Two-current model of AMR

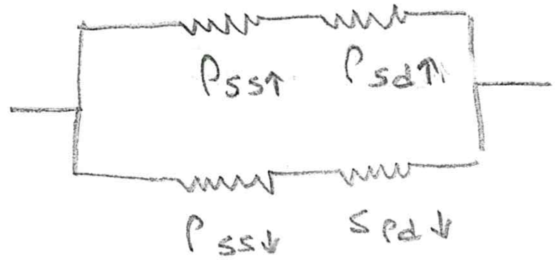
$\sigma = \sigma_{s\uparrow} + \sigma_{s\downarrow}$ since d-electrons do not participate in transport (an approx)

$\frac{1}{\rho} = \frac{1}{\rho_{s\uparrow}} + \frac{1}{\rho_{s\downarrow}}$. Assuming no spin-flip,

$\rho_{s\uparrow} = \rho_{s\uparrow s\uparrow} + \rho_{s\uparrow d\uparrow}$ and $\rho_{s\downarrow} = \rho_{s\downarrow s\downarrow} + \rho_{s\downarrow d\downarrow}$

initial state final state

$\Rightarrow \frac{1}{\rho} = \frac{1}{\rho_{s\uparrow d\uparrow} + \rho_{s\uparrow s\uparrow}} + \frac{1}{\rho_{s\downarrow s\downarrow} + \rho_{s\downarrow d\downarrow}} \Rightarrow$



As stated before $\rho_{s\uparrow d\uparrow} = 0$ since there are no pure $d\uparrow$ states available. As we will see, $s \to s$ scattering has no anisotropy.

But both $s\uparrow$ and $s\downarrow$ states can scatter into $d\downarrow$ band. It might be obvious for $s\downarrow \to d\downarrow$, but not $s\uparrow \to d\downarrow$ without spin flip which we are not considering. So why are we considering $s\uparrow \to d\downarrow$ scattering? It is because $d\downarrow$ states are mixed with a small fraction of the $d\uparrow$ states due to spin-orbit coupling. Therefore, both $s\uparrow$ (and $s\downarrow$) states can scatter into the $d\downarrow$ band without spin flip. We will see that scattering is more pronounced (i.e, higher resistivity) when the magnetic moment line up along the direction of current compared to when they are perpendicular (ie, $\rho_{\parallel} > \rho_{\perp}$)

Ordinary MR effect depends on direction and magnitude of applied field, while AMR depends on the direction of the spontaneous magnetization.

$$\frac{\Delta\rho}{\rho_{av}} = \frac{\rho_{\parallel} - \rho_{\perp}}{\frac{1}{3}\rho_{\parallel} + \frac{2}{3}\rho_{\perp}}$$

	$\frac{\Delta\rho}{\rho_{av}}$ (%) @300K
Fe	0.2
Co	1.9
Ni	2.0

We can replace ρ_{av} by $\rho_{\perp}$ or $\rho_{\parallel}$ approximately.

What causes AMR?

In the case of AMR, and any MR effect in general wrt magnetic materials, we have to consider the resistivities of up and down spin electrons separately. The justification is intuitive in the sense that the band structure of 3d ~~etc~~ ferromagnets are different due to spin-splitting (Stoner model). Therefore the current $j = -nev$ can be thought of as different for up and dn spin.

$$\vec{j} = -ne(\vec{v}_\uparrow + \vec{v}_\downarrow)$$

Similarly $\sigma = \sigma_\uparrow + \sigma_\downarrow = \frac{n_\uparrow e^2 \tau_\uparrow}{m} + \frac{n_\downarrow e^2 \tau_\downarrow}{m}$

or, $1/\rho = 1/\rho_\uparrow + 1/\rho_\downarrow = \frac{m}{n_\uparrow e^2}\frac{1}{\tau_\uparrow} + \frac{m}{n_\downarrow e^2}\frac{1}{\tau_\downarrow}$

Assuming $n_\uparrow \simeq n_\downarrow$ $\rho = \frac{m}{ne^2}\left(\frac{1}{\tau_\uparrow} + \frac{1}{\tau_\downarrow}\right)$ → This is analogous to the Mattheissen's law

$\frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2} + \cdots$ where ρ_1 and ρ_2 are resistivity due to various scattering mechanism such as electron-phonon, ~~ef~~ lattice imperfections etc.

So $\boxed{\sigma = \sigma_\uparrow + \sigma_\downarrow}$ or $\boxed{\rho = \frac{\rho_\uparrow \rho_\downarrow}{\rho_\uparrow + \rho_\downarrow}}$

Quantum mechanical treatment of AMR effect

First we need to see how d-states are mixed due to spin-orbit coupling which we treat as perturbation

In the unperturbed form, the electronic states that describe any electron inside an atom is given by

$$\phi^0_{n\ell}(r,\theta,\phi) = R_{n\ell}(r)\, Y_\ell^m(\theta,\phi)\, \chi(s) \qquad \boxed{\begin{matrix} \ell = 2 \text{ for d states} \\ m = \pm 2, \pm 1, 0 \end{matrix}}$$

where $\chi(s)$ is the spin function $= |\uparrow\rangle$ or $|\downarrow\rangle$

where $\hat{S}_z|\uparrow\rangle = +\frac{\hbar}{2}|\uparrow\rangle$ and $S_z|\downarrow\rangle = -\frac{\hbar}{2}|\downarrow\rangle$

ϕ^0_{ms} represent eigenstates of L^2, L_z, S^2, S_z operators.

for the unperturbed Hamiltonian $H_0 = \frac{p^2}{2m} + V(r) + \underbrace{H_{ex} S_z}_{\text{Zeeman-like term}}$

we have included $H_{ex} S_z$ to incorporate ferromagnetism directly into the Hamiltonian as a Zeeman-like term.

For $S_z = \pm\frac{1}{2}\hbar$ $H_{ex} S_z = \pm \frac{H_{ex}\hbar}{2}$ $\Rightarrow$ spin down electrons have lower energy as $\mu = \frac{-|e|}{m_e}\vec{S}$

This strategy avoids using a many-particle wave fn to bring about exchange interactions.

$H' = \lambda \bar{L}.\bar{S}$ the perturbation

The perturbed wave functions can be obtained by obtaining the eigenfunctions of the H' matrix elements. But in the d-orbital basis with spin included this will be a 10 X 10 matrix. Therefore perturbation theory is better suited.

Since $\lambda L\cdot S = \lambda\left(\frac{L_+S_- + L_-S_+}{2} + L_zS_z\right)$

L_+S_- and L_-S_+ will mix some states with different m and S_z values. Up to second order the perturbed wavefns are of the form

$$\phi_{+2\downarrow} = \underbrace{\left(1-\frac{\epsilon^2}{2}\right)}_{\text{Obtained from Normalization}}\phi^0_{2\downarrow} + \left(\epsilon + \overbrace{\frac{3}{2}\epsilon^2}^{2^{nd}\text{-order correction}}\right)\phi^0_{1\uparrow}$$

$\phi^0 \rightarrow$ unperturbed state

$\epsilon = \left(\frac{\lambda}{H_{ex}}\right)$

Note the probability of finding the perturbed state $\phi_{2\downarrow}$ in the unperturbed state is $\left(1-\epsilon^2/2\right)^2 < 1$

Similarly the other states for the $|\downarrow\rangle$ state

$$\phi_{+1\downarrow} = \left(1-\frac{3}{4}\epsilon^2\right)\phi^0_{1\downarrow} + \left(\sqrt{\frac{3}{2}}\,\epsilon + \frac{1}{2}\sqrt{\frac{3}{2}}\,\epsilon^2\right)\phi_{0\uparrow}$$

$$\phi_{0\downarrow} = \left(1-\frac{3}{4}\epsilon^2\right)\phi^0_{0\downarrow} + \left(\sqrt{\frac{3}{2}}\,\epsilon - \frac{1}{2}\sqrt{\frac{3}{2}}\,\epsilon^2\right)\phi_{-1\uparrow}$$

$$\phi_{-1\downarrow} = \left(1-\frac{1}{2}\epsilon^2\right)\phi_{-1\downarrow} + \left(\epsilon - \frac{3}{2}\epsilon^2\right)\phi_{-2\uparrow}$$

$$\phi_{-2\downarrow} = \phi^0_{-2\downarrow}$$

Convection: $\uparrow$ or $\downarrow$ refers to moment not spin ($\mu = -\frac{|e|}{m}\vec{S}$)
$\uparrow$ moment $\Rightarrow \downarrow$ spin
$\downarrow$ moment $\Rightarrow \uparrow$ spin
$\Rightarrow S_z|\uparrow\rangle = -\frac{\hbar}{2}|\uparrow\rangle$
$S_z|\downarrow\rangle = +\frac{\hbar}{2}|\downarrow\rangle$

Similarly we can write the states for the $|\uparrow\rangle$ states.

To proceed further (calculation of $1/\tau$), we need to look up the d orbital wavefunctions more closely, which are taken to be the real analogues of the complex spherical harmonics (Y_ℓ^m)

$$\phi_2^{+2 \to m} = \frac{1}{\sqrt{2}}\left[Y_2^{-2} + Y_2^{+2}\right] = \frac{1}{4}\sqrt{\frac{15}{\pi}}\,\frac{x^2-y^2}{r^2} = d_{x^2-y^2}$$

ℓ

$$\phi_2^{+1} = \frac{1}{\sqrt{2}}\left[Y_2^{-1} - Y_2^{+1}\right] = \frac{1}{4}\sqrt{\frac{15}{\pi}}\,\frac{zx}{r^2} = d_{xz}$$

$$\phi_2^{0} = Y_2^{0} = -\frac{1}{4}\sqrt{\frac{5}{\pi}}\,\frac{(x^2+y^2-2z^2)}{r^2} = \frac{1}{4\sqrt{3}}\sqrt{\frac{15}{\pi}}\,\frac{(r^2-3z^2)}{r^2} = d_{z^2}$$

$$\phi_2^{-1} = i\frac{1}{\sqrt{2}}\left[Y_2^{-1} + Y_2^{1}\right] = \frac{1}{2}\sqrt{\frac{15}{\pi}}\,\frac{yz}{r^2} = d_{yz}$$

$$\phi_2^{-2} = i\frac{1}{\sqrt{2}}\left[Y_2^{-2} - Y_2^{+2}\right] = \frac{1}{2}\sqrt{\frac{15}{\pi}}\,\frac{xy}{r^2} = d_{xy}.$$

Calculation of $1/\tau$:- Scattering is due to some impurity represent by a potential V_{imp}

$$\frac{1}{\tau} = T_{1\to 2} = \frac{2\pi}{\hbar}\left|\langle 2|V_{imp}|1\rangle\right|^2 g(\varepsilon_F) \qquad g(\varepsilon_F) \to \text{DOS of final state } |2\rangle$$

Here $|1\rangle$ is the s state which is the conduction electron with either $|\uparrow\rangle$ or $|\downarrow\rangle$ spin orientation

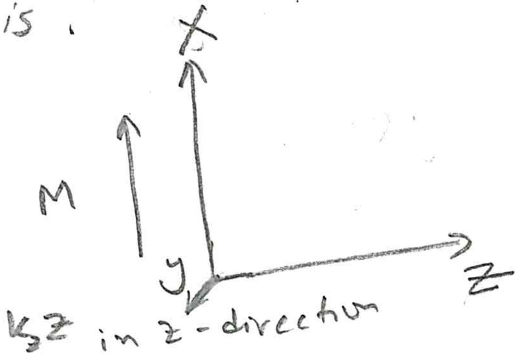
$$|1\rangle = e^{i\bar{k}\cdot\bar{r}}\,\chi(s) \qquad \chi(s) \to |\uparrow\rangle \text{ or } |\downarrow\rangle$$

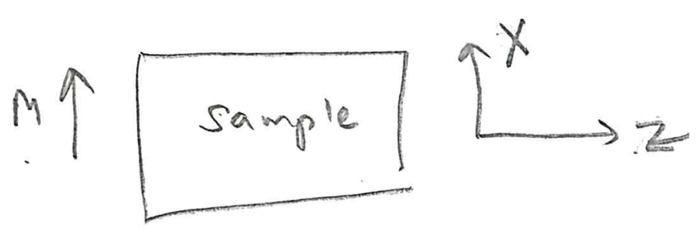
we are representing the conduction electron as a plane wave $e^{ik\cdot r}$. For the electron propagating along $+x$ direction $|1\rangle = e^{ik_x x}\,\chi(s)$. Similarly electron propagating in along z-axis $|1\rangle = e^{ik_z z}\,\chi(s)$

Lets fix Magnetization along z-axis.

parallel orientation $\Rightarrow$ z-direction

perpendicular orientation $\Rightarrow$ x-direction.





$I = e^{ik_z z}$ in z-direction

$= e^{ik_x x}$ in x-direction

Convention $\to$ majority spin $\uparrow$, spins whose moment are parallel to M.

minority spin, $\downarrow$, spin whose moment are antiparallel to M