S-d scattering calculation details

Of the two contributors to resistivity in transition metals, s-d scattering dominates s-s scattering.
The free s-electrons are scattered into localized d-states and remain trapped. This becomes the dominant contributor to ressitivity in 3-d metals such as Ni, Co, Fe, but not Cu.

Therefore $\rho = \rho_{ss} + \rho_{sd} \simeq \rho_{sd}$. since $\rho_{sd} \gg \rho_{ss}$

we are claiming here that s-d scattering is anisotropic

$\rho_{\parallel}^{\uparrow}$ $\xrightarrow{I}$ $\longrightarrow z$ $\xrightarrow{I\ \ M}$ $\rho_{\parallel}^{\downarrow}$	x $I\uparrow \rho_{\perp}^{\uparrow}$ $\longrightarrow z$ M $I\downarrow \rho_{\perp}^{\downarrow}$	$\rho_{\parallel}^{\uparrow} = \rho_{sd}^{\uparrow}(k_z)$ $\rho_{\parallel}^{\downarrow} = \rho_{sd}^{\downarrow}(k_z)$
$\rho_{\parallel} = \rho_{\parallel}^{\uparrow} + \rho_{\parallel}^{\downarrow}$	$\rho_{\perp} = \rho_{\perp}^{\uparrow} + \rho_{\perp}^{\downarrow}$	$\rho_{\perp}^{\uparrow} = \rho_{sd}^{\uparrow}(k_x)$ $\rho_{\perp}^{\downarrow} = \rho_{sd}^{\downarrow}(k_x)$

$$\rho_{sd}^{\uparrow}(k_z) \propto \sum_m \left| \int \phi_{m\downarrow} V(r) \overbrace{e^{ik_z z} \chi_{\uparrow}}^{S\uparrow} dv \right|^2$$

$\phi_{m\downarrow}$ are the spin-mixed d states. For example,

$$\phi_{2\downarrow} = \left(1 - \frac{\epsilon^2}{2}\right) \phi_2^{+2} \chi_{\downarrow} + \epsilon \phi_2^{+1} \chi_{\uparrow} \quad \left(\text{ignoring } \frac{3\epsilon^2}{2} \phi_2^{+1} \chi_{\uparrow} \text{ term.}\right)$$

$$\phi_2^{+2} \propto \frac{x^2 - y^2}{r^2} \; ; \quad \phi_2^{+1} \propto \frac{zx}{r^2}$$

Since $\langle \chi_{\downarrow} | \chi_{\uparrow} \rangle = 0$, for $m=2$ $\rho_{sd}^{\uparrow}(k_z) \propto \left| \int \epsilon \phi_2^{+1} V(r) e^{ik_z z} dv \right|^2$

But $\int \frac{zx}{r^2} V(r) e^{ik_z z} R(r) r^2 dr \sin\theta d\theta d\phi = 0$

$$\left[\begin{array}{l} x = r\sin\theta\cos\phi \\ z = r\cos\theta \end{array} \rightarrow \int V(r) R(r) e^{ik_z r\cos\theta} \sin^2\theta \cos\theta \, d\theta \xcancel{\int_0^{2\pi} \cos\phi \, d\phi}^{\,0} = 0 \right]$$

So $m = \pm 2$ term does not contribute to $\rho_{s\uparrow d}(k_z)$

Similarly $m = \pm 1$ term also since it has zx or zy terms

$m = 0$ term $\propto \frac{\overbrace{(x^2 + y^2 - 2z^2)}^{\rightarrow r^2 - 3z^2}}{r^2} \rightarrow \phi_{0\downarrow} = \left(1 - \frac{3}{4}\epsilon^2\right)\phi^0_{0\downarrow} + \sqrt{\frac{3}{2}}\,\epsilon\,\phi^0_{0\uparrow}$

$$\rho_{s\uparrow d}(k_z) \propto \left| e^{ik_z z} V(r) \sqrt{\frac{3}{2}}\,\epsilon\, R(r) \frac{1}{\sqrt{12}} (r^2 - 3z^2) \right|^2$$

normalization factor of Y_e^m From

$$\propto \underbrace{\frac{3}{2}\epsilon^2 \cdot \frac{4}{12} \left| e^{ikz} V(r) R(r) z^2 dv \right|^2}_{\rho'}$$

$$\boxed{\rho_{s\uparrow d}(k_z) \propto \frac{3}{2}\epsilon^2 \rho' = \rho_{11}^{\uparrow}}$$

Similarly $\rho_{s\downarrow d}(k_z)$ will have contributions from only the $d\downarrow$ band. Again only $m = 0$ term will be non-zero

$$\rho_{s\downarrow d}(k_z) \propto \left(1 - \frac{3}{4}\epsilon^2\right)^2 \left| \int e^{ik_z z} V(r) R(r) \frac{1}{\sqrt{12}} (r^2 - 3z^2)\, dv \right|^2$$

$$\left(1 - \frac{3}{4}\epsilon^2\right)^2 = 1 - \frac{3}{2}\epsilon^2 + \frac{9}{16}\epsilon^4$$

$$\simeq 1 - \frac{3}{2}\epsilon^2$$

$$\boxed{\rho_{s\downarrow d}(k_z) \propto \left(1 - \frac{3}{2}\epsilon^2\right)\rho'}$$ $\rho_{11}^{\downarrow} = \rho_{s\downarrow d}(k_z) \simeq \rho'$ as it should be.

Note when $\epsilon = 0$ $\rho_{s\uparrow d}(k_z) = 0$ and $\rho_{s\downarrow d}(k_z) = \rho'$

$\Rightarrow \rho'$ can be interpreted as the resistivity due to $s\downarrow$ to $d\downarrow$ scattering in absence of spin-orbit coupling.

$\curvearrowright$ perpendicular direction

When current is along x-direction both $m = \pm 2, 0$ is active since $\int x^2 e^{ik_x x} V(r) R(r)\, dv \neq 0$. But $m = \pm 1$ does not contribute.

So, $\rho_{s^\uparrow d}(k_x) \propto \underbrace{\left(\sqrt{\tfrac{3}{2}}\,\epsilon\right)^2 \left| \int \frac{x^2}{\sqrt{12}} e^{ik_x x} V(r) R(r)\, dv \right|^2}$

from $\phi_{1\downarrow} = \left(1 - \frac{3}{4}\epsilon^2\right)\phi^0_{1\downarrow} + \sqrt{\frac{3}{2}}\,\epsilon\, \phi^0_{0\uparrow}$

$+ \; \epsilon^2 \left| \int e^{ik_x x} V(r) R(r) \frac{1}{2\sqrt{2}} x^2 \right|^2$

from $\left(1 - \frac{1}{2}\epsilon^2\right)\phi_{1\downarrow} + \epsilon\, \phi_{-2\uparrow} = \phi_{-1\downarrow}$

$= \left(\frac{3}{2}\epsilon^2 + \frac{3}{2}\epsilon^2\right) \frac{1}{12} \left| \int e^{ik_x x} V(r) R(r) x^2\, dv \right|^2$

$= \frac{3}{4}\epsilon^2 \underbrace{\frac{4}{12} \left| \int e^{ik_x x} V(r) R(r) x^2\, dv \right|^2}_{\rho'}$

$\boxed{\rho_{s^\uparrow d}(k_x) = 3/4\, \epsilon^2 \rho'} \boxed{= \rho_\perp^\uparrow}$

from $\phi_{+2\downarrow}$

Similarly, $\rho_{s^\downarrow d}(k_x) = \left(1 - \frac{\epsilon^2}{2}\right)^2 \left| \int e^{ik_x x} V(r) R(r) \frac{1}{2\sqrt{2}} x^2\, dv \right|^2 \rightarrow \frac{3}{8}\left(1 - \epsilon^2\right)\rho'$

from $\phi_{0\downarrow}$ $\leftarrow$ $+ \left(1 - \frac{3}{4}\epsilon^2\right)^2 \left| \int e^{ik_x x} V(r) R(r) \frac{1}{\sqrt{12}} x^2\, dv \right|^2 \rightarrow \frac{1}{4}\left(1 - \frac{3}{2}\epsilon^2\right)\rho'$

from $-2\downarrow$ $\leftarrow$ $+ \left| \int e^{ik_x x} V(r) R(r) \frac{xy}{2\sqrt{2}}\, dv \right| \rightarrow \frac{3}{8}\rho'$

$\boxed{\rho_{s^\downarrow d}(k_x) = \left(1 - \frac{3}{4}\epsilon^2\right)\rho'} \boxed{= \rho_\perp^\downarrow} \Rightarrow \boxed{\rho_\perp^\downarrow \approx \rho'}$

From these eqns $\rho^\uparrow_{sd}(k_z) = \frac{3}{2}\epsilon^2\rho' = \frac{3}{4}\epsilon^2\rho' + \overbrace{\frac{3}{4}\epsilon^2}^{\gamma}\rho' = \rho_\perp^\uparrow + \gamma\rho' \approx \rho_\perp^\uparrow + \gamma\rho_\perp^\downarrow$

$\rho^\downarrow_{sd}(k_z) = \left(1 - \frac{3}{2}\epsilon^2\right)\rho' = \left(1 - \frac{3}{4}\epsilon^2\right)\rho' - \frac{3}{4}\epsilon^2\rho' = \rho_\perp^\downarrow - \gamma\rho' \approx \rho_\perp^\downarrow - \gamma\rho_\perp^\downarrow$

$\Rightarrow \boxed{\begin{aligned} \rho_\parallel^\uparrow &\approx \rho_\perp^\uparrow + \gamma\rho_\perp^\downarrow \\ \rho_\parallel^\downarrow &\approx \rho_\perp^\downarrow - \gamma\rho_\perp^\downarrow \end{aligned}} \Rightarrow \mathrm{AMR} = \gamma(\alpha - 1)$

Note $\rho' \approx \rho_\perp^\downarrow$ and $\rho' \approx \rho_\parallel^\downarrow$