The band structure of Bi with electron & holelike character has been verified in expt. (See Esaki, Stiles PRL 1965) in STM like measurements.

Concluding remarks on Normal MR effect ($\propto$ Kohler's rule)

1) It is positive for non-magnetic materials, $\propto B^2$ at low fields.
2) Apart from e^- and hole contribution, several other mechanism are present that lead to magnetoresistance (open (vs closed) Fermi surfaces)
3) Observation of non-saturating, often linear MR, at high fields is still under debate. Several topological semimetals (Weyl) also show such behavior.
4) MR is explained only for transverse case using e-hole effect.

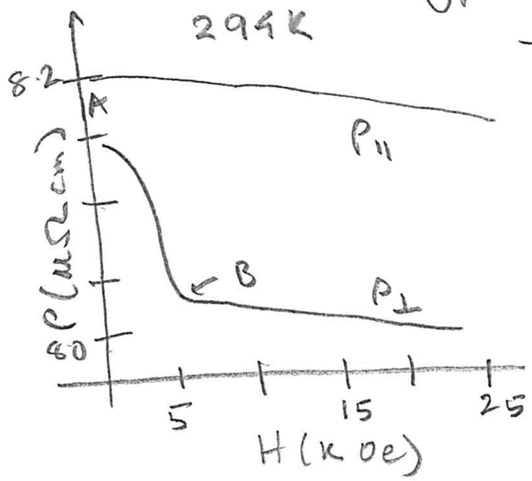
Anisotropic magnetoresistance (Magnetic materials)

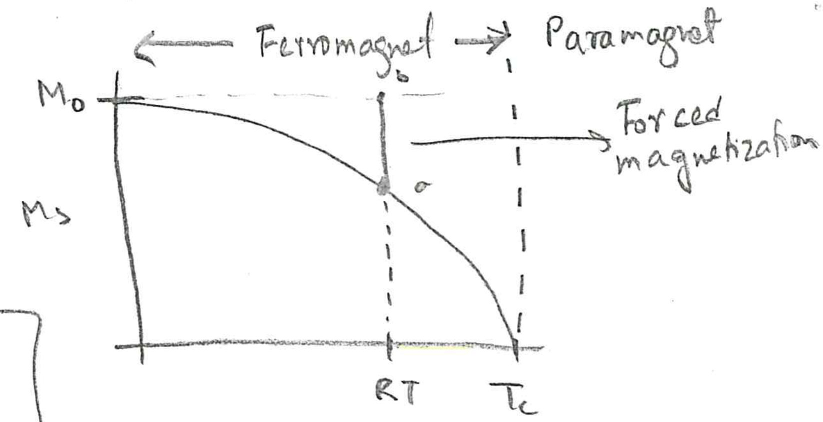
Magnetoresistance behavior in magnetic materials is strikingly different, and is typical negative ie resistivity in magnetic field is less than the value at zero field. Moreover, the resistivity when the magnetization direction is parallel to the current ($\rho_{\parallel}$) is different than the value when the magnetization vector is perpendicular to current ($\rho_{\perp}$). This difference $\Delta\rho = (\rho_{\parallel} - \rho_{\perp})$ is the anisotropic magneto-resistivity.

If we apply the current in x-direction ie $\rho_{\parallel} = \rho_{xx}$, then $\rho_{\perp} = \rho_{yy} = \rho_{zz}$, the average resistivity $= \frac{1}{3}(\rho_{xx} + \rho_{yy} + \rho_{zz})$

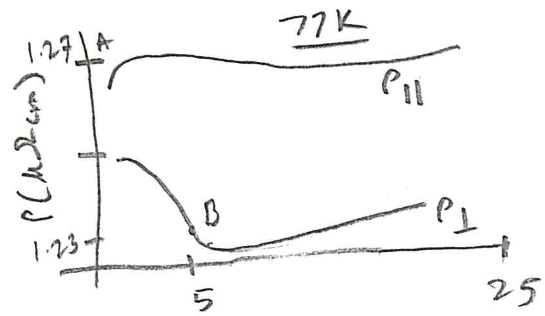
$= \frac{1}{3}\rho_{\parallel} + \frac{2}{3}\rho_{\perp} = \rho_{av}$. The AMR ratio $= \frac{\Delta\rho}{\rho_{av}} = \frac{\rho_{\parallel} - \rho_{\perp}}{\frac{1}{3}\rho_{\parallel} + \frac{2}{3}\rho_{\perp}}$

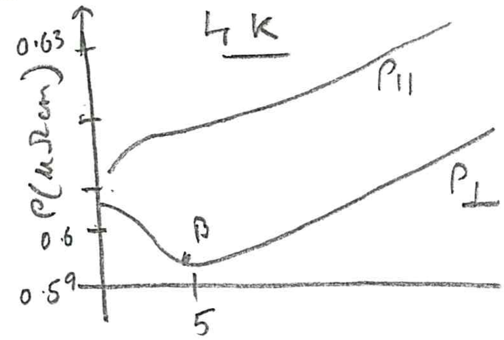
Typical AMR of bulk systems.



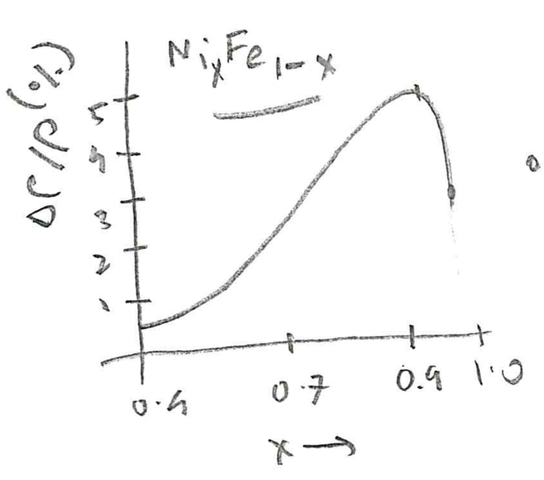


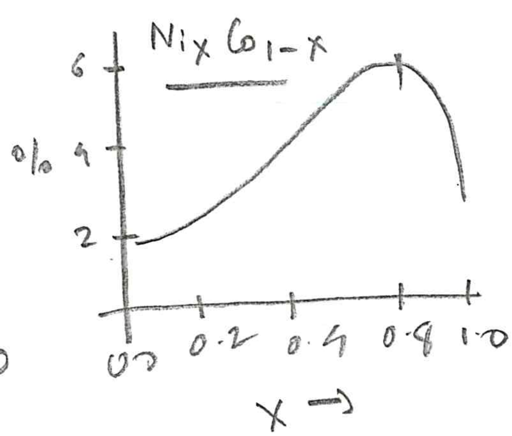
Ni





Data from McGuire and Potter (1975 IEEE)





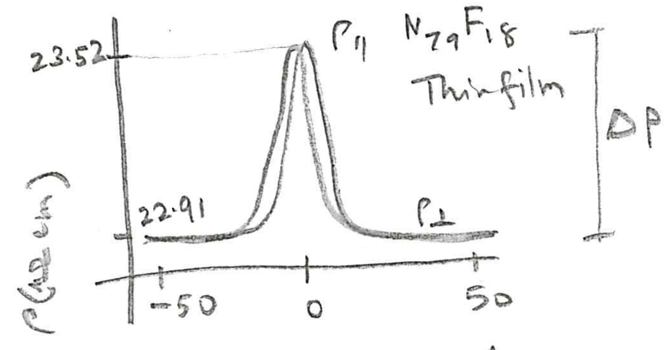
Comments

1) Beyond a particular field, both $\rho_{\parallel}$ & $\rho_{\perp}$ continues to decrease at higher fields. This is due to forced magnetization

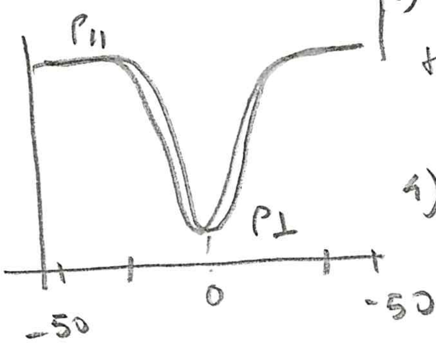
2) At low T, ordinary MR dominates, and MR is +ve

3) $\rho_{\parallel}$ is always higher than $\rho_{\perp}$, also in thin films.

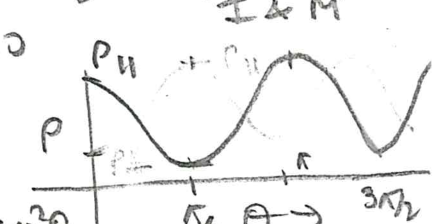
4) $\cos^2\theta$ variation
$\theta \rightarrow$ angle between I & $\vec{M}$



Transver MR plot



Longitudinal MR



$$\rho = \rho_{\perp} + (\rho_{\parallel} - \rho_{\perp})\cos^2\theta = \rho_{\perp} + \Delta\rho\cos^2\theta$$

Ordinary MR effect depends on direction and magnitude of applied field, while AMR depends on the direction of the spontaneous magnetization.

$$\frac{\Delta \rho}{\rho_{av}} = \frac{\rho_{\parallel} - \rho_{\perp}}{\frac{1}{3}\rho_{\parallel} + \frac{2}{3}\rho_{\perp}}$$

	$\frac{\Delta \rho}{\rho_{av}}$ (%) @300 K
Fe	0.2
Co	1.9
Ni	2.0
Gd	0

We can replace ρ_{av} by $\rho_{\perp}$ or $\rho_{\parallel}$ approximately.

What causes AMR ?

In the case of AMR, and any MR effect in general wrt magnetic materials, we have to consider the resistivities of up and down spin electrons separately. The justification is intuitive in the sense that the band structure of 3d ~~ele~~ ferromagnets are different due to spin-splitting (stoner model). Therefore the current $j = -nev$ can be thought of as different for up and dn spin.

$$\vec{j} = -ne(\vec{v}_{\uparrow} + \vec{v}_{\downarrow})$$

Similarly $\sigma = \sigma_{\uparrow} + \sigma_{\downarrow} = \frac{n_{\uparrow} e^2 \tau_{\uparrow}}{m} + \frac{n_{\downarrow} e^2 \tau_{\downarrow}}{m}$

or, $1/\rho = 1/\rho_{\uparrow} + 1/\rho_{\downarrow} = \frac{m}{n_{\uparrow} e^2}\frac{1}{\tau_{\uparrow}} + \frac{m}{n_{\downarrow} e^2}\frac{1}{\tau_{\downarrow}}$

Assuming $n_{\uparrow} \simeq n_{\downarrow}$ $\rho = \frac{m}{ne^2}\left(\frac{1}{\tau_{\uparrow}} + \frac{1}{\tau_{\downarrow}}\right)$ → This is analogous to the Mattheissen's law

$\frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2} + \cdots$ where ρ_1 and ρ_2 are resistivity due to various scattering mechanism such as electron-phonon, ~~et~~ lattice imperfections etc.

So $\boxed{\sigma = \sigma_{\uparrow} + \sigma_{\downarrow}}$ or $\boxed{\rho = \frac{\rho_{\uparrow}\rho_{\downarrow}}{\rho_{\uparrow} + \rho_{\downarrow}}}$

In this sense $\rho_{\parallel} = \dfrac{\rho_{\parallel}^{\uparrow}\rho_{\parallel}^{\downarrow}}{\rho_{\parallel}^{\uparrow}+\rho_{\parallel}^{\downarrow}}$ and $\rho_{\perp} = \dfrac{\rho_{\perp}^{\uparrow}\rho_{\perp}^{\downarrow}}{\rho_{\perp}^{\uparrow}+\rho_{\perp}^{\downarrow}}$

we will show that

$$\boxed{\rho_{\parallel}^{\uparrow} = \rho_{\perp}^{\uparrow} + \gamma\,\underline{\rho_{\perp}} \quad \text{and} \quad \rho_{\parallel}^{\downarrow} = \rho_{\perp}^{\downarrow} - \gamma\,\underline{\rho_{\perp}^{\downarrow}}} \quad \text{— (1)}$$

where $\gamma = \left(\lambda / H_{ex}\right)^2$ $\lambda \to$ spin-orbit interaction coefficient

$H_{ex} \to$ exchange interaction constant (J)

In Fe, Ni, Co, $\gamma \simeq 0.01$, therefore

$$\rho_{\parallel} = \frac{(\rho_{\perp}^{\uparrow} + \gamma\rho_{\perp}^{\downarrow})(\rho_{\perp}^{\downarrow} - \gamma\rho_{\perp}^{\downarrow})}{\rho_{\perp}^{\uparrow} + \rho_{\perp}^{\downarrow}} = \frac{\rho_{\perp}^{\uparrow}\rho_{\perp}^{\downarrow} - \gamma\rho_{\perp}^{\downarrow}\rho_{\perp}^{\uparrow} + \gamma\rho_{\perp}^{\downarrow}\rho_{\perp}^{\downarrow}}{\rho_{\perp}^{\uparrow} + \rho_{\perp}^{\downarrow}}$$

$$= \frac{\rho_{\perp}^{\uparrow}\rho_{\perp}^{\downarrow} - \gamma\rho_{\perp}^{\uparrow}\rho_{\perp}^{\downarrow}\left(1 - \frac{\rho_{\perp}^{\downarrow}}{\rho_{\perp}^{\uparrow}}\right)}{\rho_{\perp}^{\uparrow} + \rho_{\perp}^{\downarrow}} \quad \text{ignoring } \gamma^2 \text{ terms.}$$

$$\rho_{\parallel} - \rho_{\perp} = \frac{\gamma\rho_{\perp}^{\uparrow}\rho_{\perp}^{\downarrow}(\alpha - 1)}{\rho_{\perp}^{\uparrow} + \rho_{\perp}^{\downarrow}} = \gamma(\alpha - 1)\rho_{\perp} \quad \text{where } \alpha = \frac{\rho_{\perp}^{\downarrow}}{\rho_{\perp}^{\uparrow}}$$

$$\boxed{\frac{\Delta\rho}{\rho_{\perp}} = \gamma(\alpha - 1)} \to \text{AMR ratio}$$

$\alpha = 1$ for non-magnetic materials

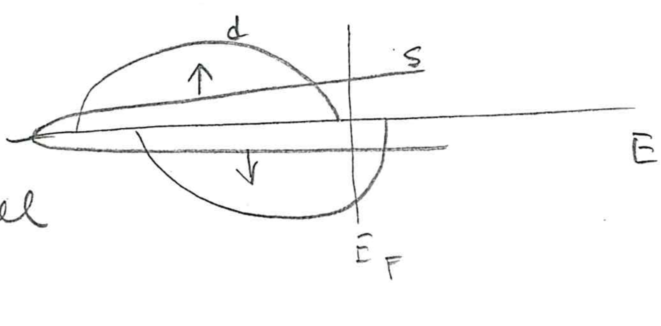
$\alpha \neq 1$ for magnetic materials.

$\alpha = \dfrac{\rho^{\downarrow}}{\rho^{\uparrow}}$ is an important parameter in AMR and GMR

$\hookrightarrow \dfrac{\Delta\rho}{\rho_0} = \dfrac{(\alpha - 1)^2}{4\alpha}$

We will now derive eqn (1) for Ni-based alloys quantum mechanically :-

1) Density of states of Ni $\longrightarrow$



At E_F $n_{d\uparrow} = 0$ ie $n_{d\uparrow}$ is full

2) Also, the d electrons are less mobile than s electrons as they have high effective mass (d-electrons are localized in space, more atomic-like)

3) As a result, the s-electrons are responsible for transport (mainly)

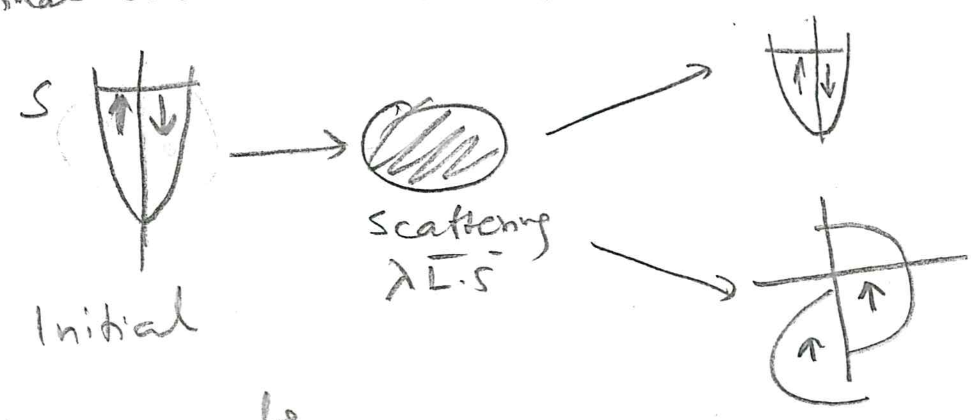
4) Quantum mechanically, if and electrons scatters from a state $|1\rangle$ to $|2\rangle$ due to a potential V, the transition probability per unit time

$$T_{1\to 2} = \frac{2\pi}{\hbar}|\langle 2|V|1\rangle|^2 g(\varepsilon_F) = \frac{1}{\tau} \quad \text{(Fermi-Golden rule)}$$

5) In our present case spin-orbit coupling $\lambda \vec{L}.\vec{S}$ is treated as the perturbation, the scattering potential

$$V = \lambda L \cdot S = \lambda (L_x S_x + L_y S_y + L_z S_z)$$

6) The initial state is a s-state (both up and down)
the final state is a down d-state or an up or down state.



3 scattering events

1) $\langle s,\downarrow|V|s,\downarrow\rangle$ (2) $\langle s,\uparrow|V|s,\uparrow\rangle$, (3) $\langle d\downarrow|V|s\downarrow\rangle$

$\langle d,\uparrow|V|s\uparrow\rangle$ is zero since there are no available $|d,\uparrow\rangle$ states at ε_F