy is conserved during a transition, $\tau(\mathbf{k})$ has to satisfy

$$\int P(\mathbf{k} \mathbf{k}') \left\{ \tau(\mathbf{k}) - \frac{k'_i}{k_i} \tau(\mathbf{k}') \right\} dS' = \text{constant}, \quad (9)$$

where the integration has to be taken over the spherical Fermi surface in \mathbf{k}' -space. The index i refers to the direction of the current. From 5, 6 and 7 it follows that $P(\mathbf{k}\mathbf{k}')$ is of the form

$$P(\mathbf{k} \mathbf{k}') = P_0 + P_1 \quad P_1 \ll P_0, \quad (10)$$

with
$$P_1 \sim (k^2 + k_z^2) (k'^2 + k_z'^2) \quad (11)$$

where P_0 is due to direct s - s -scattering, and P_1 via d -states for the electrons with parallel spin orientation.

Assuming that P_0 is isotropic, to a first approximation one finds

$$\tau_1 \sim k^2 + k_z^2, \quad (12)$$

and hence
$$R_{//} \sim 1 + 4\Delta \quad R_{\perp} \sim 1 + 3\Delta, \quad (13)$$

Δ being a positive constant and approximately equal to the relative difference between the longitudinal and the transverse resistance. In the most favourable case of large Δ we see that the ratio $(R_{//} - R_{\perp})/(\frac{1}{3}R_{//} + \frac{2}{3}R_{\perp})$ has a value of 30%, and according to this

picture this may be regarded as an upper limit of the effect. The highest experimental value is about 20%. Now $10/3 \Delta$ corresponds to the mixing of $4 (A/2\delta)^2 \varphi^+$ state per φ^- -state. Assuming that the s - d - s -scattering is four times as effective as s - s -scattering, and $(A/2\delta)^2 \sim 0.02$, we find

$$\Delta = \frac{3}{10} \times 4 \times 0.02 \times 4 = 9.6\%,$$

which is of the same order as, though somewhat lower than the higher experimental values.

The subject of this paper was suggested by the late Dr. J. L. S n o e k, who also greatly stimulated this work during its early stages. Due to his untimely death on December 3rd 1950 he has not been able to see the final version of this paper.

The author wishes to express his thanks to the Director of the Kamerlingh Onnes Laboratory, Professor C. J. G o r t e r, for the hospitality kindly given in his laboratory and for his stimulating interest and advice in the problems investigated, to Professor H. B. G. C a s i m i r and Dr D. P o l d e r for their valuable discussions, to Mr H. C. v a n E l s t, phil. nat. cand., for his help during the last part of the measurements, and to the technical staff of the Kamerlingh Onnes Laboratory, especially to Mr J. v a n W e e s e l, for the technical assistance given.

Received 27-2-51.

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