Classical Approach to Paramagnetism

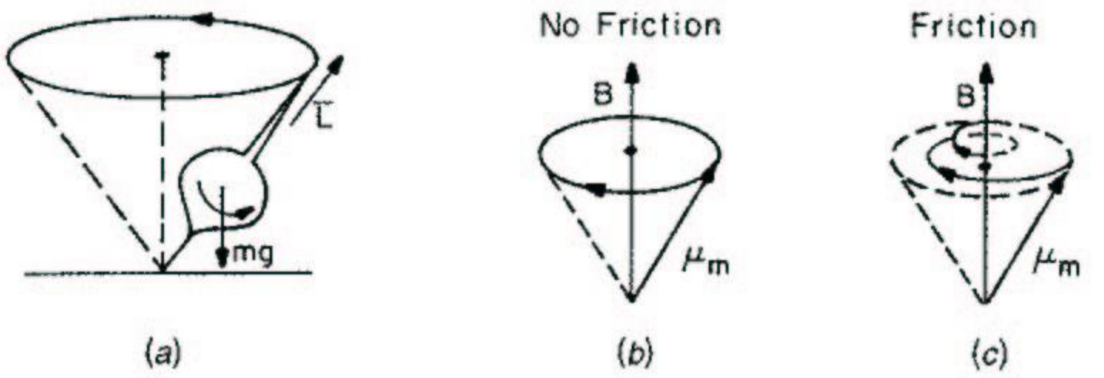


Figure 3.6 (*a*) A top precesses under the torque produced by the action of a gravitational force; (*b*) precession of magnetic moment under the action of a magnetic field; (*c*), same as center but with scattering present.

- Assume we have pre-existing magnetic dipoles in equilibrium.
- Energy of a magnetic dipole in a magnetic field:

$$E = -\mu_{\mathrm{M}} B \cos\theta$$

- Assume dissipation to end precession.

Classical Paramagnetism follows Langevin function

- Probability of finding dipole making angle θ to field:

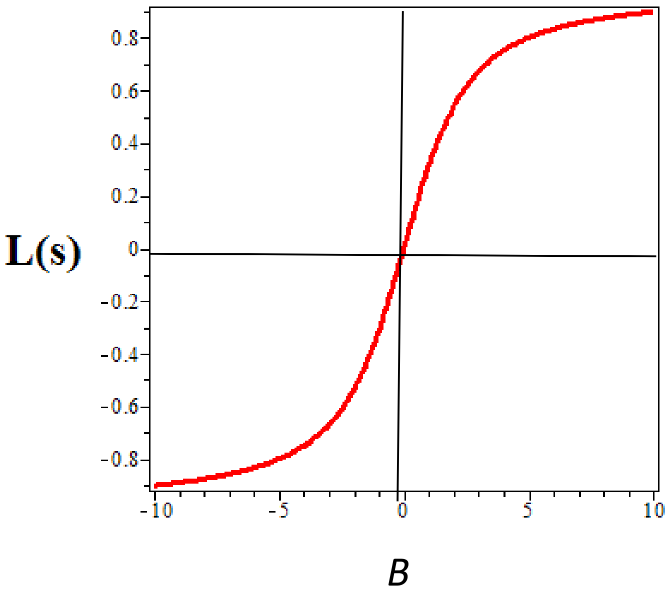
$$P = C\exp\left[\frac{-E}{k_B T}\right] = C\exp\left[\frac{\mu_M B\cos\theta}{k_B T}\right]$$

- Find average moment (component in direction of field):

$$\langle\mu_M\rangle = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta\ \mu_M\cos\theta\ \exp\left[\frac{\mu_M B\cos\theta}{k_B T}\right]}{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \exp\left[\frac{\mu_M B\cos\theta}{k_B T}\right]}$$

$$= \mu_M \frac{\int_{-1}^{1} dx\ x\exp(sx)}{\int_{-1}^{1} dx\exp(sx)} = \coth(s) - \frac{1}{s}$$

use $x = \cos\theta \quad s = \frac{\mu_M B}{k_B T}$



- May be applied to noninteracting atoms, molecules or particles
- There will be quantum corrections for low angular momentum

Notes on integration to obtain Langevin function:

$$\langle \mu_{\mathrm{M}} \rangle = \frac{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \; \mu_{\mathrm{M}} \cos\theta \; \exp\left[\frac{\mu_{\mathrm{M}} B \cos\theta}{k_B T}\right]}{\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \exp\left[\frac{\mu_{\mathrm{M}} B \cos\theta}{k_B T}\right]} = \mu_{\mathrm{M}} \frac{\int_0^{\pi} \sin\theta d\theta \; \cos\theta \; \exp\left[\frac{\mu_{\mathrm{M}} B \cos\theta}{k_B T}\right]}{\int_0^{\pi} \sin\theta d\theta \exp\left[\frac{\mu_{\mathrm{M}} B \cos\theta}{k_B T}\right]}$$

use $x = \cos\theta; \quad dx = -\sin\theta d\theta; \qquad s = \frac{\mu_{\mathrm{M}} B}{k_B T}$

$$\frac{\langle \mu_{\mathrm{M}} \rangle}{\mu_{\mathrm{M}}} = \frac{\int_{-1}^{1} dx \; x \exp(sx)}{\int_{-1}^{1} dx \exp(sx)} = \frac{1}{f(s)} \frac{df(s)}{ds}; \qquad f(s) = \int_{-1}^{1} dx \exp(sx) = \frac{1}{s}\left(e^{s} - e^{-s}\right)$$

$$\frac{\langle \mu_{\mathrm{M}} \rangle}{\mu_{\mathrm{M}}} = \frac{\frac{-1}{s^2}\left(e^{s} - e^{-s}\right) + \frac{1}{s}\left(e^{s} + e^{-s}\right)}{\frac{1}{s}\left(e^{s} - e^{-s}\right)} = \coth(s) - \frac{1}{s}$$