

## chapter 9

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# TRANSPORT PROPERTIES OF FERROMAGNETS

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## Introduction

The scope of this chapter will in fact be rather more restricted than its title might suggest. We will outline some very general properties of transport in ferromagnets and will summarize the models that have been used. We will then review in detail results on particular systems. We will not treat magnetic semiconductors, and we only occasionally mention rare earth metals and their alloys, as they will be treated in another chapter\*. Because of the lack of systematic data, we will not review the properties of magnetic intermetallic compounds as such, but will mention results on these compounds as and when they exist. A section is devoted to amorphous ferromagnets. However, we will be mostly concerned with the transport properties of the transition ferromagnets Fe, Co and Ni and their alloys, while drawing examples from other classes of magnetic metals to illustrate particular types of behaviour.

The subject has advanced so much over the last 25 years that this chapter has little to do with the equivalent one in Bozorth's book. However, a number of books and review articles have been very useful; a list of them is given at the end of the chapter (Jan 1957, Mott 1964, Hurd 1974 and 1972, Dorleijn 1976).

### 1. General properties of transport in ferromagnets

We will discuss effects which arise quite generally as a consequence of the symmetry properties of the ferromagnetic state.

#### 1.1. Resistivity and Hall effect of a monodomain polycrystal

The components  $E_i$  of the electric field inside a conductor are related to the current density  $J_j$  through

$$E_i = \sum_j \rho_{ij} J_j, \quad (1)$$

\* Vol. 1, ch. 3, by Legvold.

where the  $\rho_{ij}$  coefficients form the resistivity tensor. Suppose we have a random polycrystal with its magnetization saturated in the direction  $z$ . From symmetry arguments (Birss 1964, Hurd 1974) one finds that such a magnetized isotropic medium has a resistivity tensor of the form:

$$[\rho_{ij}] = \begin{bmatrix} \rho_{\perp}(B) & -\rho_H(B) & 0 \\ \rho_H(B) & \rho_{\perp}(B) & 0 \\ 0 & 0 & \rho_{\parallel}(B) \end{bmatrix}. \quad (2)$$

This form of the resistivity tensor corresponds to the following expression of the electric field  $\mathbf{E}$ :

$$\mathbf{E} = \rho_{\perp}(B)\mathbf{J} + [\rho_{\parallel}(B) - \rho_{\perp}(B)][\boldsymbol{\alpha} \cdot \mathbf{J}]\boldsymbol{\alpha} + \rho_H(B)\boldsymbol{\alpha} \times \mathbf{J}, \quad (3)$$

where  $\mathbf{J}$  is the current vector and  $\boldsymbol{\alpha}$  is a unit vector in the magnetization direction. The  $\rho_{ij}(B)$  are functions of the induction  $B$ , which depends on the external field  $H$  and on the demagnetizing factor  $D$  of the particular sample geometry,

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}(1 - D). \quad (4)$$

This is because of Lorentz force effects which exist in any conductor. That the effective field acting on the electron trajectories in a ferromagnet is indeed  $B$  seems physically reasonable (Kittel 1963) and has been verified experimentally (Anderson and Gold 1963, Tsui 1967, Hodges et al. 1967). Conventionally, the coefficients  $\rho_{ij}$  are split up into two parts  $\rho_{ij}(B) = \rho_{ij} + \rho_{ij}^0(B)$  where the  $\rho_{ij}$  are the "spontaneous" or "extraordinary" coefficients, and the  $\rho_{ij}^0(B)$  are the "ordinary" coefficients. It should be noted that the  $\rho_{ij}$  cannot be measured directly without some form of extrapolation because of the presence of the internal Lorentz field. In practice, this extrapolation causes little difficulty except for fairly pure samples at low temperatures where  $B/\rho$  can be high and the ordinary effects become large. Assuming that this extrapolation to zero  $B$  has been made we have the three spontaneous parameters:

- $\rho_{\parallel}$  = the resistivity for  $\mathbf{J}$  parallel to  $\mathbf{M}$  at  $\mathbf{B} = 0$ ,
- $\rho_{\perp}$  = the resistivity for  $\mathbf{J}$  perpendicular to  $\mathbf{M}$  at  $\mathbf{B} = 0$ ,
- $\rho_H$  = the extraordinary Hall resistivity.

### 1.1.1. Spontaneous resistivity anisotropy

The fact that the diagonal elements  $\rho_{\parallel}$  and  $\rho_{\perp}$  in (2) are unequal means that the resistivity depends on the relative orientation of  $\mathbf{M}$  and  $\mathbf{J}$ . Taking the geometry of fig. 1 and calling  $\theta$  the angle between  $\mathbf{M}$  and  $\mathbf{J}$ , as by definition the resistivity is

$$\rho = \mathbf{E} \cdot \mathbf{J} / |\mathbf{J}|^2,$$

we have from eq. (3),

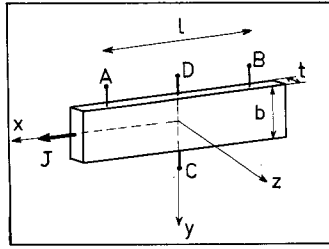


Fig. 1. Experimental geometry for transport properties. The current  $J$  is constrained to flow along the direction  $x$ , and whatever the applied field or magnetization direction, the resistivity is proportional to the voltage between probes A and B,  $\rho = (V_B - V_A)bt/Jl$ . For the conventional Hall geometry, the applied field or magnetization direction is along  $z$  and the Hall voltage is measured between C and D. Then  $\rho_H = (V_D - V_C)/J$  and  $R_0 = (V_D - V_C)t/JH$  or  $R_s = (V_D - V_C)t/J 4\pi M$ .

$$\rho_{B=0} = \frac{\rho_{\parallel} + 2\rho_{\perp}}{3} + (\cos^2 \theta - \frac{1}{3})(\rho_{\parallel} - \rho_{\perp}). \tag{5}$$

The relative spontaneous anisotropy of the resistivity is defined as:

$$\frac{\Delta\rho}{\rho} = \frac{\rho_{\parallel} - \rho_{\perp}}{\frac{1}{3}\rho_{\parallel} + \frac{2}{3}\rho_{\perp}}. \tag{6}$$

It can have either sign, and while generally values of a few percent are typical, certain systems show more than 20% anisotropy.

Figure 2 shows schematically typical resistivity changes as a function of applied field, and the method of extrapolation to obtain  $\rho_{\parallel}$  and  $\rho_{\perp}$  is indicated. Note that the resistivity of the zero field state depends on the exact domain configuration, so it is history dependent and is not well defined even for a given sample at a given temperature. In the same way, the change of resistivity with field below technical

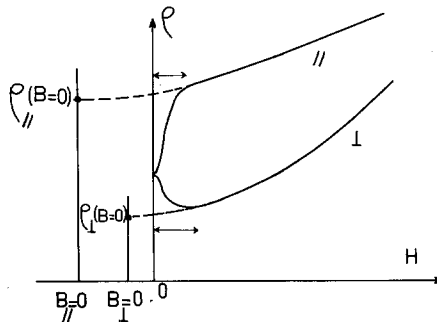


Fig. 2. Schematic extrapolation for a ferromagnetic of  $\rho(H)$  resistivity curves to  $B = 0$ . The heavy lines indicate observed resistivity as a function of field when the field is applied parallel ( $\parallel$ ) or perpendicular ( $\perp$ ) to the current direction. Arrows show the regions of incomplete technical saturation. Dotted lines indicate extrapolations from the saturated regions to the respective  $B = 0$  points ( $B = H + 4\pi M(1 - D)$  where  $D$  is the demagnetizing factor for transverse or longitudinal fields).

saturation depends on the magnetization process. We have assumed above that in the saturated region only Lorentz force effects are important for the variation of  $\rho_{\parallel}$  or  $\rho_{\perp}$  with  $B$ . In fact, the external field  $H$  may also increase the magnetization of the sample which will affect  $\rho$ , because of a reduction in spin disorder resistivity. This is particularly important near  $T_c$ .

### 1.1.2. Extraordinary Hall effect

The off-diagonal terms  $\pm\rho_H$  in (2) lead to the extraordinary Hall voltage,

$$\mathbf{E}_H(\mathbf{B} = 0) = \rho_H \boldsymbol{\alpha} \times \mathbf{J}, \quad (7)$$

perpendicular to  $\mathbf{M}$  and  $\mathbf{J}$ . This is usually measured in the conventional Hall geometry with  $\mathbf{M}$  perpendicular to  $\mathbf{J}$  (fig. 1). We can also define the extraordinary Hall angle

$$\phi_H = \rho_H / \rho_{\perp} \quad (8)$$

and, by analogy with the definition of the ordinary Hall coefficient  $R_0 = \rho_H^0 / B$  we have the extraordinary Hall coefficient

$$R_s = \rho_H / 4\pi M. \quad (9)$$

Once again, extrapolation of the Hall voltage curve as a function of  $B$  from the saturated region back to  $B = 0$  is necessary to obtain  $\rho_H$  (fig. 3). This is easy in the low field regime  $\omega_c \ll 1$  ( $\omega_c$  is the cyclotron frequency and  $\tau$  is the electronic relaxation time) but the separation between ordinary and extraordinary effects is difficult when  $\omega_c \tau \gtrsim 1$  where the ordinary Hall voltage is no longer linear in  $B$ .

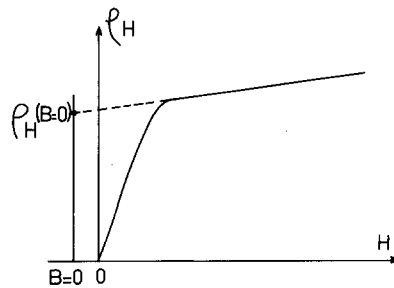


Fig. 3. Schematic extrapolation for a ferromagnet of a Hall resistivity curve to  $B = 0$ .  $\rho_H(B = 0)$  is the extraordinary Hall resistivity.

### 1.1.3. Planar Hall effect

This is a misnomer, as was already pointed out by Jan (1957). When  $\mathbf{J}$  is in the direction  $x$ , the voltage is measured perpendicular to  $\mathbf{J}$  in the direction  $y$  and  $\mathbf{M}$  is rotated in the plane  $xy$ , then from eq. (4):



$$E_y = (\rho_{\parallel} - \rho_{\perp}) \cos \theta \sin \theta J,$$

where  $\mathbf{J} \cdot \mathbf{M} = \cos \theta$ . This is a manifestation of the resistivity anisotropy and is an effect even in field which has nothing to do with the Hall effect.

1.2. Resistivity and Hall effect in single crystal ferromagnets

The symmetry arguments can be extended to single crystals (see, e.g., Hurd 1974). It is found that even for cubic crystals, the resistivity becomes dependent on the orientations of the current and the magnetization with respect to the crystal axes, and the extraordinary Hall effect depends on the magnetization direction.

For a cubic ferromagnet with  $\mathbf{M}$  in the direction  $(\alpha_1, \alpha_2, \alpha_3)$  and  $\mathbf{J}$  in the direction  $(\beta_1, \beta_2, \beta_3)$  we have the Döring expression (Döring 1938):

$$\begin{aligned} \rho = \rho_0 [ & 1 + k_1(\alpha_1^2\beta_1^2 + \alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2 - \frac{1}{3}) \\ & + k_2(2\alpha_1\alpha_2\beta_1\beta_2 + 2\alpha_2\alpha_3\beta_2\beta_3 + 2\alpha_3\alpha_1\beta_3\beta_1) + k_3(S - \frac{1}{3}) \\ & + k_4(\alpha_1^4\beta_1^2 + \alpha_2^4\beta_2^2 + \alpha_3^4\beta_3^2 + \frac{2}{3}S - \frac{1}{3}) \\ & + k_5(2\alpha_1\alpha_2\alpha_3^2\beta_1\beta_2 + 2\alpha_2\alpha_3\alpha_1^2\beta_2\beta_3 + 2\alpha_3\alpha_1\alpha_2^2\beta_3\beta_1) ], \end{aligned}$$

where  $S = \alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2$ . (10)

Other equivalent definitions have been given. Kittel and Van Vleck (1960) suggested that a physically more significant definition for the magnetoelastic coefficients (which have the same symmetry as the resistivity coefficients) would regroup the terms to give new coefficients:  $K_1 = k_1 + \frac{6}{7}k_4$ ;  $K_2 = k_2 + \frac{1}{7}k_5$ ;  $K_3 = k_3 + \frac{2}{3}k_4$ ;  $K_4 = k_4$ ;  $K_5 = \frac{2}{7}k_5$  and  $\rho'_0 = \rho_0(1 - \frac{2}{15}k_4)$ . This would separate out terms homogeneously second order ( $K_1, K_2$ ) and fourth order ( $K_3, K_4, K_5$ ) in the magnetization.

The resistivity

$$\rho = \bar{\rho} + (\rho_{\parallel} - \rho_{\perp})(\cos^2 \theta - \frac{1}{3}) \tag{11}$$

for a magnetized cubic polycrystal is related to the two types of coefficients by:

$$\bar{\rho} = \rho_0(1 - \frac{2}{15}k_3) \equiv \rho'_0$$

and

$$\rho_{\parallel} - \rho_{\perp} = \rho_0(\frac{2}{3}k_1 + \frac{3}{5}k_2 + \frac{12}{35}k_4 + \frac{3}{35}k_5) \equiv \rho_0(\frac{2}{5}K_1 + \frac{3}{5}K_2). \tag{12}$$

The resistivity of a demagnetized cubic polycrystal is

$$\rho_{\text{demag}} = \bar{\rho} \equiv \rho_0(1 - \frac{2}{15}k_3) \equiv \rho'_0 \tag{13}$$

if the magnetization is randomly oriented with respect to the crystal axes. In contrast, if there are easy axes,  $\rho_{\text{demag}}$  is different from  $\bar{\rho}$ , for example,

$$\rho_{\text{demag}} = \rho_0,$$

for (111) easy axes.

The Döring expression only gives the leading terms in an infinite expansion. Higher order terms can be necessary in interpreting experimental results.

For the extraordinary Hall effect in a cubic monocrystal, instead of defining the  $z$ -axis as the  $\mathbf{M}$  direction, we use the crystal axes to define  $x$ ,  $y$  and  $z$ . Then the asymmetric part of the resistivity tensor becomes (Hurd 1974).

$$\begin{bmatrix} 0 & -A_3 & A_2 \\ A_3 & 0 & -A_1 \\ -A_2 & A_1 & 0 \end{bmatrix} \quad \text{with} \quad A_i = \alpha_i(e_0 + e_1\alpha_1^2 + e_2\alpha_1^4 + e_3S). \quad (14)$$

This gives a Hall voltage which must be perpendicular to  $\mathbf{J}$ , but is no longer necessarily perpendicular to  $\mathbf{M}$ , and which is anisotropic.

We might note that in cubic crystals the ordinary Hall effect is isotropic in the low field limit.

For ferromagnetic monocrystals with other symmetries such as hexagonal, analogous expressions can be derived (Birss 1964, Hurd 1974).

### 1.3. Thermal and thermoelectric effects in polycrystals

The thermal conductivity and the Righi-Leduc effects are the pure thermal analogues of the resistivity and the Hall effect, with heat currents replacing electrical currents and thermal gradients replacing electrical field gradients. The phenomenological separation into spontaneous and ordinary effects carries over entirely. The only difference lies in the necessity to define whether the experimental conditions are isothermal or adiabatic (Jan 1957).

Similarly, for a magnetized ferromagnetic polycrystal subject to a temperature gradient  $\nabla T$ , the resultant electric field is

$$E_i = \sum_j S_{ij} \nabla_j T,$$

with

$$S_{ij} = \begin{bmatrix} S_{\perp} & S_{\text{NE}} & 0 \\ -S_{\text{NE}} & S_{\perp} & 0 \\ 0 & 0 & S_{\parallel} \end{bmatrix}.$$

Here,  $S_{\parallel}$  and  $S_{\perp}$  are the isothermal thermoelectric powers in parallel and perpendicular geometries.  $S_{\text{NE}}$  represents the spontaneous Nernst-Ettingshausen effect.

For both the thermal effects and the thermoelectric effects the generalization to monocrystals is just the same as for the electrical effects.

## 2. Electrical resistivity of ferromagnets

### 2.1. Theoretical models

For non-magnetic metals, current is carried by electrons which are scattered by phonons and by impurities or defects. To a first approximation the two scatterings give additive contributions to the resistivity

$$\rho(T) = \rho_0 + \rho_p(T),$$

where  $\rho_0$  is the residual resistivity and  $\rho_p(T)$  is the pure metal resistivity at temperature  $T$ . This is known as Matthiessen's rule. Deviations from this rule are small and are generally explained in terms of differences in the anisotropies of the relaxation time of impurity and of phonon scattering over the Fermi surface (Dugdale et al. 1967). In magnetic metals a number of new effects appear.

#### 2.1.1. Spin disorder scattering

On the simple model of well defined local moments in a simple conduction band, an exchange interaction between the local and conduction electron spins,  $\mathbf{J}$  and  $\mathbf{s}$  respectively, of the type  $\Gamma \mathbf{s} \cdot \mathbf{J}$  will give rise to spin disorder scattering (Kasuya 1956, 1959, De Gennes and Friedel 1958, Van Peski Tinbergen and Dekker 1963). Well above the ordering temperature there will be a temperature independent paramagnetic resistivity

$$\rho_M = \frac{k_F(m\Gamma)^2}{4\pi e^2 z \hbar^3} J(J+1), \quad (15)$$

where  $k_F$  is the Fermi wave vector and  $z$  the number of conduction electrons per atom. This paramagnetic resistivity is actually the sum of a non-spin-flip term (due to  $s_z J_z$  interactions and proportional to  $J^2$ ) and a spin-flip term (due to  $s^- J^+ + s^+ J^-$  and proportional to  $J$ ). For the region around  $T_c$  the resistivity depends on the spin-spin correlation function (see section 2.4). As  $T$  drops both the spin-flip and non-spin-flip scattering begin to freeze out. Kasuya (1956) finds

$$\rho_m = \frac{\rho_M}{J(J+1)} (J - |\langle \mathbf{J} \rangle|)(J + 1 + |\langle \mathbf{J} \rangle|). \quad (16)$$

Finally, at low temperatures magnetic scattering only remains as magnon-electron scattering. A number of authors have calculated the low temperature form of this scattering. If the spin  $\uparrow$  and spin  $\downarrow$  conduction electron Fermi surfaces are assumed to be identical spheres with the same relaxation rates, the contribution to

the resistivity is proportional to  $T^2$  (Vonsovskii 1948, Turov 1955, Kasuya 1959, Mannari 1959). Mannari (1959) finds\*

$$\rho_m = \frac{\pi^3 NJm\Gamma^2}{32 e^2 z \hbar E_F^3} \left(\frac{\mu_e}{m}\right)^2 (kT)^2, \quad (17)$$

where  $\mu_e$  is the effective mass of the magnons ( $\mu_e = \hbar^2/2D$ ). The resistivity varies as  $T^2$  because the loss of momentum of the total electron system due to collisions with the magnons is proportional to

$$\int \frac{q}{\exp(Dq^2/kT) - 1} q^2 dq,$$

which is equal to a constant times  $T^2$ . Here  $q$  is the magnon wave vector. The final  $q^2$  factor inside the integral is a small angle scattering factor, and the rest of the integrand represents the number of magnons with vector  $q$  at temperature  $T$  which undergo a collision. The reasoning is essentially the same as that leading to the well known  $T^5$  limiting dependence for electron-phonon scattering in non-magnetic metals except that:

- (i) the dispersion relation for magnons is  $E = Dq^2$  instead of  $E = \alpha q$  for phonons.
- (ii) the coupling strength for electron-magnon collisions is independent of  $q$  instead of proportional to  $q$  as it is for electron-phonon collisions.

If the conduction band is polarized or if there is scattering from s to d Fermi surfaces or if there is a magnon energy gap, the electron-magnon scattering will tend to drop off exponentially at low temperature instead of the  $T^2$  behavior (Abelskii and Turov 1960, Goodings 1963). Other mechanisms giving various temperature dependences have been also suggested (Vonsovskii 1955, Turov and Voloshinskii 1967) but appear to give extremely small contributions (see section 2.2).

The previous models generally assume that the spin  $\uparrow$  and spin  $\downarrow$  electrons have the same relaxation times and carry the same current. If there are different spin  $\uparrow$  and spin  $\downarrow$  currents, there will be a contribution of the magnons to the resistivity through spin mixing. This appears to be the major magnon contribution in many ferromagnetic alloys. This question will be treated in section 2.1.2.

Finally, in *dilute* random ferromagnets (PdFe at low concentrations is the best example of this type of system) the incoherent part of the magnon scattering – without momentum conservation – becomes predominant. The scattering rate is then simply proportional to the number of magnons giving  $\rho(T) - \rho_0 \sim T^{3/2}$  (Turner and Long 1970, Mills et al. 1971).

### 2.1.2. Two current model

All the discussion given up to now has neglected any spin independent potential. If there is a scattering potential  $V$  at magnetic sites in addition to  $\Gamma s \cdot J$ , then at

\* Note that the result of Kasuya (1959) differs from eq. (17) by a factor  $3z/\pi$ .

low temperatures the spin  $\uparrow$  and spin  $\downarrow$  electrons will be subject to potentials  $V + IJ$  and  $V - IJ$  respectively, together with weak spin-flip scattering by magnons (from  $\Gamma(s^+J^- + s^-J^+)$ ). This will give rise to different spin  $\uparrow$  and spin  $\downarrow$  currents. Alternatively, in terms of the s-d band model which is widely used for transition ferromagnets, the d  $\uparrow$  and d  $\downarrow$  densities of states at the Fermi level are different so the s to d scattering rates will be different for spin  $\uparrow$  and spin  $\downarrow$  conduction electrons. This approach was used early on to explain the  $\rho(T)$  of transition ferromagnets up to and above the Curie temperature (Mott 1936, 1964). It now appears that the simple s-d band model approach is questionable at high temperatures where the local spin aspect seems to be dominant even for a typical "band" ferromagnet such as Ni. At low temperatures however where  $s_z$  is almost a good quantum number the s-d model is more appropriate.

The electronic structure of the transition ferromagnets has been studied intensively, and low temperature measurements such as the de Haas van Alphen effect in pure metal samples show that they have complex Fermi surfaces of much the same type as those of non-magnetic transition elements except that  $k \uparrow$  and  $k \downarrow$  states are not equivalent. Detailed band structure models have been set up which are in good agreement with the measurements and which show that the electrons wave functions are s and d like, generally hybridized (Visscher and Falicov 1972). An extreme s-d model where s-d hybridization is assumed to be weak provides a good basis for the discussion of a wide range of alloy properties (Friedel 1967).

We will now consider the resistivity again. Quite generally, at low temperature the electron spin direction is well defined if we ignore magnon scattering and spin-orbit effects. Then in any model we will have conduction in parallel by two independent currents. If the corresponding resistivities are  $\rho_{\uparrow}$ ,  $\rho_{\downarrow}$  the total observed resistivity is\*:

$$\rho = \frac{\rho_{\uparrow}\rho_{\downarrow}}{\rho_{\uparrow} + \rho_{\downarrow}} \tag{18}$$

Inside each  $\rho_{\sigma}$  we can have complications such as s and d bands but eq. (18) still remains strictly valid. If now there is transfer of momentum between the two currents by spin mixing scatterings (e.g. electron-magnon or spin-orbit scattering) then again quite generally (Fert and Campbell 1976),

$$\rho = \frac{\rho_{\uparrow}\rho_{\downarrow} + \rho_{\uparrow\downarrow}(\rho_{\uparrow} + \rho_{\downarrow})}{\rho_{\uparrow} + \rho_{\downarrow} + 4\rho_{\uparrow\downarrow}} \tag{19}$$

where

$$\rho_{\uparrow} = P_{\uparrow\uparrow}/X_{\uparrow}^2 + P_{\uparrow\downarrow}/X_{\uparrow}X_{\downarrow}, \quad \rho_{\downarrow} = P_{\downarrow\downarrow}/X_{\downarrow}^2 + P_{\uparrow\downarrow}/X_{\uparrow}X_{\downarrow}, \tag{20}$$

and

\* Electrons with magnetic moment parallel to the total magnetization, i.e., electrons of the majority spin band, are indicated by  $\uparrow$ ; electrons of the minority spin band by  $\downarrow$ .

$$\rho_{\uparrow\downarrow} = -P_{\uparrow\downarrow}/X_{\uparrow}X_{\downarrow}.$$

Here  $\rho_{\sigma\sigma'}$  are integrals over the transition rates  $\rho(k\sigma, k'\sigma')$  for scattering from one electronic state to another and  $X_{\sigma}$  are integrals over driving terms:  $v_k \cdot (e \partial f_k / \partial E_k) E$  where  $E$  is the electric field,  $v_k$  the electron velocity and  $f_k$  the Fermi function (Ziman 1960).

If the occupation number of each  $k$  state under zero electric field is  $f_k^0$ , then we can write the occupation number under applied field as

$$f_k = f_k^0 - \phi_k(df_k^0/dE_k). \quad (21)$$

We can use the variational principle, using  $k \cdot u$  as a trial function for the function  $k$ , where  $u$  is a unit vector parallel to the applied field. We then get:

$$\rho_{\sigma} = \frac{1}{X_{\sigma}^2 k_B T} \sum_{\sigma'} \int (k \cdot u)[(k - k') \cdot u] P(k\sigma, k'\sigma') dk dk' \quad (22)$$

$$\rho_{\uparrow\downarrow} = \frac{1}{X_{\uparrow}X_{\downarrow}k_B T} \int (k' \cdot u)(k \cdot u) P(k_{\uparrow}k'_{\downarrow}) dk dk', \quad (23)$$

where  $P(k\sigma, k'\sigma')$  is the equilibrium scattering rate between  $(k\sigma)$  and  $(k'\sigma')$ . Matthiessens' rule is assumed for each of the three terms

$$\rho_{\sigma} = \sum_x \rho_{\sigma}^x, \quad \rho_{\uparrow\downarrow} = \sum_x \rho_{\uparrow\downarrow}^x,$$

where  $x$  refers to different types of scattering centres (phonons, magnons, each sort of impurity). There are temperature independent "pure metal" terms  $\rho_{i\uparrow}(T)$ ,  $\rho_{i\downarrow}(T)$  and  $\rho_{\uparrow\downarrow}(T)$  which go to zero at zero temperature and are assumed to be independent of impurity concentration in dilute alloys, and there are impurity terms which are assumed to be temperature independent. The spin-flip scattering by impurities (via spin-orbit coupling) is generally neglected, which leaves two impurity terms  $\rho_{0\uparrow}$  and  $\rho_{0\downarrow}$ .

The model then predicts a deviation from Matthiessens' rule in the residual resistivity of ternary alloys (Fert and Campbell 1976):

$$\Delta\rho = \rho_{AB} - (\rho_A + \rho_B) = \frac{(\alpha_A - \alpha_B)^2 \rho_A \rho_B}{(1 + \alpha_A)^2 \alpha_B \rho_A + (1 + \alpha_B)^2 \alpha_A \rho_B}, \quad (24)$$

where

$$\alpha_A = \rho_{A\downarrow}/\rho_{A\uparrow} \quad \text{and} \quad \alpha_B = \rho_{B\downarrow}/\rho_{B\uparrow}.$$

The analysis of the deviations in alloys with different relative concentrations of A and B can be used to determine  $\alpha_A$ ,  $\alpha_B$  (Campbell et al. 1967, Cadeville et al.

1968, Dorleijn and Miedema 1975a, Fert and Campbell 1976, Dorleijn 1976).

For a binary alloy at finite temperature the general equation (19) can be used, with

$$\rho_{\sigma} = \rho_{0\sigma} + \rho_{i\sigma}(T).$$

For the low temperature range where

$$\rho_{0\uparrow}, \rho_{0\downarrow} \gg \rho_{i\uparrow}(T), \rho_{i\downarrow}(T), \rho_{\uparrow\downarrow}(T),$$

this reduces to:

$$\rho(T) = \rho_0 \left[ 1 + \frac{(\alpha - \mu)^2}{(1 + \alpha)^2} \right] \rho_i(T) + \left( \frac{\alpha - 1}{\alpha + 1} \right)^2 \rho_{\uparrow\downarrow}(T), \quad (25)$$

where

$$\mu = \frac{\rho_{i\downarrow}(T)}{\rho_{i\uparrow}(T)},$$

$$\rho_0 = \frac{\rho_{0\uparrow}\rho_{0\downarrow}}{\rho_{0\uparrow} + \rho_{0\downarrow}} \quad (\text{residual resistivity})$$

and

$$\rho_i(T) = \frac{\rho_{i\uparrow}(T)\rho_{i\downarrow}(T)}{\rho_{i\uparrow}(T) + \rho_{i\downarrow}(T)} \quad (\text{which is not the ideal pure metal resistivity}).$$

The term of eq. (25) proportional to  $\rho_{\uparrow\downarrow}(T)$  will give a strong variation of the resistivity as a function of the temperature when  $\alpha$  is very different from unity; in nickel, for example, Co, Fe or Mn impurities enhance the low temperature resistivity variation by almost an order of magnitude. The form of the electron-magnon contribution to  $\rho_{\uparrow\downarrow}(T)$  has been calculated by Fert (1969) and Mills et al. (1971) using a spin-split spherical conduction band model. In alloys with  $\alpha \approx 1$  the temperature dependence of the resistivity will nearly be that of  $\rho_i(T)$ .

Using experimental data on binary and ternary alloys, it turns out to be possible to obtain consistent values for the parameters for a number of impurities, together with estimates of the temperature behaviour of the pure metal terms (see section 2.3). It is important to note that it is not possible to obtain a full description of the pure metal behaviour, i.e., the three pure metal terms, without analyzing alloy data.

Another way of treating the two current conduction has been presented by Yamashita and Hayakawa (1976). They start from a realistic band structure model for Ni and calculate the resistivity by numerically solving coupled spin  $\uparrow$  and spin  $\downarrow$  Boltzmann equations for series of  $k$  vectors. They find that the electron-magnon contribution to the resistivity is very small when there is no impurity or

phonon scattering but becomes important when impurity or phonon scattering makes the spin  $\uparrow$  and spin  $\downarrow$  mean free paths different; in the latter case, there is no more "cancellation between outgoing and incoming scatterings". This is another way of describing the spin-mixing effect of the magnons.

## 2.2. Resistivity of pure metals

### 2.2.1. Tabular results

The resistivity (and also the thermoelectric power and thermal conductivity) of pure Fe, Co and Ni are given in tabular form over a wide range of temperatures by Laubitz et al. (1973, 1976) and Fulkerson et al. (1966).

Data on polycrystalline hexagonal Co should be treated with caution, as the transport properties of monocrystals are highly anisotropic. At room temperature (Matsumoto et al. 1966)

$$\rho_c = 10.3 \mu\Omega\text{cm}, \quad \rho_p = 5.5 \mu\Omega\text{cm}$$

where  $\rho_c$  is the resistivity measured along the  $c$  axis while  $\rho_p$  is measured in the plane perpendicular to the  $c$  direction. Texture effects in polycrystals will certainly be important.

### 2.2.2. Resistivity at low temperatures

With high purity samples at low temperatures the presence of the induction in each ferromagnetic domain means that the Lorentz  $\omega_c\tau$  is not negligible even in zero applied field. There is an associated "internal" magnetoresistance and to get meaningful results for the intrinsic low field resistivity this effect must be eliminated as well as possible. Careful extrapolations to  $B = 0$  using Kohlers' law  $\Delta\rho/\rho_0 = f(B/\rho_0)$  have been done for Ni (Schwerer and Silcox 1968), Co (Volkenshtein et al. 1973) and Fe (Volkenshtein and Dyakina 1971). For Fe in particular the "internal" magnetoresistance is very important, partly because of the high value of  $4\pi M$  and partly because Fe behaves as a compensated metal (see section 2.2.4) so that the transverse magnetoresistance can become very strong, whereas the longitudinal magnetoresistance is relatively much weaker. This means that the observed resistivity of a high purity Fe sample at low temperature is strongly dependent on the domain configuration, which regulates how much of the sample is submitted to transverse magnetization and how much to longitudinal; the domain configuration is a function of applied field, stresses and measuring current. The resistivity behaviour arising from such effects of internal magnetoresistance has been studied in detail in Fe whisker monocrystals (Taylor et al. 1968, Shumate et al. 1970, Berger 1978); an example of experimental results is given in fig. 4. A contribution to the resistivity from the internal Hall effect has also been found in Co monocrystals; this contribution is associated with the zig-zag path of the conduction electrons which is induced by the Hall effect in a polydomain sample (Ramanan and Berger 1978, Berger 1978).

The low temperature resistivity of Ni, Co and Fe has been found to vary as



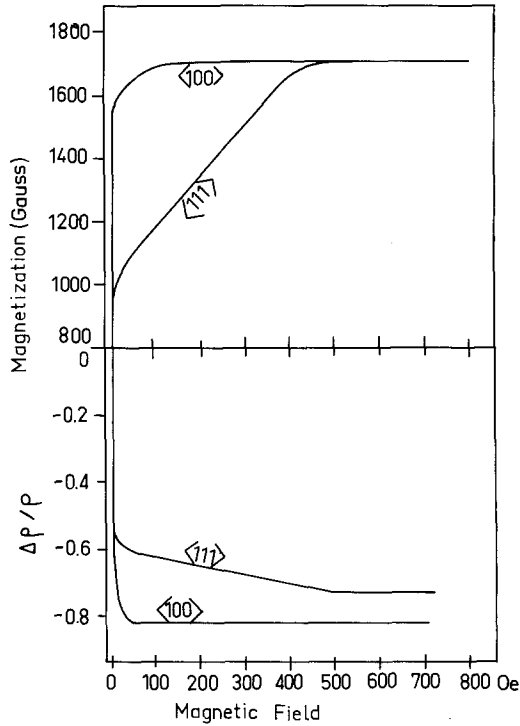


Fig. 4. Magnetization of iron single crystals (above) and magnetoresistance (below) of (100) and (111) iron whiskers at 4.2 K as a function of applied field (after Taylor et al. 1968).

$\rho_0 + AT^2$  by most workers; above 10 K an additional term in  $T^4$  is generally needed (White and Woods 1959, Greig and Harrison 1965, White and Tainsh 1967, Schwerer and Silcox 1968, Beitcham et al. 1970). In early work no magnetoresistance corrections were made, with the result that rather varied values of  $A$  were obtained. In a number of samples a term linear in  $T$  was also needed, but careful experiments on the effect of magnetic fields show that this linear term is not intrinsic but is a result of the internal magnetoresistance (Volkenstein et al. 1971, 1973).

The best values of  $A$  from data to which the magnetoresistance correction was applied to obtain values at  $B = 0$  are:

$A = 9.5 \times 10^{-12} \Omega\text{cmK}^{-2}$  for Ni (Schwerer and Silcox 1968)

$A = 16 \times 10^{-12} \Omega\text{cmK}^{-2}$  for Co (Volkenstein et al. 1973)

$A = 15 \times 10^{-12} \Omega\text{cmK}^{-2}$  for Fe (Volkenstein et al. 1971, 1973).

These values however are not quite consistent with measurements on other samples, even if internal magnetoresistance effects are taken into account. It is likely that  $\rho(T)$  depends in some way on the nature of the residual impurities. As the scattering by residual impurities is still predominant up to 10 K in the purest samples,  $\rho(T)$  is actually expected to show deviations from Matthiessens' rule similar to those observed in alloys and explained in the two-current model

(section 2.3). More precisely, the two-current model relates  $\rho(T)$  to the parameter  $\alpha$  characteristic of the impurity scattering (eq. (25)). According to whether  $\alpha$  is large or close to unity  $\rho(T) - \rho_0$  is large and varying as  $\rho_{\uparrow\downarrow}(T)$  or small and varying as  $\rho_i(T)$ . The values of  $\rho(T) - \rho_0$  for pure metals, although scattered, are relatively small, which suggest that  $\alpha$  is generally close to unity. The variation in  $T^2$  can be then ascribed to  $\rho_i(T)$ .

The variation in  $T^2$  has been attributed either to electron-magnon scattering or to s-d electron-electron scattering (Baber 1937) of the same type as leads to a  $T^2$  term in the resistivity of non-magnetic transition metals at low temperatures, and which is much the same magnitude as the  $T^2$  term in the ferromagnets. If the thermal conductivity of the ferromagnetic metals is also measured at low temperatures the Lorentz ratio corresponding to the non-impurity scattering is about  $1 \times 10^{-8} \text{ W}\Omega\text{K}^{-2}$  both in Ni (White and Tainsh 1967) and in Fe (Beitcham et al. 1970). This is close to the value estimated theoretically (Herring 1967) for s-d electron-electron scattering. However the way in which the experimental data are analyzed has been criticized (Farrel and Greig 1969). Secondly, we can consider data on  $\rho_{i\uparrow}(T)$ ,  $\rho_{i\downarrow}(T)$  and  $\rho_{\uparrow\downarrow}(T)$  obtained from an analysis of dilute Ni alloys (see section 2.3). The spin-flip magnon-electron resistivity  $\rho_{\uparrow\downarrow}(T)$  is roughly  $5 \times 10^{-9} \Omega\text{cm}$  at 10 K in Ni (see fig. 9); because of the small angle scattering factor the electron-magnon contributions to  $\rho_{i\uparrow}(T)$ ,  $\rho_{i\downarrow}(T)$  should be lower by a factor of the order of  $T/T_c$ , giving  $\rho_{i\sigma}$  (magnons)  $\sim 10^{-10} \Omega\text{cm}$ ; as the "observed"  $\rho_{i\sigma}$  values in Ni at 10 K are much higher (these are higher than  $10^{-9} \Omega\text{cm}$ ), we can infer that the electron-magnon contributions to the  $\rho_{i\sigma}$  are not dominant. This actually is in agreement with predictions of calculations based on a realistic band structure model of Ni (Yamashita et al. 1975). It thus turns out that electron-electron collisions play the major role in the low temperature  $T^2$  term of the pure metals.

### 2.2.3. Residual resistivity

In pure ferromagnetic metals the "internal magnetoresistance" enhances the resistivity which is no longer proportional to the concentration of impurities. This effect is particularly important for Fe which has a high value of  $4\pi M$  and a high transverse magnetoresistance (fig. 5). For demagnetized Fe polycrystals it was pointed out that the apparent residual resistivity ratio  $\rho(300 \text{ K})/\rho(4.2 \text{ K})$  would never increase beyond about 300 however pure the sample (Berger and de Vroomen 1965). It is now standard practice to measure the low temperature resistivity of Fe samples in a saturating longitudinal magnetic field so as to eliminate transverse magnetoresistance. This can reduce the apparent resistivity by a factor of 5 or more. In principle a correction should still be made for the longitudinal magnetoresistance. In Ni samples the enhancement of the residual resistivity by the internal magnetoresistance is less important than in Fe but still significant (Fujii 1970, Schwerer and Silcox 1970).

The contribution of the domain walls to the residual resistivity of ferromagnetic metals has been subject to many discussions. It now appears that domain walls are too thick to scatter electrons appreciably. However, as it has been pointed out in

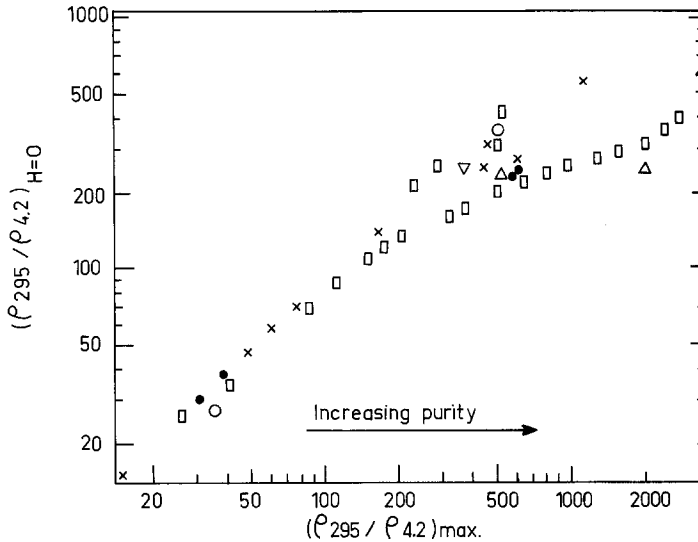


Fig. 5. Residual resistance ratio in zero field and in longitudinal applied field for iron samples of increasing purity (after Berger 1978).

the preceding section, the resistivity depends indirectly on the domain walls as it depends on the domain configuration in the sample (Berger 1978).

#### 2.2.4. High field behaviour

It is well known that the magnetoresistance and Hall effect of pure metals under high fields such that  $\omega_c\tau \gg 1$  give information on the Fermi surface.

In Ni under high fields applied along a non-symmetry direction of a monocrystal, the transverse magnetoresistance saturates, and  $R_0$  corresponds to an effective carrier density of 1 electron per atom, even though Ni has an even number of electrons and so would normally be expected to behave as a compensated metal. Reed and Fawcett (1964a) showed that a ferromagnetic metal did not have to obey the same rules as non-magnetic metals because of the inequivalence of spin  $\uparrow$  and spin  $\downarrow$ . They deduced from their results that the minority d band in Ni was electron-like in character. The behaviour of the magnetoresistance for certain field directions indicated the presence of open orbits for certain field orientations. The results could be compared with de Haas–Van Alphen data (Hodges et al. 1967, Tsui 1967, Ruvalds and Falicov 1968). No obvious transition corresponding to a major difference in mobility for d-like and s-like parts of the Fermi surface was observed.

In Co the transverse magnetoresistance is again saturated (Coleman et al. 1973) with open orbit behaviour for certain special directions.

In Fe up to fields of about 100 kG, the magnetoresistance tends to a  $B^2$  dependence indicating that the metal is compensated (Reed and Fawcett 1964b). There appears to be a considerable spread of  $\omega_c\tau$  values. In the same field range at low temperatures the ordinary Hall coefficient  $R_0$  is strong, negative and

weakly field dependent (Klaffky and Coleman 1974). This also is consistent with a metal having compensated character, for which  $R_0$  bears no relationship to any effective number of electrons per atom.

At still higher values of  $\omega_c\tau$  the magnetoresistance increases much more slowly than  $B^2$  (Coleman 1976); magnetic breakdown and intersheet scattering have been invoked.

### 2.3. Alloys: residual resistivity and temperature dependence of resistivity

The residual resistivity per atomic percent impurity has been measured for a wide range of impurities in Ni, Co and Fe ( $\rho_0$  in tables 1, 2, 3) and the deviations from Matthiessens' rule have been studied both for ternary alloys (fig. 6) and for binary alloys as a function of temperature. Using the two current model equations of section 2.1.2 the experimental data have been used to determine the spin  $\uparrow$  and spin  $\downarrow$  residual resistivities  $\rho_{0\uparrow}$  and  $\rho_{0\downarrow}$  for each impurity in each host (tables 1, 2, 3).

TABLE 1  
Values of  $\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$ ,  $\rho_0$ ,  $\rho_{0\uparrow}$ ,  $\rho_{0\downarrow}$  for dilute impurities in nickel\*.

Impurity in nickel	$\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$	$\rho_0$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\uparrow}$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\downarrow}$ ( $\mu\Omega\text{cm}$ )
Co	13 <sup>(a)</sup> , 30 <sup>(b)</sup> , 20 <sup>(c)</sup> , 13 <sup>(d)</sup> , 20 <sup>(f)</sup> , 20 <sup>(g)</sup>	$0.16 \pm 0.03$	$0.18 \pm 0.03$	$3.5 \pm 1$
Fe	11 <sup>(a)</sup> , 20 <sup>(b)</sup> , 7.3 <sup>(d)</sup>	$0.36 \pm 0.04$	$0.4 \pm 0.04$	$6 \pm 1.5$
Mn	6.3 <sup>(a)</sup> , 15 <sup>(b)</sup> , 5.4 <sup>(d)</sup>	$0.7 \pm 0.1$	$0.75 \pm 0.2$	$7.5 \pm 2.5$
Cr	0.21 <sup>(a)</sup> , 0.45 <sup>(b)</sup> , 0.4 <sup>(c)</sup> , 0.21 <sup>(d)</sup> , 0.2 <sup>(f)</sup> , 0.4 <sup>(g)</sup>	$5 \pm 0.1$	$22 \pm 6$	$6.5 \pm 0.5$
V	0.45 <sup>(a)</sup> , 0.55 <sup>(b)</sup> , 2.3 <sup>(d)</sup>	$4.4 \pm 0.2$	$13 \pm 1$	$6.7 \pm 0.5$
Ti	0.9 <sup>(a)</sup> , 4 <sup>(b)</sup> , 2.7 <sup>(d)</sup>	$3.3 \pm 0.6$	$5.6 \pm 2$	$10.5 \pm 4$
Pd	1 <sup>(d)</sup>	$0.2 \pm 0.05$	0.3 <sup>(d)</sup>	0.3 <sup>(d)</sup>
Rh	0.3 <sup>(a)</sup> , 0.17 <sup>(c)</sup> , 0.29 <sup>(f)</sup>	$1.8 \pm 0.1$	$10 \pm 2$	$2.1 \pm 0.2$
Ru	0.075 <sup>(a)</sup> , 0.15 <sup>(c)</sup>	$4.8 \pm 0.2$	$56 \pm 15$	$5.8 \pm 0.5$
Mo	0.28 <sup>(c)</sup> , 0.37 <sup>(f)</sup>	$6.4 \pm 0.6$	$25 \pm 4$	$8 \pm 1$
Nb	0.44 <sup>(c)</sup> , 0.47 <sup>(f)</sup>	$5 \pm 0.2$	$16 \pm 1$	$7 \pm 0.2$
Zr	7.5 <sup>(c)</sup>	$2.8 \pm 0.5$	4 <sup>(c)</sup>	30 <sup>(c)</sup>
Pt	0.24 <sup>(a)</sup> , 0.17 <sup>(c)</sup>	$0.85 \pm 0.2$	$5.3 \pm 1.6$	$1 \pm 0.2$
Ir	0.24 <sup>(a)</sup> , 0.13 <sup>(c)</sup>	$3.8 \pm 0.2$	$28 \pm 7$	$4.8 \pm 0.2$
Os	0.13 <sup>(c)</sup> , 0.13 <sup>(f)</sup>	$5.5 \pm 0.5$	$50 \pm 2$	$6.4 \pm 0.5$
Re	0.3 <sup>(a)</sup> , 0.26 <sup>(c)</sup>	$5.8 \pm 0.5$	$26 \pm 3$	$7.5 \pm 0.5$

TABLE 1 (continued)

Impurity in nickel	$\alpha = \rho_{0\downarrow} / \rho_{0\uparrow}$	$\rho_0$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\uparrow}$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\downarrow}$ ( $\mu\Omega\text{cm}$ )
W	0.4 <sup>(e)</sup> , 0.5 <sup>(f)</sup>	6 ± 0.5	16.5 ± 1	7 ± 0.5
Ta	0.53 <sup>(e)</sup> , 0.46 <sup>(f)</sup>	5.2 ± 0.5	16 ± 1	7.5 ± 0.5
Hf	8.6 <sup>(e)</sup> , 8.1 <sup>(f)</sup>	3.6 ± 0.5	3.5 ± 0.5	30 ± 1
Cu	2.9 <sup>(a)</sup> , 3.7 <sup>(d)</sup>	0.9 ± 0.1	1.1 ± 0.2	3.7 ± 0.2
Au	5.9 <sup>(a)</sup>	0.36 <sup>(a)</sup>	0.44 <sup>(a)</sup>	2.6 <sup>(a)</sup>
Al	1.7 <sup>(a)</sup>	2.13 <sup>(a)</sup>	3.4 <sup>(a)</sup>	5.8 <sup>(a)</sup>
Si	1.3 <sup>(a)</sup>	2.83 <sup>(a)</sup>	5 <sup>(a)</sup>	6.4 <sup>(a)</sup>
Zn	2.2 <sup>(a)</sup>	1 ± 0.1	1.3 <sup>(a)</sup>	2.9 <sup>(a)</sup>
Ga	1.7 <sup>(g)</sup>	1.91 <sup>(g)</sup>	3 <sup>(g)</sup>	5.2 <sup>(g)</sup>
Ge	1 <sup>(g)</sup>	2.84 <sup>(g)</sup>	5.7 <sup>(g)</sup>	5.7 <sup>(g)</sup>
In	1.5 <sup>(h)</sup>	3.6 <sup>(h)</sup>	6 <sup>(h)</sup>	9 <sup>(h)</sup>
Sn	1.6 <sup>(a)</sup> , 1.35 <sup>(h)</sup>	3.2 ± 0.4	5.2 ± 0.8	7.7 ± 0.5
Sb	0.8 <sup>(h)</sup>	1.6 <sup>(h)</sup>	3.6 <sup>(h)</sup>	2.9 <sup>(h)</sup>

\* For  $\alpha$  we give the values found by:

<sup>(a)</sup> Dorleijn and Miedema (1975a), Dorleijn (1976);

<sup>(b)</sup> Fert and Campbell (1976);

<sup>(c)</sup> Leonard et al. (1969);

<sup>(d)</sup> Farrell and Greig (1969);

<sup>(e)</sup> Durand and Gautier (1970);

<sup>(f)</sup> Cadeville et al. (1968);

<sup>(g)</sup> Hugel (1973);

<sup>(h)</sup> Ross et al. (1978);

<sup>(i)</sup> Durand (1973).

The values of  $\alpha$  given in (a), (b), (f), (g) have been mostly derived from measurements of the residual resistivity of ternary alloys, which is the most direct method. The values of  $\alpha$  given in (d), (e), (h), (i) have been obtained on binary alloys from the deviations from Matthiessen's rule at low temperature (h), 77 K (i), 300 K with the assumption of complete spin mixing (d) or 300 K with the assumption of no spin mixing (e). The values that we give for  $\rho_0$ ,  $\rho_{0\uparrow}$ ,  $\rho_{0\downarrow}$  have been estimated from the spread of the values found in the literature.

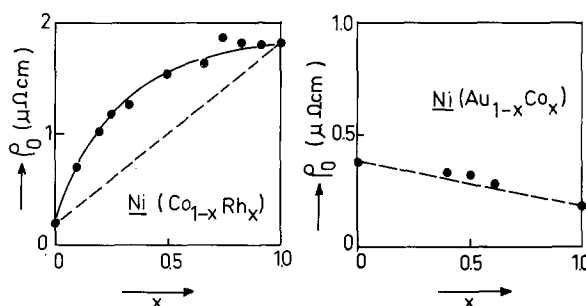


Fig. 6. Residual resistivities of  $\text{Ni}(\text{Co}_{1-x}\text{Rh}_x)$  and  $\text{Ni}(\text{Au}_{1-x}\text{Co}_x)$  alloys. The large deviations from Matthiessen's rule (broken line) for the  $\text{Ni}(\text{Co}_{1-x}\text{Rh}_x)$  alloys are accounted for by very different values of  $\alpha_{\text{Co}}$  and  $\alpha_{\text{Rh}}$ ; the solid curve is calculated from eq. (24) with  $\alpha_{\text{Co}} = 13$  and  $\alpha_{\text{Rh}} = 0.3$ . The very small deviations from MR in the  $\text{Ni}(\text{Au}_{1-x}\text{Co}_x)$  are associated with values of  $\alpha_{\text{Au}}$  and  $\alpha_{\text{Co}}$  both much larger than 1 (after Dorleijn 1976).

TABLE 2  
 Values of  $\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$ ,  $\rho_0$ ,  $\rho_{0\uparrow}$ ,  $\rho_{0\downarrow}$  for dilute impurities in cobalt\*.

Impurity in cobalt	$\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$	$\rho_0$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\uparrow}$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\downarrow}$ ( $\mu\Omega\text{cm}$ )
Fe <sup>(a)</sup>	12	0.5	0.54	6.7
Mn <sup>(b)</sup>	0.8	5.5	12	10
Cr <sup>(b)</sup>	0.3	1.8	7.3	2.4
V <sup>(b)</sup>	1	3.8	7.7	7.7
Ti <sup>(b)</sup>	1.4	4.5	7.6	11
Rh <sup>(b)</sup>	1	1.4	2.8	2.8
Ru <sup>(a)</sup>	0.22	4.0	22.4	4.86
Mo <sup>(b)</sup>	0.7	6.0	14.4	10
Nb <sup>(b)</sup>	1	6.5	13	13
Zr <sup>(b)</sup>	3.3	4.0	5.2	17
Ir <sup>(a)</sup>	0.33	2.9	11.7	3.82
Os <sup>(a)</sup>	0.29	5.3	23.5	6.84
Re <sup>(b)</sup>	0.43	5.3	18	7.7
W <sup>(b)</sup>	0.84	5.7	10.5	12.5
Ta <sup>(b)</sup>	1.23	5.5	10	12.3
Hf <sup>(b)</sup>	2.5	4.0	5.5	14
Sn <sup>(c)</sup>	1.2	2.9	5.3	6.4
Sb <sup>(c)</sup>	0.9	2	4.2	3.8

\*<sup>(a)</sup>Loegel and Gautier 1971; <sup>(b)</sup>Durand 1973; <sup>(c)</sup>Ross et al. 1978. The data have been obtained from deviations from Matthiessen's rule in the residual resistivity of ternary alloy (a) or in the resistivity of binary alloys at low temperature (c) or at 77 K (b). We have preferred the results given by Durand (1973) to slightly different ones given previously by Durand and Gautier (1970).

### 2.3.1. Nickel host

The general picture of  $\alpha$  ( $=\rho_{0\downarrow}/\rho_{0\uparrow}$ ) values for impurities in Ni estimated by different groups is consistent, although numerical values are not in perfect agreement (table 1, fig. 7). It is found that Co, Fe, Mn, Au and Cu have  $\alpha \gg 1$  while Cr, V and a number of other transition impurities have  $\alpha < 1$ . As has been pointed out (Durand and Gautier 1970, Fert and Campbell 1971, 1976, Hagakawa and Yamashita 1975) there is a very clear connection between the electrical and the magnetic properties of the impurities. Those impurities with high values of  $\alpha$  are those which, on the Friedel analysis of the magnetic properties (Friedel 1967), do not have  $d \uparrow$  virtual bound states at or near the spin  $\uparrow$  Fermi energy. These impurities have low  $\rho_{0\uparrow}$  values because the  $d \uparrow$  phase shift at the Fermi energy is small; in contrast, when the impurity is such that a  $d \uparrow$  virtual bound state is close to the Fermi energy,  $\rho_{0\uparrow}$  is large so  $\alpha$  is small. The difference in  $\rho_{0\uparrow}$  values between these two types of impurities can be quite striking:  $\rho_{0\uparrow} \approx 0.16 \mu\Omega\text{cm}/\%$  for Co impurities while  $\rho_{0\uparrow} \approx 56 \mu\Omega\text{cm}/\%$  for Ru impurities! Detailed comparisons between calculated and experimental values of spin  $\uparrow$  and spin  $\downarrow$  resistivities have been made.

The temperature dependence of the resistivity of binary alloys of Ni can be

TABLE 3  
 Values of  $\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$ ,  $\rho_0$ ,  $\rho_{0\uparrow}$ ,  $\rho_{0\downarrow}$  for dilute impurities in Fe\*.

Impurity in Fe	$\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$	$\rho_0$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\uparrow}$ ( $\mu\Omega\text{cm}$ )	$\rho_{0\downarrow}$ ( $\mu\Omega\text{cm}$ )
Ni	3 <sup>(a)</sup> , 7 <sup>(b)</sup>	2 ± 0.2	2.4 ± 0.2	12 ± 5
Co	1 <sup>(a)</sup> , 3.7 <sup>(b)</sup>	0.9 ± 0.1	1.6 ± 0.4	3.3 ± 1.3
Mn	0.09 <sup>(a)</sup> , 0.17 <sup>(b)</sup>	1.5 ± 0.2	13 ± 5	1.7 ± 0.2
Cr	0.17 <sup>(a)</sup> , 0.37 <sup>(b)</sup>	2.2 ± 0.3	12.5 ± 6	2.8 ± 0.2
V	0.12 <sup>(a)</sup> , 0.13 <sup>(b)</sup>	1.1 ± 0.3	10.5 ± 3	1.3 ± 0.3
Ti	0.25 <sup>(a)</sup> , 0.66 <sup>(b)</sup>	2.75 ± 0.25	10.5 ± 4	4 ± 0.4
Rh	5.8 <sup>(b)</sup>	0.95 <sup>(b)</sup>	1.1 <sup>(b)</sup>	6.4 <sup>(b)</sup>
Ru	0.38 <sup>(b)</sup>	2 ± 0.1	7.3 <sup>(b)</sup>	2.8 <sup>(b)</sup>
Mo	0.21 <sup>(b)</sup>	1.75 ± 0.2	11 <sup>(b)</sup>	2.3 <sup>(b)</sup>
Pt	8 <sup>(b)</sup>	1.3 <sup>(b)</sup>	1.5 <sup>(b)</sup>	12 <sup>(b)</sup>
Ir	9 <sup>(b)</sup>	2 <sup>(b)</sup>	2.2 <sup>(b)</sup>	20 <sup>(b)</sup>
Os	0.33 <sup>(b)</sup>	3.5 ± 0.5	13 <sup>(b)</sup>	4.3 <sup>(b)</sup>
Re	0.31 <sup>(b)</sup>	2.7 ± 0.5	8.7 <sup>(b)</sup>	2.7 <sup>(b)</sup>
W	0.24 <sup>(b)</sup>	1.6 ± 0.1	7.5 <sup>(b)</sup>	1.8 <sup>(b)</sup>
Be	6.2 <sup>(b)</sup>	4 <sup>(b)</sup>	4.7 <sup>(b)</sup>	29 <sup>(b)</sup>
Al	8.6 <sup>(b)</sup>	5.3 ± 0.2	5.6 <sup>(b)</sup>	48 <sup>(b)</sup>
Si	5.6 <sup>(b)</sup>	6 ± 0.6	6.4 <sup>(b)</sup>	36 <sup>(b)</sup>
Ga	8.1 <sup>(b)</sup>	4.8 <sup>(b)</sup>	5.4 <sup>(b)</sup>	44 <sup>(b)</sup>
Ge	6.2 <sup>(b)</sup>	6.8 ± 0.2	7.9 <sup>(b)</sup>	49 <sup>(b)</sup>
Sn	≈ 1 <sup>(c)</sup>	8.7 ± 1		
Sb	≈ 1 <sup>(c)</sup>	9.8 ± 0.4		

\* For  $\alpha$  we give the values derived by: <sup>(a)</sup> Fert and Campbell (1976); <sup>(b)</sup> Dorleijn and Miedema (1977), Dorleijn (1976); <sup>(c)</sup> Ross et al. 1979, from the residual resistivity of ternary alloys (b) or from  $\rho(T)$  of binary alloys (a) and (c). When there are data from several authors we have estimated mean values of  $\rho_0$ ,  $\rho_{0\uparrow}$ ,  $\rho_{0\downarrow}$ .

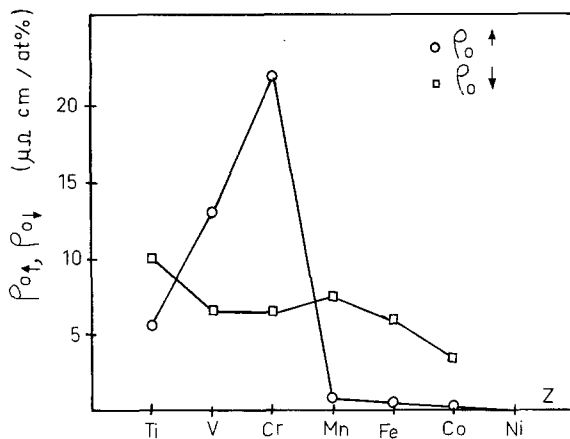


Fig. 7. Sub-band residual resistivities  $\rho_{0\uparrow}$  and  $\rho_{0\downarrow}$  of 3d impurities in nickel. (References in footnote to table 1.)

analyzed by using the two-current model equations to estimate the pure metal parameters  $\rho_{\uparrow\downarrow}(T)$ ,  $\rho_{i\uparrow}(T)$  and  $\rho_{i\downarrow}(T)$ . Figure 8 shows the agreement between experimental results below 50 K for series of Ni alloys and curves obtained from eq. (25) by using  $\alpha$  values derived from independent measurements on ternary alloys,  $\mu = 3.6$ ,  $\rho_i(T) = 9.5 \times 10^{-12}T^2 + 1.7 \times 10^{-14}T^4$  (in  $\Omega\text{cm}$  if  $T$  is expressed in K) and  $\rho_{\uparrow\downarrow}(T)$  of fig. 9 (dashed line). At temperatures up to about 50 K the analysis can be done unambiguously but at higher temperatures different sets of solutions fitting the experimental data are possible. At 300 K a reasonable estimate is  $\rho_{\uparrow\downarrow}(300) = 11 \mu\Omega\text{cm}$ ,  $\rho_{i\uparrow}(300) = 6.7 \mu\Omega\text{cm}$ ,  $\rho_{i\downarrow}(300) = 27 \mu\Omega\text{cm}$  (Fert and Campbell 1976).

The contribution to  $\rho_{\uparrow\downarrow}(T)$  from electron-magnon collisions has been calculated by Fert (1969) and Mills et al. (1971) in a model of spin-split spherical Fermi surfaces. The calculation gives the correct order of magnitude. The variation obtained for  $\rho_{\uparrow\downarrow}(T)/T^2$  as a function of  $T$  is shown in fig. 9 (solid line) together with the variation needed to fit the experimental results (dashed line). The calculated curve drops at low temperature, which results from a freezing out of electron-magnon scattering in the presence of a gap between spin  $\uparrow$  and spin  $\downarrow$  Fermi surfaces; the experimental curve shows a similar drop below about 30 K and then an upturn below 5 K; this upturn seems to be associated to a variation in  $T^{3/2}$  at very low temperature and has been ascribed to electron-magnon scattering

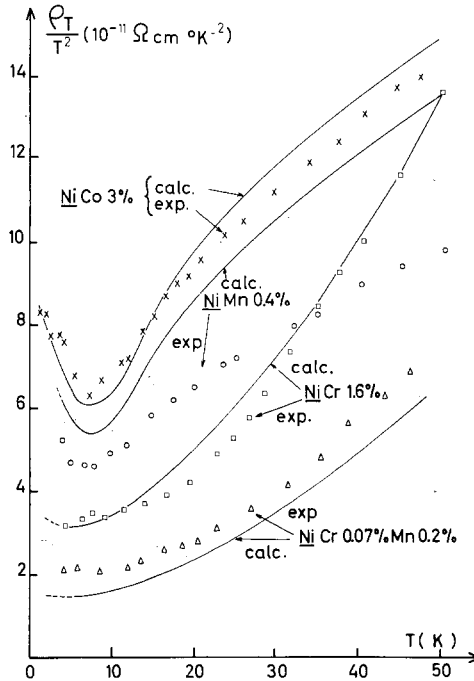


Fig. 8.  $\rho_T/T^2 = (\rho(T) - \rho(0))/T^2$  against  $T$  for several nickel based alloys. The solid curves are calculated from eq. (25) in the way described in the text (after Fert and Campbell 1976).



in regions where the spin  $\uparrow$  and spin  $\downarrow$  Fermi surfaces touch or are very near (Fert and Campbell 1976).

The resistivities  $\rho_{i\sigma}(T)$  are expected to include contributions from electron-electron, electron-phonon and electron-magnon collisions. Because of the small angle scattering factor the electron-magnon contributions to  $\rho_{i\uparrow}$ ,  $\rho_{i\downarrow}$  should each be equal to roughly  $(T/T_c)\rho_{\uparrow\downarrow}$  and therefore relatively small at low temperatures. If then the electron-electron or electron-phonon contributions are dominant, the possibility of scattering of the spin  $\downarrow$  electrons to the  $d\downarrow$  band makes  $\rho_{i\downarrow}(T)$  larger than  $\rho_{i\uparrow}(T)$ , in agreement with  $\mu > 1$ . At very low temperatures the variation of  $\rho_i(T)$  in  $T^2$  can be attributed to electron-electron scattering, as it has been concluded in section 2.2.2. Above 10 K the electron-phonon collisions become progressively more important. When approaching room temperature the electron-magnon collisions should begin to make a substantial contribution to  $\rho_{i\sigma}(T)$ . Theoretical estimates of the electron-phonon contributions to  $\rho_{i\uparrow}$  and  $\rho_{i\downarrow}$  at 300 K are  $4.25 \mu\Omega\text{cm}$  and  $19.2 \mu\Omega\text{cm}$  respectively (Yamashita and Hayakawa 1976); we can reasonably infer that additional contributions of a few  $\mu\Omega\text{cm}$  from electron-magnon scattering account for the experimental  $\rho_{i\sigma}(300)$ . Without magnon contributions to  $\rho_{i\sigma}(300)$  and without  $\rho_{\uparrow\downarrow}$  term the resistivity of pure nickel at 300 K would be predicted to amount to roughly  $4.25 \times 19.2 / (4.25 + 19.2) \sim 3.5 \mu\Omega\text{cm}$ , instead of  $7 \mu\Omega\text{cm}$  experimentally. We conclude that:

- (i) at low temperature the main contributions to  $\rho_{i\sigma}(T)$  arise from electron-electron and electron-phonon scatterings; electron-magnon collisions come into play through  $\rho_{\uparrow\downarrow}(T)$  and are important in alloys with  $\alpha$  very different from unity;
- (ii) at near room temperature the electron-magnon collisions contribute to both  $\rho_{i\sigma}$  and  $\rho_{\uparrow\downarrow}$ ; they will become increasingly important as temperature increases.

The analysis of the experimental data on Ni alloys by Yamashita and Hayakawa (1976), although based on a different treatment of the two current conduction, arrives at similar conclusions.

### 2.3.2. Cobalt host

The  $\alpha$  values of a large number of impurities have been obtained in Co metal (Durand and Gautier 1970, Loegel and Gautier 1971, Durand 1973, Ross et al. 1978), table 2. They are again consistent with the magnetic structures of the impurities. The parameters  $\rho_{i\uparrow}(T)$ ,  $\rho_{i\downarrow}(T)$  and  $\rho_{\uparrow\downarrow}(T)$  of Co have been evaluated by Loegel and Gautier (1971); the behaviour of  $\rho_{\uparrow\downarrow}(T)$  is similar to that of Ni.

### 2.3.3. Iron host

Extensive work has been done on Fe based alloys (Campbell et al. 1967, Fert and Campbell 1976, Dorleijn 1976, Dorleijn and Miedema 1977, Ross et al. 1979), table 3. The resulting  $\alpha$  values from different authors, both from ternary alloy data or from temperature dependence, are in reasonable agreement with each other. The range of  $\alpha$  values is very great.  $\rho_{0\downarrow}/\rho_{0\uparrow}$  varies from 0.13 for  $\text{FeV}$  to 9 for  $\text{FeIr}$  (table 3). There is a good correlation between the resistivity  $\rho_0$  in each band and the charge screening in that band for each impurity (Dorleijn 1976).

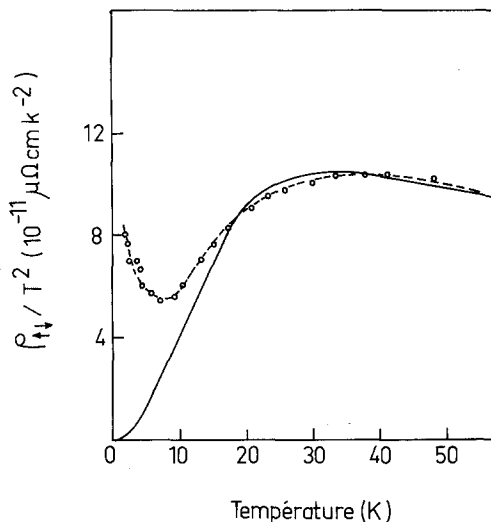


Fig. 9. Experimental (dashed line) and calculated (solid line) curves for  $\rho_{\uparrow\downarrow}/T^2$  in nickel. The experimental curve is after Fert and Campbell (1976); the calculated curve is obtained from the model calculation of Fert (1969) with  $\theta_1 = 38$  K.

The behaviour of  $\rho_{\uparrow\downarrow}(T)$  in Fe is similar to that in Ni and the value of  $\rho_{i\downarrow}/\rho_{i\uparrow}$  seems to be near 1 (Fert and Campbell 1976). More complete low temperature measurements would be necessary to decide this. As in Ni, the low temperature  $\rho(T)$  data cannot be understood without including the  $\rho_{\uparrow\downarrow}$  term.

#### 2.3.4. Alloys containing interstitial impurities

Ni, Co or Fe based alloys containing small concentrations of interstitial impurities of B or C can be prepared by rapid quenching. Swartz (1971), Schwerer (1972) and Cadeville and Lerner (1976) have investigated the resistivity of  $\underline{\text{Ni}}\text{C}$ ,  $\underline{\text{Co}}\text{C}$ ,  $\underline{\text{Ni}}_{1-x}\underline{\text{Fe}}_x\text{C}$  alloys. The residual resistivity of these alloys is equal to about  $3.4 \mu\Omega\text{cm/at.}\%$  for C in Ni and  $6.6 \mu\Omega\text{cm/at.}\%$  for C in Co. From the deviations from Matthiessens' rule in  $\underline{\text{Ni}}\text{CrC}$  and  $\underline{\text{Co}}\text{CuC}$  alloys, Cadeville and Lerner (1976) have estimated that the resistivity  $\rho_{0\downarrow}$  was about twice as large as  $\rho_{0\uparrow}$ . This result, together with magnetization and thermo-electric data by the same authors, are consistent with a predominant screening by the electrons of the  $d\downarrow$  band. In  $\underline{\text{Ni}}_{1-x}\underline{\text{Fe}}_x\text{C}$  alloys the resistivities  $\rho_{0\uparrow}$  and  $\rho_{0\downarrow}$  of the C impurities are found to become nearly equal for  $x > 0.4$ , which has been ascribed to the change from strong to weak ferromagnetism (Cadeville and Lerner 1976).

The resistivity of B impurities in Ni and Co have been found to be fairly small ( $\sim 1 \mu\Omega\text{cm/at.}\%$ ). This has been ascribed to a predominant screening by the  $d\downarrow$  electrons resulting in a small resistivity for the spin  $\uparrow$  electrons (Cadeville and Lerner 1976).

2.4. High temperature and critical point behaviour

It was observed a long time ago that the resistivities of ferromagnetic metals changed slope as a function of temperature at the Curie temperature. For Ni this was originally interpreted by Mott (1936) as indicating a reduction of the spin  $\uparrow$  resistivity on ordering. Later work (Kasuya 1956, Yoshida 1957, Coles 1958, Weiss and Marotta 1959) showed that spin disorder scattering provided a more general explanation. When the resistivities of the 3d ferromagnetic metals are compared with those of their non-magnetic 4d and 5d counterparts it can be seen clearly that there is an extra magnetic scattering contribution which is approximately constant above  $T_c$  and which decreases gradually below  $T_c$  (fig. 10). The simplest disorder model shows that the paramagnetic term above  $T_c$  is equal to

$$\rho_m = \frac{k_F(m\Gamma)^2}{4\pi e^2 z \hbar^3} J(J + 1), \tag{26}$$

where  $J$  is the effective local spin and  $\Gamma$  the local spin conduction electron spin coupling parameter. De Gennes and Friedel (1958) suggested that the critical magnetic scattering near  $T_c$  was similar in type to the critical scattering of neutrons and that it should lead to a peak in  $\rho(T)$  at  $T_c$ . Later work by Fisher and

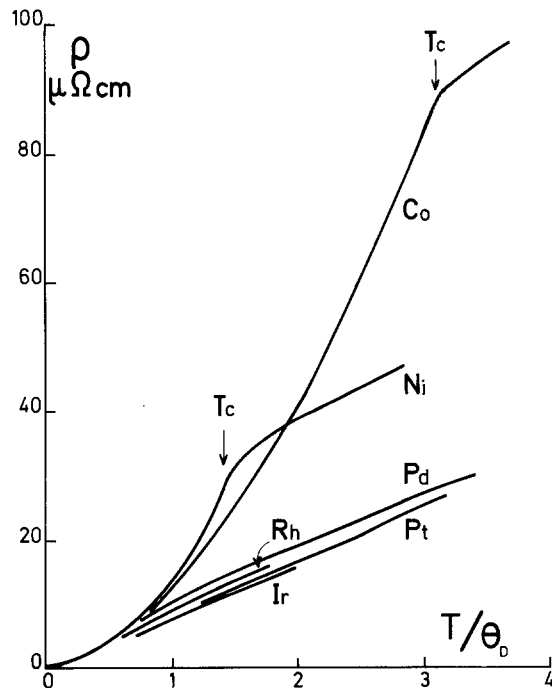


Fig. 10. Resistivity of several transition metals as a function of  $T/\theta_D$ .  $\theta_D$  is the Debye temperature.

Langer (1968), using a better approximation for the spin-spin correlation function near  $T_c$ , modified this prediction to that of a peak in  $d\rho/dT$  at  $T_c$ . They also made the important remark that just above  $T_c$  the same leading term in the spin-spin correlation should dominate  $d\rho/dT$  and the magnetic specific heat, so that these two parameters should have the same critical behaviour as  $T$  tends to  $T_c$  from above. Both magnetic entropy  $S$  and magnetic scattering rate should be proportional to

$$\int_0^{2k_F} \Gamma(k, T)k^3 dk, \tag{27}$$

where  $\Gamma(k, T)$  is the spin-spin correlation function. Later theoretical work showed that the same correspondence should hold equally in the region just below  $T_c$  (Richard and Geldart 1973).

Renormalization theory can predict the critical coefficients for  $d\rho/dT$  (Fisher and Aharony, 1973) but it is difficult to decide over what range of temperature each side of  $T_c$  the strictly ‘‘critical behaviour’’ should be observed; Geldart and Richard (1975) discussed the cross-over from a regime near  $T_c$  where the short-range correlations dominate to a long-range correlation regime. The theory of resistivity behaviour at  $T_c$  in weak ferromagnets has been developed by Ueda and Moriya (1975), Der Ruenn Su and Wu (1975).

Experimentally, the critical behaviour of  $d\rho/dT$  has been studied very carefully for Ni, Fe, Gd and the compound  $GdNi_2$  (Craig et al. 1967, Zumsteg and Parks 1970, Shaklette 1974, Kawatra et al. 1970, Zumsteg and Parks 1971, Parks 1972, Zumsteg et al. 1970). For Ni (Zumsteg and Parks 1970) and Fe (Shaklette 1974) it is found that  $d\rho/dT$  and the specific heat do indeed show the same  $\lambda$  point type of behaviour around  $T_c$  (fig. 11). The data are parameterized using

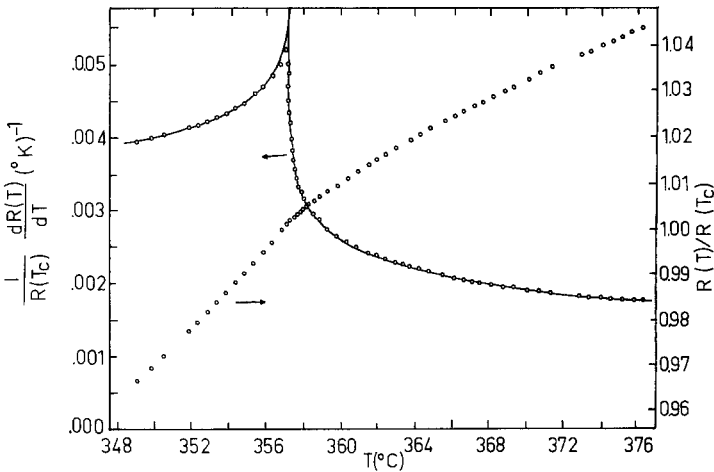


Fig. 11. Resistivity  $R(T)$  of nickel and  $dR/dT$  versus temperature in the region of the Curie point (after Zumsteg and Parks 1970).

$$\frac{1}{\rho_c} \frac{d\rho}{dT} = \frac{A}{\lambda} (\varepsilon^{-\lambda} - 1) + B, \quad T > T_c, \quad (28)$$

and

$$\frac{1}{\rho_c} \frac{d\rho}{dT} = \frac{A'}{\lambda} (|\varepsilon|^{-\lambda'} - 1) + B', \quad T < T_c, \quad (29)$$

where

$$\varepsilon = (T - T_c)/T_c. \quad (30)$$

Renormalization theory predicts  $\lambda = \lambda' \approx 0.10$  and  $A/A' \approx 1.3$  (Zumsteg et al. 1970) for a 3 dimensional exchange ferromagnet. The accurate determination of  $\lambda$  and  $\lambda'$  is extremely delicate especially as  $T_c$  must be fitted self-consistently from the data and it appears essential to have the theoretical predictions as a guide.

In pure Fe, Kraftmakher and Pinegina (1974) find  $\lambda, \lambda' = 0 \pm 0.1$  while Shaklette (1974) observes  $\lambda, \lambda' = -0.12 \pm 0.01$  by imposing  $\lambda \equiv \lambda'$ . Agreement with the magnetic specific heat data in Fe is very good (Shaklette 1974, Connelly et al. 1971). For Ni, the values obtained were  $\lambda = 0.1 \pm 0.1, \lambda' = 0.3 \pm 0.1$  (Zumsteg and Parks 1970) but within the fitting accuracy this is presumably also consistent with theoretical values.

In Gd which is hexagonal the critical behaviour looks very different when measured along the  $c$ - and the  $a$ -axes. Zumsteg et al. (1970) suggest that the resistivity changes are complicated by the critical behaviour of the lattice parameters, but this has been questioned (Geldart and Richard 1975).

GdNi<sub>2</sub> was investigated in the hope that it would correspond to a simple local moment system.  $d\rho/dT$  shows similar critical behaviour to Fe and Ni but has more complicated temperature dependence a few degrees above  $T_c$  (Kawatra et al. 1970, Zumsteg and Parks 1971). The significance of this has been discussed (Geldart and Richard 1975). The resistivity variation has also been measured at the structural and ferromagnetic transition in TbZn (Sousa et al. 1979).

The critical behaviour of  $d\rho/dH$  has been studied for Ni (Schwerer 1974) and for Gd with the current in the basal plane (Simons and Salomon 1974).

The behaviour of transport properties near  $T_c$  can also be studied in alloys, but local inhomogeneity leads to a spread in the local values of  $T_c$  at different parts of the sample and so the critical behaviour is smeared out. This has been observed in NiCu alloys (Sousa et al. 1975) and in PdFe (Kawatra et al. 1970, Kawatra et al. 1969).

Finally, behaviour at the critical concentration for ferromagnetism (the concentration at which  $T_c \rightarrow 0$ ) can be studied. Very varied behaviour has been found in different alloy systems. In NiCu alloys there is a peak in  $d\rho/dT$  at  $T_c$  as long as  $T_c$  exists and there is a maximum in  $\rho(T)$  some degrees higher, while for  $c > c_{\text{crit}}$  a minimum in  $\rho(T)$  is observed (Houghton and Sarachik 1970). In NiAu (splat cooled to avoid segregation)  $\rho(T)$  shows a maximum at  $T_c$  for  $c < c_{\text{crit}}$  (Tyler et al.

1973). For NiCr, Yao et al. (1975) find weak minima in  $\rho(T)$  for  $c > c_{\text{crit}}$  while Smith et al. find giant minima in the region  $c \sim c_{\text{crit}}$  (Smith et al. 1975). In NiPd alloys, Tari and Coles (1971) express the low temperature resistivity behaviour as  $\rho = \rho_0 = AT^n$  and find  $A$  is sharply peaked at  $c_{\text{crit}}$  while  $n$  has a minimum with  $n \approx 1$ . The Curie point "is not easy to detect on the  $\rho(T)$  curves". Amamou et al. (1975) using the same way of expressing the resistivity behaviour found  $n \rightarrow 1$  and strong peaks in  $A$  at the critical concentrations of a large number of alloys systems.

The transition from low temperature two current behaviour to high temperature spin disorder behaviour has been studied in Fe based alloys (Schwerer and Cuddy 1970). The high temperature resistivity behaviour of the alloy seems to depend essentially on the local impurity moment.

### 3. Other transport properties of Ni, Co, Fe and their alloys

Here we will summarize results on different transport properties in these metals and alloys and outline the interpretations which have been given. We will generally find that Ni has been studied in most detail while rather less is known about Fe and Co. In the interpretation of the results, we will refer to what has been learnt about the different systems from the resistivity measurements which we have already discussed.

#### 3.1. Ordinary magnetoresistance

We have outlined the situation for pure metals in section 2.2. For non-magnetic alloys the low temperature magnetoresistance behaviour generally follows Kohler's rule  $(\rho(B) - \rho(0))/\rho(0) = f(B/\rho(0))$ , where  $f$  is a function which varies from metal to metal but which is rather insensitive to the type of impurity present for a given host. In a ferromagnet above technical saturation the same effect, due to the Lorentz force on the electrons, can be observed but as  $B$  includes the magnetization term  $4\pi M$ ,  $\rho(0)$  cannot be attained except by extrapolation. Schwerer and Silcox (1970) showed by a careful study of dilute Ni alloy samples that for a given series of alloys (e.g. NiFe samples) the ordinary magnetoresistance follows a Kohler's rule, but that the Kohler function  $f$  varied considerably with the type of scatterer (fig. 12). Other work (Fert et al. 1970, Dorleijn 1976) is consistent with these data.

It can be seen in fig. 12 that the strongest magnetoresistances are associated with impurities having large values of  $\rho_{0\downarrow}/\rho_{0\uparrow}$  (i.e. NiFe, NiCo...). The longitudinal magnetoresistance of these alloys is also high [ $\Delta\rho_{\parallel}/\rho(0)$  saturates at about 10 in NiFe (Schwerer and Silcox 1970)] considerably greater than that observed for Cu based alloys for instance, where  $\Delta\rho_{\parallel}/\rho(0)$  saturates at about 0.7 (Clark and Powell 1968). Attempts have been made to understand this behaviour in the two current model. In its simplest form the two types of electron (spin  $\uparrow$  and spin  $\downarrow$ ) can be represented by electron-like spheres in  $k$  space with different relaxation

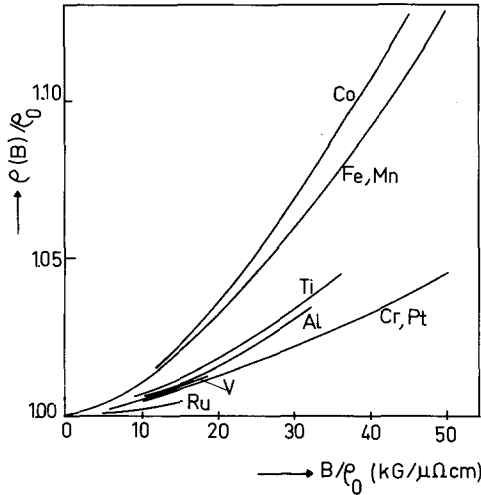


Fig. 12. Kohler plots for the transverse magnetoresistance at 4.2 K of nickel containing Co, Fe, Mn, Ti, Al, Cr, Pt, V or Ru impurities (after Dorleijn 1976).

times. In this approximation (Fert et al. 1970) the transverse magnetoresistance is indeed an increasing function of  $\rho_{\downarrow}(0)/\rho_{\uparrow}(0)$ , but the model is not satisfactory as it predicts a zero longitudinal magnetoresistance in disagreement with experiment. As a next step, it is possible to invoke relaxation time anisotropy within each spin band. Dorleijn (1976) suggests that the intrinsic magnetoresistivity of the spin  $\uparrow$  band of Ni is much greater than that of the spin  $\downarrow$  band so that the longitudinal and transverse magnetoresistances are much greater when the current is carried mainly by the spin  $\uparrow$  electrons. Jaoul (1974) proposes that there is a mixing between spin  $\uparrow$  and spin  $\downarrow$  currents which is an increasing function of  $B/\rho(0)$ . This is because spin-orbit effects mean that an electron on a given orbit on the Fermi surface passes continuously between spin  $\uparrow$  and spin  $\downarrow$ , progressively mixing currents as  $B/\rho(0)$  increases. This model predicts the saturation magnetoresistances of the different alloy series reasonably well.

The ordinary magnetoresistance in Fe based alloys is more difficult to express in the form of Kohler curves, because the much higher value of  $4\pi M$  in Fe means that extrapolations to  $B = 0$  are always very extended. Data given by Dorleijn (1976) again indicate different Kohler curves for Fe samples containing different impurities, but the correlation with the value of  $\rho_{\downarrow}(0)/\rho_{\uparrow}(0)$  is much less clear than in the case of Ni based alloys.

There is an additional effect that appears under similar experimental conditions as the Lorentz force ordinary magnetoresistance, but which is due to the high field susceptibility of the ferromagnetic metal. This high field susceptibility can have two origins. First, there is an increasing magnetic order in an applied field which can also be thought of as a reduction in the number of magnons with increasing field. This term is maximum around  $T_c$  and goes to zero as  $T$  goes to zero. Secondly, for a band ferromagnet, the local magnetic moments can be altered by an applied field at any temperature, even  $T = 0$  (Van Elst 1959).

Insofar, as an increasing field produces increasing magnetic order and hence lower spin disorder scattering,  $d\rho/dH$  due to the first term will be negative. The second type of effect can in principle give either positive or negative magnetoresistance depending on the electronic structure of the system. Van Elst (1959) measured at 300 K  $(1/\rho)(d\rho/dH)_{\parallel} \approx (1/\rho)(d\rho/dH)_{\perp}$  with effects of the order of  $10^{-4}/\text{kG}$  and with significant variations from one alloy to another. This behaviour is due to the first effect. At low temperatures the Lorentz-force magnetoresistance dominated except for NiMn alloys which showed negative  $d\rho/dH$  even at low temperature; this is probably due to an unusual band susceptibility in these alloys.

### 3.2. Ordinary Hall coefficient

In non-magnetic metals it is known that the ordinary Hall coefficient  $R_0$  behaves to a rough approximation as  $R_0 \propto 1/en^*$  where  $n^*$  is the effective density of current carriers and  $e$  is their charge ( $e$  is negative for electron-like carriers and positive for hole-like carriers). The actual values of  $R_0$  can be considerably modified by Fermi surface and scattering anisotropy effects (Hurd 1972); for the high field condition  $\omega_c \gg 1$ ,  $R_0$  depends only on the Fermi surface geometry and can be highly anisotropic in single crystals.

In ferromagnetic metals the ordinary Hall effect can be separated from the extraordinary Hall effect by measurements above technical saturation, as long as the susceptibility of the sample in high fields is negligible so that there is no paramagnetic extraordinary Hall effect correction (see section 3.4).

The ordinary Hall coefficient in Ni at room temperature is  $R_0 \approx -6 \times 10^{-13} \Omega\text{cm/G}$  (Lavine 1961), which corresponds to conduction by electron-like carriers with an effective electron density  $n^*$  of about 1 electron per atom.  $R_0$  varies by about 20% between room temperature and 50 K; at lower temperatures the low field condition  $\omega_c \tau \ll 1$  no longer holds for high purity Ni samples so  $R_0$  tends towards the high field value (Reed and Fawcett 1964).

Pugh and coworkers (Pugh et al. 1955, Sandford et al. 1961, Ehrlich et al. 1964) and Smit (1955) showed that for a number of Ni based alloys, in particular NiFe and NiCu, the low temperature Hall coefficients in concentrated samples correspond to much lower effective carrier concentrations,  $n^* \sim 0.3$  electrons per atom. They pointed out that this low number of carriers was probably associated with a regime where only the conduction band for one direction of spin was carrying the current. Later work on Ni and NiCu alloys (Dutta Roy and Subrahmanyam 1969) showed that  $R_0$  is very temperature dependent in the alloys, and that above the Curie point  $n^*$  returns to a value of about 1 electron-atom, i.e., to a situation where both spin directions carry current.

This would seem to fit in well with other data on the two current model. However, careful measurements by Huguenin and Rivier (1965) and by Miedema and Dorleijn (1977) on a wide range of Ni based alloys have shown that the situation is more complicated. The data can be summarized as follows: the low temperature  $R_0$  is very close to zero in dilute alloys (concentration  $\sim 0.5\%$ ) for



which  $\rho_{0\downarrow}/\rho_{0\uparrow} \gg 1$  (i.e. NiFe, NiCu, NiCo...) but then increases rapidly with impurity concentration to a value corresponding to  $n^* \sim 0.3$  in samples where the impurity resistivity is greater than about  $5 \mu\Omega\text{cm}$ . For alloys for which  $\rho_{0\downarrow}/\rho_{0\uparrow} < 1$ ,  $R_0$  is essentially independent of impurity concentration at about  $-6 \times 10^{-13} \Omega\text{cm/G}$  (note that only samples of this type having  $\rho > 2 \mu\Omega\text{cm}$  were studied).

Now in a two current model  $R_0$  is given by

$$R_0 = \rho^2 R_{0\uparrow}/\rho_{\uparrow}^2 + R_{0\downarrow}/\rho_{\downarrow}^2, \quad (31)$$

where  $R_{0\uparrow}$ ,  $R_{0\downarrow}$  are the ordinary Hall coefficients for the two spin directions taken separately. From the experimental data it can be concluded that  $R_{0\downarrow}$  is reasonably constant, while  $R_{0\uparrow}$  varies strongly with  $\rho_{\uparrow}$ . Dorleijn and Miedema suggested that the effect is due to a "smudging out" of the details of the spin  $\uparrow$  Fermi surface of Ni with increasing  $\rho_{\uparrow}$  and they associated this with the observed changes of the magnetocrystalline anisotropy with alloy concentration (Miedema and Dorleijn 1977). As we will see in section 3.3, the resistivity anisotropy of the same alloys changes similarly with impurity concentration until a certain residual resistivity value is reached. The  $R_0$  data suggest that the "smudged out" Fermi surface situation corresponds more closely to the extreme s-d model with conduction entirely by an s  $\uparrow$  like band containing about 0.3 electrons per atom.

The results on  $R_0$  in Fe based alloys are less clear, partly because the separation into ordinary and extraordinary Hall components is more difficult because of the large value of  $4\pi M$ . Fe has a positive ordinary Hall coefficient, as have the dilute Fe based alloys except for FeCo (Beitel and Pugh 1958) although  $R_0$  for FeNi alloys changes sign with temperature and with concentration (Soffer et al. 1965). There appears to be evidence (Carter and Pugh 1966) that alloys for which  $\rho_{\uparrow}(0)/\rho_{\downarrow}(0) \gg 1$  such as FeCr, behave similarly to Ni in that  $R_0$  is high at low temperatures and drops considerably at higher temperatures as both spin directions begin to participate in the conduction.

### 3.3. Spontaneous resistivity anisotropy

This was defined in section 1 and is a spin orbit effect. The mechanism can vary from system to system. The simplest case to understand, at least in principle, is that of dilute rare earth impurities (Fert et al. 1977). Because of the unclosed f shell, the magnetic rare earths can be regarded as ion-like with a non-spherical distribution of charge (apart from the spherical ion  $\text{Gd}^{3+}$ ). A conduction electron plane wave encounters an object with a different cross section depending on whether it arrives with its  $k$  vector parallel or perpendicular to the rare earth moment, which provides an axis for the non-spherical charge distribution. The anisotropy of the resistivity is proportional to the electronic quadrupole moment of the particular rare earth. The theory of this effect has been worked out in detail (Fert et al. 1977).

In transition metals, the spin orbit coupling is usually a weak perturbation on the spin magnetization. The lowest order terms leading to a resistivity anisotropy

will be either mixing terms of the type  $(L^+S^-)^2$  or polarization terms of the type  $(L_zS_z)^2$ . Smit (1951) calculated the resistivity anisotropy to be expected on an s-d model from the mixing terms acting between spin  $\uparrow$  and spin  $\downarrow$  d bands. When data became available for both the anisotropy and the  $\rho_{\downarrow}/\rho_{\uparrow}$  ratios in various Ni alloys, it was found that there was good agreement between the results and predictions which could be made using the Smit approach (Campbell et al. 1970). Agreement is however less good for impurities having a virtual bound d state near the Fermi surface, and an additional  $(L_zS_z)^2$  mechanism was suggested for these cases (Jaoul et al. 1977).

The relative anisotropy of the resistivity  $(\rho_{\parallel} - \rho_{\perp})/\bar{\rho}$  defined in section 1 has been measured for Ni and a large number of Ni alloys as a function of concentration and temperature (Smit 1951, Van Elst 1959, Berger and Friedberg 1968, Campbell et al. 1970, Vasilyev 1970, Campbell 1974, Dedić 1975, Dorleijn 1976, Dorleijn and Miedema 1976, Kaul 1977, Jaoul et al. 1977) and for many dilute Fe based alloys, mainly at He temperature (Dorleijn and Miedema 1976). We will first discuss the Ni data.

The anisotropy ratio for pure Ni is near +2% from nitrogen temperature up to room temperature, and then gradually drops as the temperature is increased up to the Curie point (Smit 1951, Van Elst 1959, Kaul 1977). Below nitrogen temperature the anisotropy is difficult to estimate for pure samples because of the rapidly increasing ordinary magnetoresistance, but it appears to remain fairly constant.

For most dilute NiX alloy series the limiting low temperature anisotropy ratio is relatively concentration independent for a given type of impurity X over a fairly wide concentration range but the value depends strongly on the type of impurity, table 4. For NiCo, NiFe and NiCu (fig. 13) the anisotropy ratio increases continuously with concentration up to concentrations corresponding to residual resistivities of about  $2 \mu\Omega\text{cm}$ . It is a disputed point as to whether the appropriate characteristic value of the anisotropy ratio for these alloys is the plateau value (Jaoul et al. 1977) or a value at some lower concentration (Dorleijn and Miedema 1975b, 1976).

When the temperature is increased, the anisotropy ratio of a given sample tends towards the pure Ni value and finally becomes zero at the Curie point of the alloy (Vasilyev 1970, Kaul 1977).

There is a clear correlation between the value of  $\alpha$  and the low temperature anisotropy ratio (Campbell et al. 1970). Alloys having high values of  $\alpha$  (NiCo, NiFe, NiMn . . .) have high positive resistivity anisotropies while alloys with  $\alpha \lesssim 1$  have small positive or negative anisotropies. A spin-orbit mixing model originally suggested by Smit (1951) gives a convincing explanation of the overall variation of the anisotropy ratio with the value of  $\alpha$ . As Ni metal has a fully polarized d band, there are no d  $\uparrow$  states at the Fermi surface for the conduction electrons to be scattered to. However because of the spin-orbit mixing by the matrix element  $\lambda L^+S^-$  some d  $\uparrow$  character is mixed into the d  $\downarrow$  band. The resulting weak s  $\uparrow$  to d  $\downarrow$  scattering can be shown to depend strongly on the relative orientation of the  $\mathbf{k}$  vector of the s conduction electron and the sample magnetization. This leads to a resistivity anisotropy of the form

TABLE 4  
Anisotropy of the residual resistivity of dilute nickel based alloys\*.

Impurity	Co	Fe	Mn	Cr	V	Pd		
$\frac{\rho_{\parallel} - \rho_{\perp}}{\bar{\rho}} \times 10^2$	20 <sup>(a)</sup>	13.6 <sup>(b)</sup>	9.9 <sup>(a)</sup>	-0.35 <sup>(b)</sup>	0.6 <sup>(a)</sup>	2 <sup>(c)</sup>		
	14.8 <sup>(b)</sup>	14 <sup>(d)</sup>	7.8 <sup>(b)</sup>	-0.28 <sup>(a)</sup>	0.15 <sup>(b)</sup>			
	28 <sup>(c)</sup>	19.5 <sup>(c)</sup>	9.5 <sup>(c)</sup>	-0.23 <sup>(d)</sup>				
Impurity	Rh	Ru	Mo	Nb	Pt	Ir		
$\frac{\rho_{\parallel} - \rho_{\perp}}{\bar{\rho}} \times 10^2$	0.05 <sup>(b)</sup>	-0.60 <sup>(b)</sup>	0.1 <sup>(a)</sup>	0.15 <sup>(c)</sup>	0.4 <sup>(b)</sup>	-1.52 <sup>(c)</sup>		
	0.05 <sup>(c)</sup>	-0.82 <sup>(c)</sup>	0.05 <sup>(c)</sup>	0.4 <sup>(c)</sup>				
Impurity	Re	W	Cu	Au	Al	Si	Zn	Sn
$\frac{\rho_{\parallel} - \rho_{\perp}}{\bar{\rho}} \times 10^2$	-0.50 <sup>(b)</sup>	0.4 <sup>(a)</sup>	6.8 <sup>(b)</sup>	7.5 <sup>(b)</sup>	4.7 <sup>(a)</sup>	2.5 <sup>(a)</sup>	5.7 <sup>(a)</sup>	3.4 <sup>(a)</sup>
	-0.45 <sup>(c)</sup>	0.8 <sup>(c)</sup>	7.8 <sup>(c)</sup>	7.9 <sup>(c)</sup>	3.9 <sup>(c)</sup>	2.1 <sup>(b)</sup>	4.7 <sup>(b)</sup>	2.9 <sup>(b)</sup>
						4.6 <sup>(c)</sup>	2.8 <sup>(c)</sup>	6.5 <sup>(c)</sup>

\* After: <sup>(a)</sup> Van Elst 1959, <sup>(b)</sup> Dorleijn and Miedema 1974, Dorleijn 1976, <sup>(c)</sup> Jaoul et al. 1977, <sup>(d)</sup> Schwerer and Silcox 1970. We indicate - when this is possible - the resistivity anisotropy of alloys in the concentration range where the concentration dependence is weak (see fig. 13). The experimental data on  $(\rho_{\parallel} - \rho_{\perp})/\rho_{\parallel}$  has been re-expressed in terms of  $(\rho_{\parallel} - \rho_{\perp})/\bar{\rho}$ .

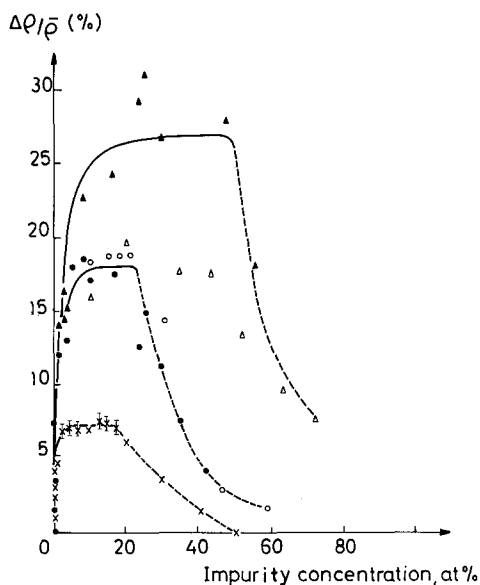


Fig. 13. Concentration dependence of the resistivity anisotropy at 4.2 K for several nickel based alloys.  $\blacktriangle$ :  $\underline{\text{NiCo}}$ ,  $\bullet$ :  $\underline{\text{NiFe}}$ ,  $\times$ :  $\underline{\text{NiCu}}$  (after Jaoul et al. 1977).

$$\rho_{\parallel} - \rho_{\perp} / \bar{\rho} = \gamma(\alpha - 1), \quad (32)$$

where  $\gamma$  is a spin-orbit constant which can be estimated to be about 0.01 from the Ni  $g$  factor. This model explains the sign, the magnitude and the general variation of the anisotropy with  $\alpha$  (fig. 14). In addition, it has been shown (Ehrlich et al. 1964, Dorleijn and Miedema 1976, Jaoul et al. 1977) that an analysis of the anisotropy ratio of ternary alloys can lead to estimates of the individual anisotropies for the spin  $\uparrow$  and spin  $\downarrow$  currents and that for alloys with  $\alpha > 1$  the results are in agreement with the predictions of the Smit mechanism.

However, for a number of alloys of Ni for which  $\alpha < 1$ , although the resistivity anisotropies remain small as would be expected from the Smit mechanism, eq. (32) is not accurately obeyed and the anisotropies of the two spin currents do not obey the Smit rules (Ehrlich et al. 1955, Jaoul et al. 1977). A further mechanism needs to be invoked for these systems, which are characterized by virtual bound states at the spin  $\uparrow$  Fermi level. A mechanism has been proposed involving the  $\lambda L_z S_z$  spin-orbit interaction on the impurity site, particularly for impurities which have strong spin-orbit interactions (Jaoul et al. 1977). Dorleijn and Miedema (1976) pointed out that for most impurities, whatever the value of  $\alpha$ ,  $(\Delta\rho/\rho)_{\uparrow} > 1$  and  $(\Delta\rho/\rho)_{\downarrow} < 1$  but they did not explain this regularity.

The temperature variation of the anisotropy ratio can also be understood using the Smit model (Campbell et al. 1970). As phonon and magnon scattering increases with increasing temperature, the effective value of  $\alpha$  for an alloy tends to approach the pure metal value. Data on NiCu alloys have been analyzed in this

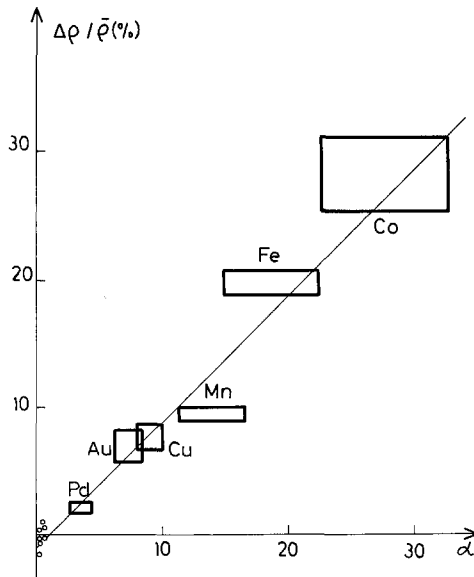


Fig. 14. Resistivity anisotropy of Ni based alloys at 4.2 K as a function of  $\alpha = \rho_{0\downarrow}/\rho_{0\uparrow}$ . The straight line is  $\Delta\rho/\bar{\rho} = 0.01(\alpha - 1)$  (after Jaoul et al. 1977).

way over a wide temperature and concentration range (Kaul 1977) so as to estimate  $\rho_{i\uparrow}(T)$ ,  $\rho_{i\downarrow}(T)$  and  $\rho_{\uparrow\downarrow}(T)$ .

High concentration effects in certain alloy series have been interpreted as due to characteristic changes in the electronic structure with concentration (Campbell 1974).

The resistivity anisotropy of a large number of Fe based alloys has also been studied (Dorleijn and Miedema 1976). Here, the alloys having  $\rho_{\uparrow}(0)/\rho_{\downarrow}(0) \gg 1$  have strong positive resistivity anisotropies while those with  $\rho_{\uparrow}(0)/\rho_{\downarrow}(0) < 1$  have small anisotropy ratios (table 4). Again, an analysis in terms of the anisotropies of the spin  $\uparrow$  and spin  $\downarrow$  currents has been carried out and the predictions of the Smit approach seem well borne out (Dorleijn 1976).

As we have seen in section 1 the resistivity anisotropy in cubic ferromagnetic monocrystals can be expanded in a series of Döring coefficients  $k_1 \dots k_5$ . Once again, Ni and Ni alloys have been the most studied [pure Ni (Bozorth 1951), Ni 15% Fe (Berger and Friedberg 1968), Ni 1.6% Cr and Ni 3% Fe (Jaoul 1974), Ni 0.5% Fe, Ni 0.55% Pt and Ni 4% Pd (Dedié 1975)]. Very roughly the individual  $k_i$  coefficients are simply proportional to the average polycrystal anisotropy with the exception of  $k_3$  (table 5). This coefficient may behave differently from the others because it does not strictly represent an anisotropy – it corresponds to an average change of the sample resistivity with the moment direction which is independent of the current direction.

TABLE 5  
Magnetoresistance anisotropy in Ni and Ni alloy single crystals. Döring coefficients  $k_i$  are given in percent. References: <sup>(a)</sup> Döring 1938, <sup>(b)</sup> Berger and Friedberg 1968, <sup>(c)</sup> Jaoul 1974, <sup>(d)</sup> Dedié 1975.

	$k_1$	$k_2$	$k_3$	$k_4$	$k_5$
Ni, 300 K <sup>(a),(d)</sup>			-3.4	-5.2	+1.7
NiFe 15% 4.2 K <sup>(b)</sup>	55.0	14.5	-26.3	-37.8	+24.7
NiCr 1% 4.2 K <sup>(c)</sup>	-3.0	-0.3	-1.2	+2.3	0 ± 0.7
NiPd 4% 4.2 K <sup>(d)</sup>	4.0	1.0	-5.5	-4.0	-3.0

There are also measurements of the Döring coefficients for Fe at room temperature (Bozorth 1951).

No convincing model has been proposed to explain the monocrystal anisotropy coefficients which presumably depend on the detailed band structure of the metal. The fact that the terms which are fourth order in the direction cosines of the magnetization ( $k_3, k_4, k_5$ ) are as large as the second order terms ( $k_1, k_2$ ) is remarkable.

### 3.4. Extraordinary Hall effect

Apart from the resistivity, the property of ferromagnetic metals which has attracted the greatest theoretical attention is the extraordinary Hall effect,  $R_s$ ; the extraordinary Hall voltage is remarkable in being both strong and rapidly varying

with temperature and impurity concentration. The fundamental mechanisms which are believed to produce this effect were proposed some years ago by Smit (1955) and Luttinger (1958) but the physical understanding of these effects has been considerably improved quite recently (Berger 1970, Lyo and Holstein 1972, Nozières and Lewiner 1973). We will outline the discussion given by Nozières and Lewiner (1973); although this theory was developed specifically for semiconductors the same physics can broadly be used for ferromagnetic metals.

An electron in a band submitted to the spin-orbit interaction acquires an effective electric dipole moment

$$\mathbf{p} = -\lambda \mathbf{k} \times \mathbf{s},$$

where  $\lambda$  is a spin-orbit parameter,  $\mathbf{k}$  is the  $\mathbf{k}$  vector and  $\mathbf{s}$  the spin of the electron. If there were no scattering centres, the effective Hamiltonian would be

$$\mathcal{H}_{\text{eff}} = k^2/2m - e\mathbf{E} \cdot (\mathbf{r} + \mathbf{p})$$

(where  $\mathbf{r}$  is the centre of the electron wave packet) for a metal in a uniform electric field  $\mathbf{E}$ . Local scattering potentials give local terms in the Hamiltonian

$$V(\mathbf{r}) - \lambda(\mathbf{k} \times \mathbf{s}) \cdot \nabla V.$$

Here, the second term arises from spin-orbit coupling in the lattice. An additional contribution to  $\lambda$  can also arise from a local spin-orbit interaction.

There are two distinct effects:

(a) the scattering matrix elements between plane wave states are expressed as

$$\langle \mathbf{k}' | V - \lambda(\mathbf{k} \times \mathbf{s}) \cdot \nabla V | \mathbf{k} \rangle = V_{\mathbf{k}\mathbf{k}'} [1 - i\lambda(\mathbf{k} \times \mathbf{k}') \cdot \mathbf{s}]$$

(by applying the general commutator rule  $[f(x), k_x] \equiv i \partial f(x)/\partial x$  to  $V(\mathbf{r})$ ). This means that the probability of scattering  $\mathbf{k} \rightarrow \mathbf{k}'$  is not the same as the probability  $\mathbf{k}' \rightarrow \mathbf{k}$  because of interference between the spin-orbit term and the potential scattering. For a weak  $\delta$  function potential,

$$W_{\mathbf{k}\mathbf{k}'} = V^2 [1 + 2\lambda V \pi n(\mathbf{k} \times \mathbf{k}') \cdot \mathbf{s}],$$

where  $n$  is the density of states at the Fermi level.

This "skew scattering" leads to a Hall current such that the Hall angle  $\phi_H \propto \lambda V$ , which is independent of scattering centre concentration, but which can be of either sign, depending on the sign of  $V$ .

(b) Now we come to the "side jump" term. The total Hamiltonian is

$$\mathcal{H} = k^2/2m + V(\mathbf{r}) - e\mathbf{E} \cdot \mathbf{r} + \mathbf{p} \cdot [\nabla V - e\mathbf{E}],$$

and the total velocity is:

$$v = \dot{p} - i[r, \mathcal{H}] = k/m + \lambda[\nabla V - eE] \times s + \dot{p}.$$

Without scattering,  $p$  changes as  $k$  increases under the influence of  $E$ , and secondly, the energies of the different  $k$  states are altered by the second term in  $v$ . However, when scattering is introduced, both of these currents are exactly cancelled out in static conditions; the first because

$$\langle \dot{p} \rangle = -\lambda \langle \dot{k} \rangle \times s = 0,$$

and the second because the electron distribution readjusts itself to minimize energy, and this new distribution automatically has an average velocity perpendicular to  $E$  equal to zero.

It would thus appear that the spin orbit terms do not lead to any extra current. But, during each scattering event there is also a “side jump” or shift of the centre of gravity of a scattered wave packet

$$\delta r = \int \delta v dt = -\lambda \Delta k \times s$$

(as  $\delta v = \lambda \nabla V \times s = -\lambda \dot{k} \times s$  during the scattering event).

Now, there are two side jump contributions:

(i) electrons travelling with a component of  $k$  parallel to  $E$  jump sideways on being scattered; the resultant of these jumps is a current.

(ii) electrons with a component of  $k$  perpendicular to  $E$  gain or lose an energy  $-e\delta r \cdot E$  on scattering. This shifts the total electron distribution to provide a second current.

These terms are not cancelled out by any compensating terms. They lead to a total Hall current of  $2\lambda Ne^2 E \times \langle s \rangle$ , which is proportional to the electric field  $E$  but independent of the scattering rate. The definition of  $R_s$  is  $V_y/I_x M_z$ , where  $y$  is the Hall probe direction,  $x$  the current direction and  $z$  the moment direction. Putting  $V_y = \rho I_y$  and  $E = \rho I_x$ , with the Hall current just given we clearly obtain  $R_s \propto \lambda \rho^2$ .

Note that the parameter  $\lambda$  represents the rate of change of the spin-orbit dipole  $-\lambda k \times s$  with  $k$ . This is a band property. However, local spin-orbit interactions on scattering centres can give an additional contribution to  $\lambda$  and complicate the picture.

We can now turn to the experimental data. The skew scattering term can be expected to dominate in dilute alloys at low temperatures, and indeed in Ni based alloys for which  $\rho(0) \leq 1 \mu\Omega\text{cm}$  at helium temperatures (fig. 15) it has been shown that the Hall angle  $\phi_H$  is independent of impurity concentration, but depends strongly on the type of impurity (Jaoul 1974, Fert and Jaoul 1972, Dorleijn 1976). It is possible to define Hall angles for each direction of spin,  $\phi_{H\uparrow}$  and  $\phi_{H\downarrow}$  and experiments on ternary alloys (Dorleijn 1976) or on the temperature dependence

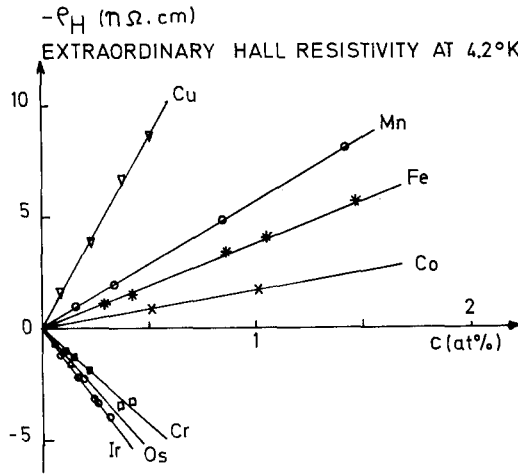


Fig. 15. Extraordinary Hall resistivity of several types of Ni based alloys as a function of their impurity concentration. The data are limited to alloys having a resistivity smaller than about  $1 \mu\Omega\text{cm}$ ; in more concentrated alloys, a side-jump contribution progressively appears and becomes predominant for  $\rho = 10 \mu\Omega\text{cm}$  (see fig. 16) (Jaoul 1974).

of the Hall angle (Jaoul 1974) allow one to estimate these two Hall angles for each impurity. Results are given in table 6. The values of the skew scattering Hall angles can be discussed in terms of the electronic structure of the various impurities (Fert and Jaoul 1972, Jaoul 1974).

For samples with higher resistivities (either because of higher impurity concentration or because they are measured at higher temperatures) the side jump term becomes important. Considering only data taken at low temperatures, results for a given alloy series can generally be fitted (Jaoul 1974, Dorleijn 1976) by the

TABLE 6

Skew scattering Hall effect in dilute Ni based alloys. For each impurity,  $\phi_H$  is the dilute limit Hall angle in millirad, and  $\phi_{H\uparrow}$ ,  $\phi_{H\downarrow}$  are the corresponding spin  $\uparrow$  and spin  $\downarrow$  Hall angles. References: \* Dorleijn 1976, † Jaoul 1974.

Impurity	Ti	V	Cr	Mn
$\phi_H$	+1.5*, -4.5	-3*, -2.5 <sup>†</sup>	+2.8*, +2 <sup>†</sup>	-6.5*, -9.5 <sup>†</sup>
$\phi_{H\uparrow}$	-3.4*	-4*, -7 <sup>†</sup>	-3*, -7 <sup>†</sup>	-10 <sup>†</sup>
$\phi_{H\downarrow}$	+5.5*	+6*, -3*	+4*, -3 <sup>†</sup>	+1.5 <sup>†</sup>

Impurity	Fe	Co	Cu	Ru	Rh
$\phi_H$	-6.2, -10 <sup>†</sup>	-6.2*, -10.5 <sup>†</sup>	-10*, -23 <sup>†</sup>	+2.5*, +3 <sup>†</sup>	0*, -4 <sup>†</sup>
$\phi_{H\uparrow}$	-7*, -10 <sup>†</sup>	-6*, -10 <sup>†</sup>	-14*, -24 <sup>†</sup>	-4.7*, +3 <sup>†</sup>	-1.4*, -3 <sup>†</sup>
$\phi_{H\downarrow}$	+6*, +10 <sup>†</sup>	+2.5*, +7 <sup>†</sup>	+3.5*, +10 <sup>†</sup>	+3*, +3 <sup>†</sup>	+1.3*, -5 <sup>†</sup>



expression

$$R_s = a\rho + b\rho^2, \tag{33}$$

or alternatively (fig. 16)

$$\phi_H = \phi_H^0 + B\rho, \tag{34}$$

if the variation of the magnetization with impurity concentration is neglected. It is usually assumed that this represents a separation into the skew scattering term and the side jump term. For most Ni based alloy series, as we have seen the values of  $\phi_H^0$  vary considerably, but the values of  $B$  hardly vary from one impurity to another, with  $B \approx -1$  millirad/ $\mu\Omega\text{cm}$ . However, for those Ni based alloys with  $\rho_{\downarrow}(0)/\rho_{\uparrow}(0) \gg 1$ , the data as a function of concentration cannot be represented by eq. (33) unless only a very restricted range of concentration is considered. It is interesting to note that these particular alloys are those which also show anomalous  $R_0$  and resistivity anisotropy behaviour as a function of concentration.

At room temperature,  $\rho$  in Ni and Ni alloys is always "high" so that the side jump mechanism can be assumed to dominate. The experimental value of the ratio  $R_s/\rho^2$  increases from the pure Ni value,  $R_s/\rho^2 \sim 0.1 (\Omega\text{cmG})^{-1}$ , as a function of impurity concentration and rapidly saturates at a plateau value of about  $0.15 (\Omega\text{cmG})^{-1}$  for a wide range of Ni alloys (Köster and Gmöhling 1961, Köster and Romer 1964), (fig. 17). The room temperature  $R_s/\rho^2$  values for the alloys are close to the values at low temperatures for the same alloys (Dorleijn 1976). However, for certain alloy systems  $R_s/\rho^2$  measured at room temperature changes steadily with impurity concentration. Thus for NiFe,  $R_s$  changes sign at about 15% Fe (Smit 1955, Kondorskii 1964). Alloys with this concentration of Fe show low values of  $R_s/\rho^2$  even if a second high resistivity impurity is introduced (Levine 1961).

In pure Fe and FeSi alloys,  $R_s/\rho^2$  is remarkably constant over a wide range of concentrations and temperatures (Kooi 1954, Okamoto et al. 1962, where this ratio remains constant although  $R_s$  varies over three decades) (fig. 18). For other Fe based alloys the ratio generally approaches the pure Fe value at moderate or

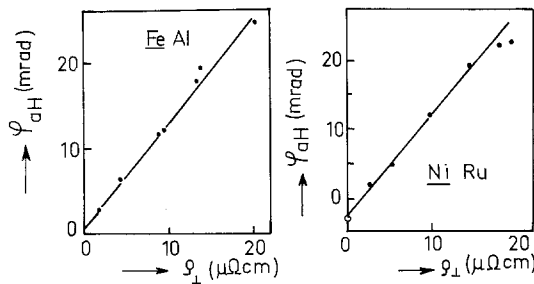


Fig. 16. The extraordinary Hall angle at 4.2 K as a function of the residual resistivity of FeAl and NiRu alloys (Dorleijn 1976).

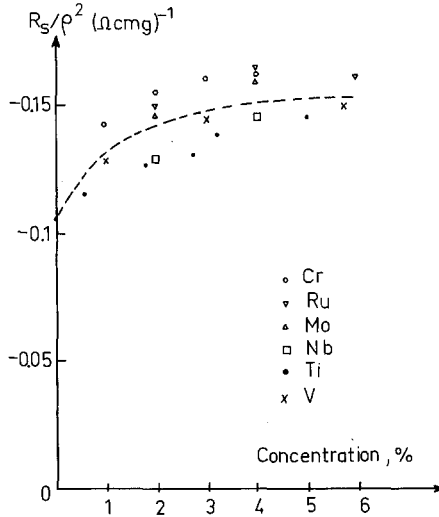


Fig. 17. The ratio  $R_s/\rho^2$  in Ni and Ni alloys at room temperature (after Köster et al. 1961 and 1964).

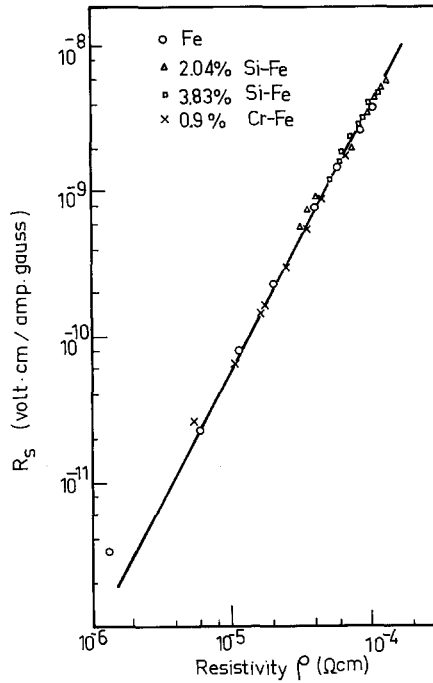


Fig. 18. Log-log plot of  $R_s$  against  $\rho$  for Fe and some Fe alloys above nitrogen temperature (after Okamoto et al. 1962).

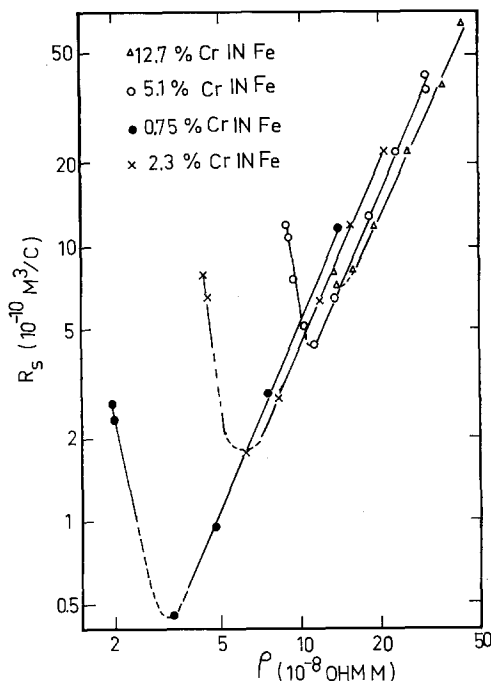


Fig. 19. Log-log plot of  $R_s$  against  $\rho$  for FeCr alloys, with temperature as an implicit variable (after Carter and Pugh 1966).

high temperatures (Soffer et al. 1965, Carter and Pugh 1966). However, at low temperatures where skew scattering can be important, the behaviour can be completely different (fig. 19) (Carter and Pugh 1966). It seems that in the FeCr case, there is a strong skew scattering effect at low temperatures which has disappeared by room temperature (but see Majumdar and Berger 1973). Dorleijn (1976) has made an analysis in terms of skew scattering, side jump and ordinary Hall effect in Fe alloys at helium temperatures, but the interpretation is tricky, particularly because samples frequently show a field dependent Hall coefficient.

The extraordinary Hall coefficient has been measured as a function of temperature in pure Co (Cheremushkina and Vasileva 1966).

Kondorskii (1969) suggested that the sign of the side jump effect was related to the charge and polarization of the dominant carriers, which can be compared with the model outlined above. No satisfactory quantitative estimates of the size of the effect seem to have been made for ferromagnetic metals, and other basic questions concerning this mechanism remain open.

The anisotropy of the Hall effect in single crystals is technically difficult to study, and, as a result, the existence of an anisotropy in the extraordinary Hall coefficient of cubic metals has been uncertain. Now evidence has been provided for the anisotropy in  $R_s$  for Fe (Hirsch and Weissmann 1973) and for Ni (Hiraoka 1968) at room temperature. In hexagonal Co both  $R_0$  and  $R_s$  are highly anisotro-

pic (Volkenshtein et al. 1961) which means that measurements on hcp Co polycrystals are subject to severe texture problems.

3.5. Thermoelectric power

In non-magnetic metals under elastic scattering conditions, the thermoelectric power (TEP) coefficient depends on the differential of the resistivity at the Fermi surface through the Mott formula:

$$S = \frac{\pi^2}{3} \frac{k_B^2}{|e|} T \frac{1}{\rho} \left[ \frac{d\rho}{dE} \right]_{E_F}$$

In ferromagnets the situation is complicated by the existence of the two spin currents at low temperatures and by magnetic scattering at higher temperatures.

The TEP curves as a function of temperatures for Fe, Co and Ni metals show effects which are clearly due to ferromagnetic ordering (fig. 20). For Co and Ni, the curve of  $S(T)$  shows a bulge towards negative values of  $S$  in the ferromagnetic temperature range, and a distinct change of slope at  $T_c$ . For Fe, the behaviour is similar but complicated by a positive hump in  $S(T)$  just below room temperature.

The critical behaviour of  $S(T)$  has attracted considerable attention. In Ni, the curve for  $dS/dT$  near  $T_c$  resembles the specific heat curve in the same way as does  $d\rho/dT$  (Tang et al. 1971). Although it has been argued that the TEP anomaly represents strictly the specific heat of the itinerant electrons (Tang et al. 1972) a more reasonable interpretation is in terms of the critical behaviour of the elastic scattering (Thomas et al. 1972). Combining the Mott formula and the expression

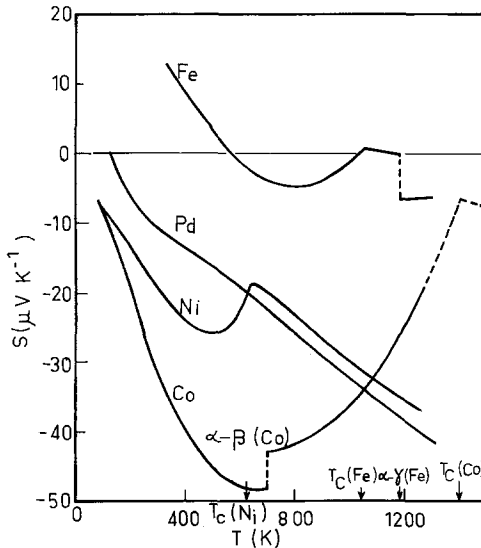


Fig. 20. The absolute thermoelectric power of Ni, Fe, Pd and Co (Laubitz et al. 1976).

for the resistivity as a function of  $k$  near  $T_c$  leads to

$$S = S_p - \frac{1}{2}A_0T(1 + \rho_n/\rho),$$

where  $A_0 = 2\pi^2k_B^2/3|e|E_F$ , and  $S_p$  is the background non-magnetic TEP. Results on  $GdNi_2$  were discussed in terms of this approach (Zoric et al. 1973).

The systematics of  $S(T)$  were studied at room temperature and above in a number of Ni based alloys (Vedernikov and Kolmets 1961, Kolmets and Vedernikov 1962, Köster and Gmöhling 1961, Köster and Romer 1964).  $S$  at room temperature becomes rapidly more positive with impurity concentration for those alloys for which  $\rho_{\downarrow}(0)/\rho_{\uparrow}(0) \leq 1$  (NiV, NiCr . . .) while  $S$  becomes more negative for alloys with  $\rho_{\downarrow}(0)/\rho_{\uparrow}(0) \geq 1$  (fig. 21). The negative bulge in  $S(T)$  remains very strong for a wide range of NiFe alloys measured up to  $T_c$  (Basargin and Zakharov 1974), but tends to disappear in NiV alloys (Vedemikov and Kolmets 1961).

The low temperature TEP of Ni based alloys has been analyzed using the two current model (Farrell and Greig 1969, 1970, Cadeville and Roussel 1971). If the intrinsic TEP coefficients for the two spin directions are  $S_{\uparrow}$  and  $S_{\downarrow}$  then the observed value of  $S$  should be  $S = (\rho_{\downarrow}S_{\uparrow} + \rho_{\uparrow}S_{\downarrow})/(\rho_{\uparrow} + \rho_{\downarrow})$  at low temperatures; at high temperatures where the two currents are mixed, the impurity diffusion thermopower becomes  $S = \frac{1}{2}(S_{\uparrow} + S_{\downarrow})$ . Using these two expressions, Farrell and Greig (1969) extracted  $S_{\uparrow}$ ,  $S_{\downarrow}$  for a number of impurities in Ni and similar analyses have been done in Ni and Co based alloys (Cadeville et al. 1968, Cadeville 1970, Cadeville and Roussel 1971). A detailed discussion has been given

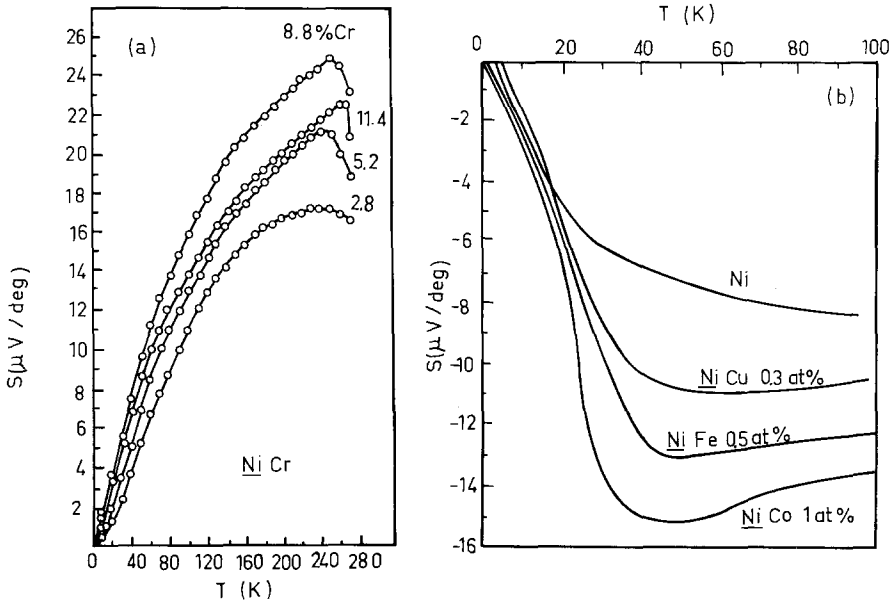


Fig. 21. The absolute thermoelectric power of some nickel based alloys as a function of temperature (after Beilin et al. 1974 and Farrell and Greig 1970). (a) NiCr; (b) Ni alloys.

of the relationship between the electronic structure of the impurity and the TEP coefficients (Cadeville and Roussel 1971).

Another aspect of the two current situation is the influence of magnon-electron scattering (Korenblit and Lazarenko 1971). Scattering of a spin  $\downarrow$  electron to a spin  $\uparrow$  state involves the creation of a magnon, which needs positive energy, while spin  $\uparrow$  to spin  $\downarrow$  scattering is through the destruction of a magnon. The electron-magnon scattering will then lead to a positive term in  $S$  at moderate temperatures in alloys where the spin  $\downarrow$  current dominates, and a negative term in alloys where the spin  $\uparrow$  current dominates. The TEP due to this effect will be superimposed on the elastic electron-impurity term except at very low temperatures, and will complicate the analysis of the diffusion terms. Results on Ni alloys have been interpreted with this mechanism (Beilin et al. 1974).

A magnon drag effect has been suggested (Bailyn 1962, Gurevich and Korenblit 1964, Blatt et al. 1967). Measurements on the TEP in a  $\text{NiCu}$  and a  $\text{NiFe}$  alloy in applied fields appear to be consistent with this mechanism (Granneman and Berger 1976). However, the strong positive TEP hump in pure Fe does not have this origin (Blatt 1972).

The value of  $S$  is anisotropic with respect to the magnetization direction in a ferromagnet. Measurements on Fe and Ni single crystals at room temperature (Miyata and Funatogawa 1954) gave

$$\Delta S_{100} = +0.70 \mu\text{V/K}, \quad \Delta S_{111} = -0.13 \mu\text{V/K} \quad \text{in Fe,}$$

and

$$\Delta S_{100} = +0.57 \mu\text{V/K}, \quad \Delta S_{111} = +0.69 \mu\text{V/K} \quad \text{in Ni.}$$

The Fe result was confirmed by Blatt (1972).

### 3.6. Nernst-Ettingshausen effect

This is the thermoelectric analogue of the Hall effect. It has been studied in the pure ferromagnetic metals and in a number of alloys (Ivanova 1959, Kondorskii and Vasileva 1964, Cheremushkina and Vasileva 1966, Kondorskii et al. 1972, Vasileva and Kadyrov 1975). Like  $R_s$ , this coefficient varies strongly with temperature in ferromagnets. Kondorskii (1964) proposed the phenomenological relationship

$$Q = -(\alpha + \beta\rho)T,$$

and the origin of the effect was discussed in terms of the side jump mechanism by Berger (1972) and Campbell (1979).

### 3.7. Thermal conductivity

This is not a purely electron transport effect, as heat can be carried also by phonons and even magnons, and separating out the different contributions is difficult. Farrell and Greig (1969) in careful measurements on Ni and Ni alloys have shown that a coherent analysis of the alloy data needs to take into account

the two current character of the conduction. They found that it was not possible to decide for or against the presence of any electron–electron term in pure Ni at low temperatures (White and Tainsh 1967).

At higher temperatures, Tursky and Koch (1970) have shown that it is possible to use the spontaneous resistivity anisotropy to separate out phonon and electron thermal conductivity.

By measurements in strong fields, Yelon and Berger (1972) identified a magnon contribution to the low temperature thermal conductivity in NiFe.

The thermal conductivity of Ni shows an abrupt change of slope at  $T_c$  (Laubitz et al. 1976). This property is very difficult to measure with high precision.

## 4. Dilute ferromagnetic alloys

### 4.1. Palladium based alloys

It has been known for some time that PdFe, PdCo, PdMn and PdNi alloys are “giant moment” ferromagnets at low concentrations; the transport properties of these systems have been well studied.

#### 4.1.1. Resistivity and isotropic magnetoresistance

PdFe alloys are soft ferromagnets down to at least 0.15% Fe. The Fe magnetization at  $T \ll T_c$  saturates completely in small applied fields (Chouteau and Tournier 1972, Howarth 1979). The magnetic disorder at relatively low temperatures is in the form of magnons; for the dilute alloys ( $C < 2\%$  Fe), it appears that the magnon–electron scattering is essentially incoherent so the magnetic resistivity is proportional to the number of magnons present, leading to a temperature dependent resistivity proportional to  $T^{3/2}$  for  $T \ll T_c$  and a characteristic temperature dependent negative magnetoresistance (Long and Turner 1970, Williams and Loram 1969, Williams et al. 1971, Hamzić and Campbell 1978). At higher concentrations a  $T^2$  resistivity variation replaces the  $T^{3/2}$  behaviour (Skalski et al. 1970). At the Curie temperature there is a change in slope of the  $\rho(T)$  curve but it is difficult to analyze the results in terms of critical scattering behaviour because of smearing due to the spread of  $T_c$  values in the samples (Kawatra et al. 1969).

PdMn alloys are “ferromagnets” below 4% Mn concentration in that they show a high initial susceptibility below a well defined ordering temperature (Rault and Burger 1969, Coles et al. 1975). In fact, high field magnetization measurements (Star et al. 1975) show that the Mn magnetization only becomes truly saturated when very strong magnetic fields are applied. The temperature dependence of the resistivity of these alloys is qualitatively similar to that observed in PdFe, with a change of slope in  $\rho(T)$  at  $T_c$  and a  $T^{3/2}$  variation of the resistivity at low temperatures (Williams and Loram 1969). In contrast to the PdFe alloys the magnetoresistance remains strongly negative even when  $T$  tends to zero (Williams et al. 1973).

PdCo alloys have very similar ordering temperatures and total magnetic moments per atom as the PdFe alloys (Nieuwenhuys 1975), and the temperature dependence of the resistivity is again of the same type (Williams 1970). However the paramagnetic resistivity at  $T > T_c$  is proportional to the Co concentration (Colp and Williams 1972) whereas in PdFe alloys it increases as the square of the Fe concentration (Skalski et al. 1970). The PdCo alloys below 5% Co show a negative magnetoresistance at  $T \ll T_c$  which indicates that they are not true ferromagnets (Hamzić et al. 1978a)\*.

PdNi alloys are ferromagnets above a critical concentration of 2.3% Ni (Tari and Coles 1971). Near this concentration the low temperature variation of the resistivity of the alloys becomes particularly strong (Tari and Coles 1971). Both the paramagnetic and ferromagnetic alloys show a large positive magnetoresistance due to an increase in the local moments at the Ni sites with the applied field (Genicon et al. 1974, Hamzić et al. 1978a).

#### 4.1.2. Magnetoresistance anisotropy

PdFe, PdCo and PdNi alloys all show positive anisotropies  $\rho_{\parallel} > \rho_{\perp}$  at moderate magnetic impurity concentrations. At low concentrations PdFe samples show vanishingly small anisotropies (Hamzić et al. 1978a). From this and other evidence it has been concluded that the Co and Ni impurities carry local orbital moments.

#### 4.1.3. Extraordinary Hall effect

Over a broad concentration range the Hall coefficient in PdFe alloys behaves similarly to that in concentrated NiFe alloys, changing sign near 20% Fe (Matveev et al. 1977, Dreesen and Pugh 1960). At low concentrations the Hall angle tends to zero for PdFe and PdMn but takes on a concentration independent value for PdNi and PdCo (Hamzić et al. 1978b, Abramova et al. 1974). This should be related to the local orbital moments of Co and Ni impurities.

#### 4.1.4. Thermoelectric power

In the concentrated ferromagnets, features clearly associated with the ferromagnetic ordering are visible in the temperature dependence of the TEP. For the Pd based alloys this does not seem to be the case except perhaps when the magnetic impurity concentration is greater than 5% (Gainon and Sierro 1970). At 1%, or lower, concentrations PdFe and PdMn show weak negative or positive TEP below 20 K varying in a rather complex way with concentration and temperature (Gainon and Sierro 1970, Macdonald et al. 1962, Schroeder and Uher 1978). Pd 1% Co shows a negative TEP hump at 20 K (Gainon and Sierro 1970); this hump becomes more pronounced and goes to lower temperatures as the concentration is decreased (Hamzić, 1980). Below the critical concentration PdNi alloys show a strong negative hump in the TEP around 15 K which disappears once the concentration exceeds the critical value (Foiles 1978).

\* They can be considered to be "quasiferromagnets", i.e., systems having an overall magnetic moment but where the local moments are each somewhat disoriented with respect to the average moment direction.



#### 4.2. Platinum based alloys

Again, PtFe and PtCo are giant moment ferromagnets at concentrations of a few percent, but at lower concentrations the behaviour is more complicated. For PtFe below about 0.8% spin glass order sets in (Ododo 1979).

In the ferromagnetic concentration range there is the usual step in  $\rho(T)$  at the ordering temperature, but below 0.8% Fe this step disappears (Loram et al. 1972). The isotropic magnetoresistance is strongly negative at concentrations less than about 5% Fe (Hamzić et al. 1981).

PtCo alloys below 1% Co show resistivity variations which are complex because of competing tendencies to Kondo condensation and to magnetic ordering (Rao et al. 1975, Williams et al. 1975). At concentrations above about 1% Co a step can be seen in  $\rho(T)$  at  $T_c$ . The isotropic magnetoresistance is positive at low concentrations, becoming negative by 2% Co (Lee et al. 1978, Hamzić et al. 1980).

Both PtFe and PtCo alloys show concentration independent resistivity anisotropies and extraordinary Hall angles at low concentrations (Hamzić et al. 1979).

The low temperature thermoelectric power of PtCo alloys becomes strongly negative below about 2% Co concentration (Lee et al. 1978). This TEP is sensitive to applied magnetic fields.

PtMn alloys are spin glasses (Sarkissian and Taylor 1974), and PtNi alloys are not magnetically ordered below 42% Ni.

### 5. Amorphous alloys

Since the early 1970s considerable effort has been devoted to the study of the electrical and magnetic properties of amorphous alloys. The resistivity minimum observed in many systems has been subject to much controversy.

#### 5.1. Resistivity of amorphous alloys

The amorphous alloys have a very high resistivity ( $\rho \gtrsim 100 \mu\Omega\text{cm}$ ) which changes relatively little as a function of temperature. Figure 22 shows that, in series of NiP alloys, the temperature coefficient changes from positive to negative as the concentration of P increases. This behaviour is well explained in the Ziman model of the resistivity of liquid metals (Ziman 1961) and its extension to amorphous alloys (Nagel 1977). In the Ziman model the resistivity turns out to be proportional to  $a(2k_F)$  where  $k_F$  is the Fermi wave vector and  $a(q)$  the atomic structure factor. If  $2k_F$  is close to the first peak of  $a(q)$ , the resistivity is high and decreases as a function of  $T$  owing to the thermal broadening of the peak. In contrast, if  $2k_F$  lies well below (or well above) the peak, the resistivity is relatively low and increases as a function of  $T$ . In the NiP alloys (fig. 22) the additional conduction electrons provided by the higher concentrations of P raise  $2k_F$  to the first peak of  $a(q)$ , which accounts for the experimental behaviour (Cote 1976). On the other hand, the small resistivity upturns observed in NiP at low temperature (fig. 22)

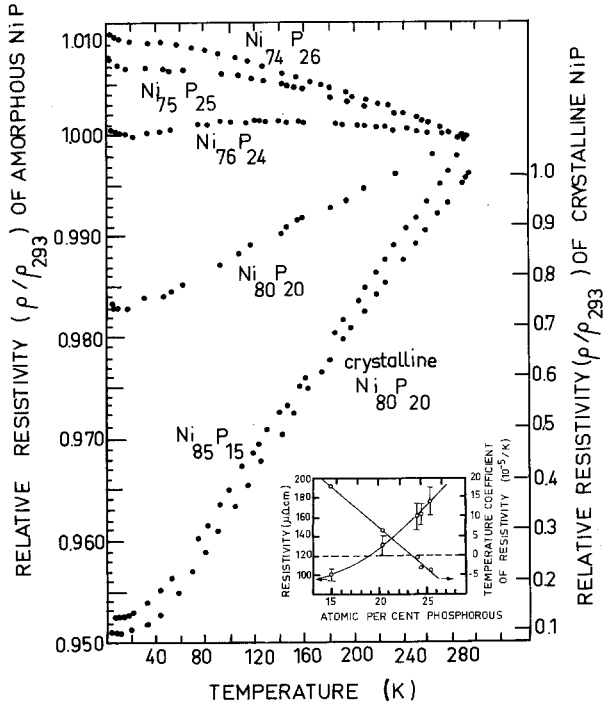


Fig. 22. Relative resistivities of amorphous and crystalline Ni-P alloys. Nominal compositions are indicated;  $(\rho_{\text{am}}/\rho_{\text{cryst}})_{293} \approx 3$ . The inset shows  $\rho$  and  $d\rho/dT$  as a function of the composition (after Cote 1976). Similar results are given by Boucher (1973) for Ni-Pd-P amorphous alloys.

cannot be explained by the Ziman model. Such resistivity upturns, which generally give rise to a resistivity minimum, have been found in many amorphous systems. They have been found in both ferromagnetic and non-ferromagnetic amorphous alloys and, up to now, only in alloys containing transition (or rare-earth) metals. Their origin has been subject to much controversy.

Resistivity minima have been first found by Hasegawa and Tsui (1971a, b) in amorphous PdSi containing Cr, Mn, Fe or Co impurities (fig. 23). The classical features of the Kondo effect are observed: the resistivity varies logarithmically over a large temperature range and becomes constant in the low temperature limit; at low concentration of magnetic impurities the logarithmic term increases with the concentration, there is a negative magnetoresistance. But, surprisingly, the resistivity minimum still exists in the most concentrated alloys which are ferromagnetic. These results seem to indicate that weakly coupled moments subsist in amorphous ferromagnets and can give rise to Kondo scattering. Results on many other systems have suggested that the coexistence of ferromagnetism and Kondo effect is quite general in amorphous alloys; thus large logarithmic upturns have been observed (fig. 24) in ferromagnets of the series FeNiB, FeNiPB, FeNiPC, FeNPBS (Cochrane et al. 1978, Babić et al. 1978, Steward and Phillips 1978), FeNiPBAl, FeMnPBAl, CoPBAl (Rao et al. 1979), PdCoP (Marzwell 1977); in

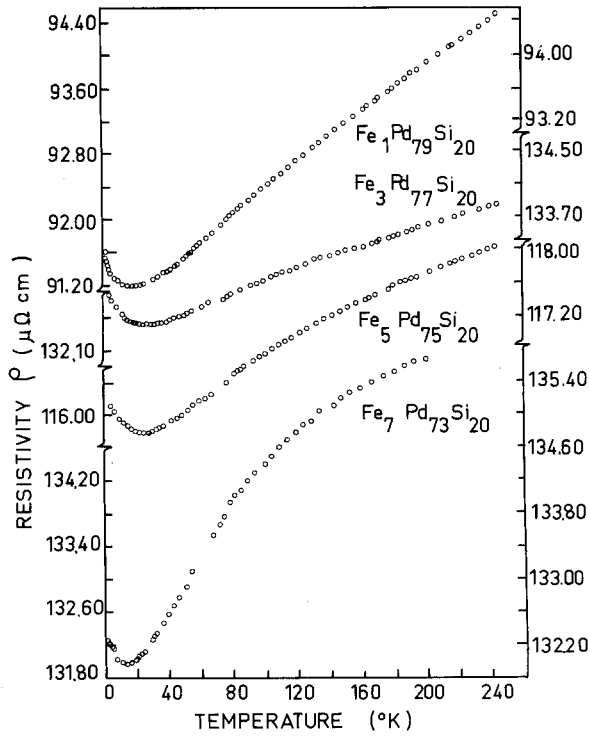


Fig. 23. Resistivity versus temperature for  $\text{Fe}_x\text{Pd}_{80-x}\text{Si}_{20}$  alloys.  $T_c \approx 28$  K for  $x = 7$  (after Hasegawa and Tsuei 1970b).

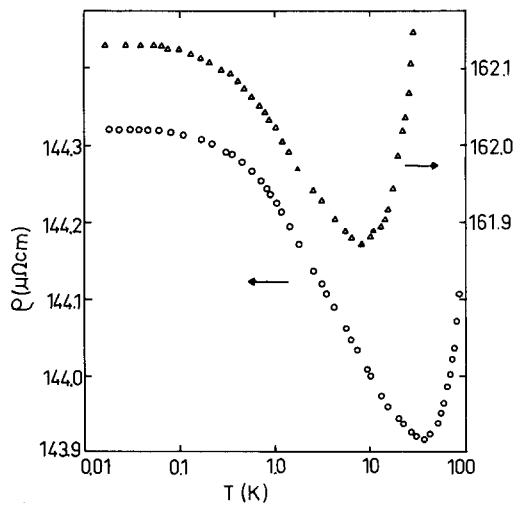


Fig. 24. Resistivity versus  $\log T$  plot for two ferromagnetic amorphous alloys: ( $\Delta$ )  $\text{Fe}_{75}\text{P}_{16}\text{B}_6\text{Al}_3$  and ( $\circ$ )  $\text{Fe}_{60}\text{Ni}_{15}\text{P}_{16}\text{B}_6\text{Al}_3$  (after Rapp et al. 1978).

many cases the addition of small amounts of Cr strongly enhances the resistivity upturn.

On the other hand, Cochrane et al. (1975) found that the logarithmic resistivity upturn of several amorphous alloys was field independent, in contrast to what is generally observed in Kondo systems. They also noticed a logarithmic upturn in NiP alloys with high P concentration in which the Ni atoms were not supposed to carry a magnetic moment. On the basis of these observations they ruled out the explanation by the Kondo effect and proposed a non-magnetic mechanism. Their model treats the electron scattering by the two level systems which are supposed to be associated with structural instabilities in amorphous systems; a variation of the resistivity in  $-\ln(T^2 + \Delta^2)$  is predicted, where  $\Delta$  is a mean value of the energy difference between the two levels. The resistivity curves of several amorphous alloys fit rather well with such a variation law.

At the present time (1979) however the trend is in favour of an explanation of the resistivity minima by the Kondo effect rather than by a non-magnetic mechanism. Clear examples of logarithmic resistivity upturns in non-magnetic systems are still lacking: alloys such as NiP or YNi can be suspected to contain magnetic Ni clusters (Berrada et al. 1978). On the other hand, systematic studies of the resistivity of FeNiPB (Babić et al. 1978), FeNiPBAl, FeMnPBAl (Rao et al. 1979) have shown definite correlations between the resistivity anomalies and the magnetic properties (logarithmic term large when  $T_c$  is small, etc.); it has been also found in several systems that the logarithmic upturn is lowered by an applied field. Finally, Mössbauer experiments on FeNiCrPB alloys have found very small hyperfine fields on a significant number of Fe sites, which seems to confirm the coexistence of ferromagnetism and Kondo effect (Chien 1979).

What we have written up to now concerned the metal-metalloid alloys which have been the most studied amorphous alloys. Studies of metal-metal amorphous alloys of rare-earths with transition or noble metals have been also developed recently. Resistivity minima have been again observed in these systems but appear to be generally due to contributions from magnetic ordering and not to Kondo effect. In  $\text{Ni}_3\text{Dy}$  (fig. 25) the resistivity increases either if a magnetic field is applied or if the temperature is lowered below the ordering temperature  $T_c$ . This suggests a positive contribution from magnetic ordering to the resistivity, in contrast to what is observed in crystalline ferromagnetics. This has been ascribed by Asomoza et al. (1977a, 1978) to coherent exchange scattering by the rare-earth spins (Ni has no magnetic moment in these alloys). The model calculation predicts a resistivity term proportional to  $m(2k_F)$  where  $m(q)$  is the spin correlation function

$$m(q) = \frac{1}{NC_1^2 J(J+1)} \sum_{\mathbf{R}, \mathbf{R}' \neq \mathbf{R}} \exp[i\mathbf{q} \cdot (\mathbf{R} - \mathbf{R}')] \mathbf{J}_{\mathbf{R}} \cdot \mathbf{J}_{\mathbf{R}'}$$

Here  $C_1$  is the concentration of magnetic ions, having local moments  $J$  and placed at  $\mathbf{R}, \mathbf{R}'$ ; the sum is over the pairs of magnetic ions.

The resistivity will depend on the magnetic order through  $m(2k_F)$ ; for example,

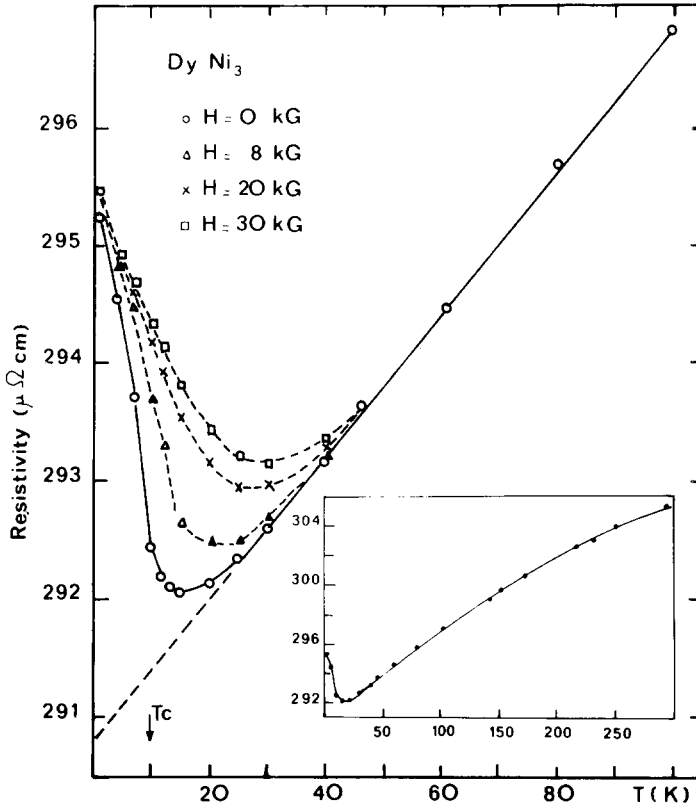


Fig. 25. The resistivity of a  $\text{DyNi}_3$  amorphous alloy in several applied fields is plotted as a function of temperature (after Asomoza et al. 1977).

ferromagnetic correlations will increase or decrease  $\rho$  according to whether the interferences are constructive or destructive. The  $\text{Ni}_3$ -RE alloys should correspond to the case of ferromagnetic correlations and constructive interferences. The Ag-RE, Au-RE and Al-RE amorphous alloys also show a clear contribution from magnetic ordering to the resistivity, but the interpretation seems to be a little more complicated than for the  $\text{Ni}_3$ -RE alloys (Asomoza et al. 1979, Fert and Asomoza 1979). Finally, alloys of the series Fe-RE and Co-RE generally show a monotonic decrease of the resistivity from the helium range to room temperature (Cochrane et al. 1978, Zen et al. 1979). In these alloys of high  $T_c$  the variation of the resistivity due to magnetic ordering must be displayed over a wide temperature range and is certainly difficult to separate from the normal variation due to the phonons and the thermal variation of the structure factor. We believe that this normal variation should be predominant, specially at low temperature. Similarly, in the alloys such as FeNiPB discussed above, a contribution from magnetic ordering to  $\rho(T)$  certainly exists but is likely covered up by other contributions (Kondo or structural effects) at low temperature.

### 5.2. Hall effect and resistivity anisotropy of amorphous alloys

The amorphous ferromagnetic alloys have a very large extraordinary Hall effect which generally covers up the ordinary Hall effect. This is because the extraordinary Hall resistivity, in contrast to the ordinary one, is an increasing function of the scattering rate (the contributions from skew scattering and side-jump are roughly proportional to  $\rho$  and  $\rho^2$  respectively). Thus  $\rho_H(B)$  is practically proportional to the magnetization in many systems and, for example, is frequently used to record hysteresis loops (McGuire et al. 1977a, b, Asomoza et al. 1977b).

The extraordinary Hall effect of ferromagnetic alloys of gold with nickel, cobalt or iron has been studied by Bergmann and Marquardt (1979) and ascribed to skew scattering; the change of sign of  $\rho_H$  between Ni and Fe has been accounted for by a model based on a virtual bound state picture of the 3d electrons. On the other hand, the extraordinary Hall effect of FeNiPB alloys rather suggest a side-jump mechanism (Malmhäll et al. 1978). The extraordinary Hall effect has been also studied in amorphous alloys of transition metals with rare-earths and related to the magnetization of the transition and rare-earth sublattices in phenomenological models (Kobliska and Gangulee 1977, McGuire et al. 1977, Asomoza et al. 1977b).

The spontaneous resistivity anisotropy is rather large in amorphous alloys of gold with nickel or cobalt ( $\rho_{\parallel} - \rho_{\perp} \approx 1 \mu\Omega\text{cm}$ ) and has been interpreted in a model of virtual bound state for the 3d electrons (Bergmann and Marquardt 1979). The resistivity anisotropy seems to be smaller in alloys of the FeNiP type (Marohnić et al. 1977). The resistivity anisotropy has been also studied in amorphous alloys of nickel or silver with rare-earths and turns out to be mainly due to electron scattering by the electric quadrupole of the 4f electrons (Asomoza et al. 1979).

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