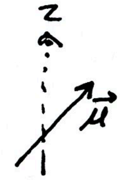
i) Magnetic moment of single electron is,



$$\vec{\mu} = -g_s \frac{e}{2m_e} \vec{S} = -\frac{e}{m} \vec{S} \qquad [g_s \approx 2]$$

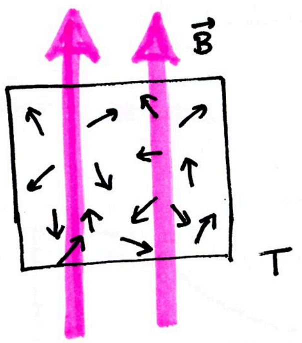
$$\mu_z = -\frac{e}{m} S_z = -\frac{e}{m} m_s \hbar$$

$$\mu_z = -\frac{e\hbar}{2m} \qquad \text{for } m_s = \tfrac{1}{2}$$

$$= \frac{e\hbar}{2m} \qquad \text{for } m_s = -\tfrac{1}{2}$$

$$\therefore \mu_z = \pm \mu_B$$

ii)



Let us consider N no. of electron spins (isolated, non interacting) pointing in different directions. $\vec{B}$ be the uniform magnetic field in z direction.

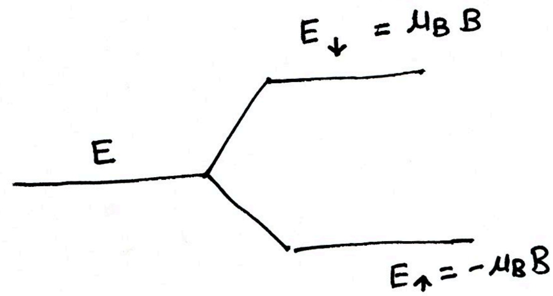
In the presence of magnetic field, the energy of electron spi[n]

given as $E = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta = -\mu_z B$

For parallel ~~axis~~ alignment, $E_\uparrow = -\mu_B B$

For antiparallel alignment, $E_\downarrow = \mu_B B$

Considering the spins are localized, we use the Boltzmann statistics,



Now,

fraction of spin in state $|\uparrow\rangle$,

$$N_\uparrow = \frac{e^{-E_\uparrow/k_B T}}{Z}$$

$$N_\uparrow = \frac{e^{\mu_B B/k_B T}}{e^{\mu_B B/k_B T} + e^{-\mu_B B/k_B T}}$$

$$N_\downarrow = \frac{e^{-E_\downarrow/k_B T}}{e^{\mu_B B/k_B T} + e^{-\mu_B B/k_B T}} = \frac{e^{-\mu_B B/k_B T}}{e^{\mu_B B/k_B T} + e^{-\mu_B B/k_B T}}$$

Magnetization of N spins is,

$$M = \frac{\sum_i \mu_i}{V}$$

$$= \frac{N N_\uparrow \mu_B + N N_\downarrow (-\mu_B)}{V}$$

$$= \frac{N}{V} \mu_B (N_\uparrow - N_\downarrow)$$

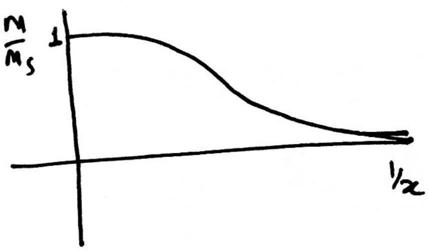
$$= \frac{N}{V} \mu_B \left(\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \right) \qquad \text{where } x = \frac{\mu_B B}{K_B T}$$

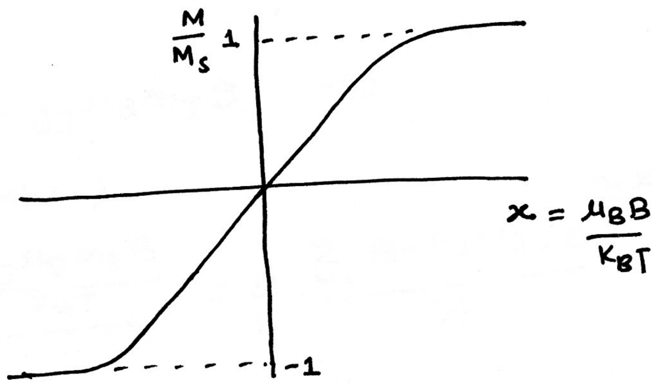
$$M = \left(\frac{N}{V} \mu_B\right) \tanh x$$

$$\boxed{M = M_s \tanh\left(\frac{\mu_B B}{K_B T}\right)}$$

[$M_s = \frac{N}{V}\mu_B$ gives maximum possible magnetization]

a) For $x >> 1$





(b) For $x << 1$ i.e, $K_B T >> \mu_B B$

$$\frac{M}{M_s} = \tanh\left(\frac{\mu_B B}{K_B T}\right) \approx \frac{\mu_B B}{K_B T}$$

$$M = M_s \frac{\mu_B B}{K_B T}$$

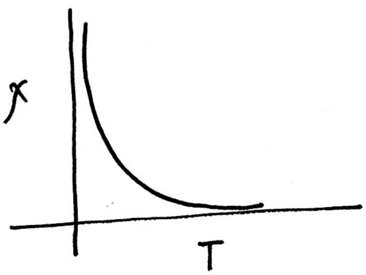
$$\frac{M}{H} = \frac{N}{V} \mu_B \frac{\mu_0 \mu_B}{K_B T}$$

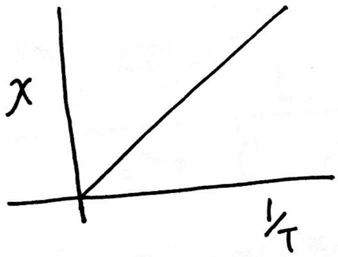
$$\chi = \frac{M}{H} = \left(\frac{n \mu_0 \mu_B^2}{K_B} \right) \frac{1}{T}$$

$$\boxed{\chi = \frac{C}{T}} \qquad \text{where} \quad C = \frac{n\mu_0\mu_B^2}{k_B}$$

Curie's law

Temperature dependence of susceptibility,





Now,

we consider the case for arbitrary angular momentum J,

There are $m_J = -J, -J+1, \ldots, J-1, +J$ values (i.e J is quantized)

$$\mu_z = g_J\mu_B m_J$$

$$E = -\vec{\mu}\cdot\vec{B} = -\mu_z B = -g_J\mu_B m_J B \quad -(1)$$

(Energy splits into $(2J+1)$ states)

$$M = \frac{N}{V}\frac{\sum_{m_J}(g_J\mu_B m_J)\, e^{\frac{g_J\mu_B m_J B}{k_B T}}}{Z} = \frac{N}{V}\frac{\sum_{m_J}(g_J\mu_B m_J)\, e^{m_J x}}{Z}, \quad x = \frac{g_J\mu_B B}{k_B T} \quad (2)$$

The partition function,
$$Z = \sum_{m_J} e^{\frac{g_J\mu_B m_J B}{k_B T}} = \sum_{m_J} e^{m_J x} \quad -(3)$$

$$\frac{\partial Z}{\partial x} = \sum_{m_J} m_J e^{m_J x} \quad -(4)$$

$$M = \frac{N}{V} g_J\mu_B \frac{\sum_{m_J} m_J e^{m_J x}}{Z}$$

$$= \frac{N}{V} g_J\mu_B \frac{1}{Z}\frac{\partial Z}{\partial x}$$

$$M = \frac{N}{V} g_J\mu_B \frac{\partial}{\partial x}(\ln Z) \quad -(5)$$