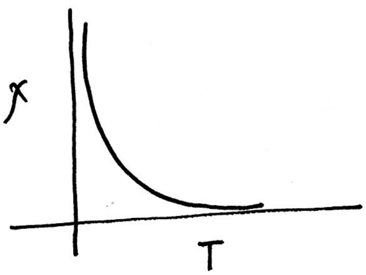
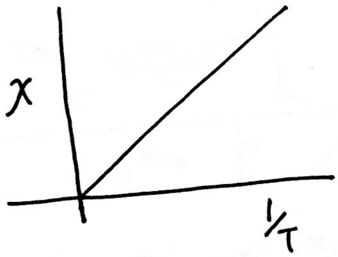
$$\boxed{\chi = \frac{C}{T}} \qquad \text{where} \quad C = \frac{n \mu_0 \mu_B^2}{k_B}$$

Curie's law

Temperature dependence of susceptibility,





Now,

we consider the case for arbitrary angular momentum J,

There are $m_J = -J, -J+1, \ldots, J-1, +J$ values (i.e J is quantized)

$$\mu_z = g_J \mu_B m_J$$

$$E = -\vec{\mu}\cdot\vec{B} = -\mu_z B = -g_J \mu_B m_J B \quad -(1)$$

(Energy splits into $(2J+1)$ states)

$$M = \frac{N}{V} \frac{\sum_{m_J} (g_J \mu_B m_J)\, e^{\frac{g_J \mu_B m_J B}{k_B T}}}{Z} = \frac{N}{V} \frac{\sum_{m_J} (g_J \mu_B m_J)\, e^{m_J x}}{Z} \quad , x = \frac{g_J \mu_B B}{k_B T} \quad (2)$$

The partition function,

$$Z = \sum_{m_J} e^{\frac{g_J \mu_B m_J B}{k_B T}} = \sum_{m_J} e^{m_J x} \quad -(3)$$

$$\frac{\partial Z}{\partial x} = \sum_{m_J} m_J e^{m_J x} \quad -(4)$$

$$M = \frac{N}{V} g_J \mu_B \frac{\sum_{m_J} m_J e^{m_J x}}{Z}$$

$$= \frac{N}{V} g_J \mu_B \frac{1}{Z} \frac{\partial Z}{\partial x}$$

$$M = \frac{N}{V} g_J \mu_B \frac{\partial}{\partial x} (\ln Z) \quad -(5)$$

rom eqⁿ (3),

$$Z = \sum_{m_J} e^{m_J x} = \sum_{m_J=-J}^{J} (e^{x})^{m_J}$$

[sum of Geometric series $S_n = \frac{a_0(1-r^n)}{1-r}$

$$= \frac{e^{-Jx}\left[1 - e^{(2J+1)x}\right]}{1-e^{x}}$$

$$= \frac{e^{-Jx}\, e^{\frac{(2J+1)x}{2}}\left[e^{-\frac{(2J+1)x}{2}} - e^{\frac{(2J+1)x}{2}}\right]}{e^{x/2}\left[e^{-x/2} - e^{x/2}\right]}$$

$$= e^{-\cancel{Jx} + \cancel{Jx} + \cancel{x/2} - x/2} \frac{\left[e^{-(J+\frac{1}{2})x} - e^{(J+\frac{1}{2})x}\right]}{\left[e^{-x/2} - e^{x/2}\right]}$$

$$Z = \frac{\sinh\left[(J+\frac{1}{2})x\right]}{\sinh(x/2)} \quad \text{---} \ (6)$$

~~Substi~~ Taking log of Z,

$$\ln Z = \ln \sinh\left[(J+\tfrac{1}{2})x\right] - \ln \sinh(x/2)$$

$$\frac{\partial(\ln Z)}{\partial x} = \frac{\cosh\left[(J+\frac{1}{2})x\right]}{\sinh\left[(J+\frac{1}{2})x\right]}\cdot\left(\frac{2J+1}{2}\right) - \frac{\cosh(x/2)}{\sinh(x/2)}\cdot\left(\frac{1}{2}\right) \quad -(7)$$

substituting (7) in (5)

$$M = \left(\frac{N}{V} g_J \mu_B\right)\cdot J\left[\left(\frac{2J+1}{2J}\right)\coth\,(J+\tfrac{1}{2})x - \frac{1}{2J}\coth\left(\frac{x}{2}\right)\right]$$

$$= \left(\frac{N}{V} g_J J \mu_B\right)\left[\left(\frac{2J+1}{2J}\right)\coth\left(\frac{2J+1}{2J}\right)Jx - \frac{1}{2J}\coth\left(\frac{xJ}{2J}\right)\right]$$

$$M = M_S\left[\left(\frac{2J+1}{2J}\right)\coth\left(\frac{2J+1}{2J}\right)y - \frac{1}{2J}\coth\frac{1}{2J}y\right]$$

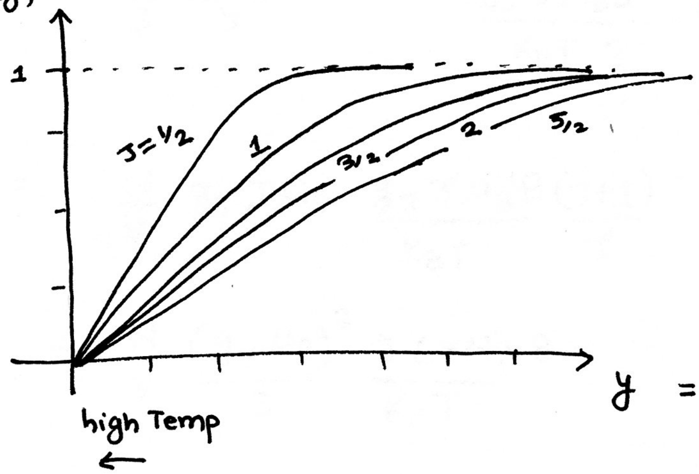
where $y = xJ = \frac{g_J J \mu_B B}{K_B T}$

$$\frac{M}{M_S} = B_J(y)$$

, where $B_J(y)$ is the Brillouin function.

Plot of the Brillouin function $B_J(y)$ for various values of the spin J.

$\frac{M}{M_S} = B_J(y)$



$y = \frac{g_J J \mu_B B}{k_B T}$

(a) For fixed B, as $T \to 0$, $M = M_S$ i.e. each ion is completely aligned by the field. However this case arises only when $k_B T \ll g_J \mu_B B$

~~i.e. $T < 1K$ in a field.~~ $T \ll \frac{g_J \mu_B B}{k_B} \approx 1K$ for field of 1T.

(b) At high temperatures, $g_J J \mu_B B \ll k_B T$,

$$\coth x \approx \frac{1}{x} + \frac{1}{3}x + O(x^3),$$

$$B_J(y) \approx \left(\frac{2J+1}{2J}\right)\left\{\frac{1}{\left(\frac{2J+1}{2J}\right)y} + \frac{1}{3}\left(\frac{2J+1}{2J}\right)y\right\} - \frac{1}{2J}\left\{\frac{2J}{y} + \frac{1}{3}\frac{y}{2J}\right\}$$

$$\approx \cancel{\frac{1}{y}} + \frac{1}{3}\left(\frac{2J+1}{2J}\right)^2 y - \cancel{\frac{1}{y}} - \frac{1}{3}\frac{y}{(2J)^2}$$

$$\approx \frac{y}{3}\left[\left(\frac{2J+1}{2J}\right)^2 - \left(\frac{1}{2J}\right)^2\right] = \frac{y}{3}\left[\left(\frac{2J+1}{2J} + \frac{1}{2J}\right)\left(\frac{2J+1-1}{2J}\right)\right]$$

$$B_J(y) \approx \frac{y}{3}\frac{(J+1)}{J}$$

$$\frac{M}{M_S} = \frac{y}{3}\frac{(J+1)}{J} = \frac{g_J J \mu_B B}{K_B T \; 3}\frac{(J+1)}{J}$$

$$M = \frac{N}{V} g_J J \mu_B \frac{g_J \cancel{J} \mu_B B}{K_B T}\frac{(J+1)}{\cancel{J}}$$

$$M = \frac{N}{V}\frac{(g_J \mu_B)^2}{3}\frac{J(J+1)\,B}{K_B T}$$

$$\chi = \frac{M}{H} = \frac{N}{V}\frac{(g_J \mu_B)^2 \mu_0}{3}\frac{J(J+1)}{K_B T}$$

Susceptibilities are usually quoted as molar susceptibilities, based on the magnetization per mole, rather than per cubic centimeter. Thus, χ_{molar} is given by multiplying χ by the volume of a mole, $\frac{N_A}{[\frac{N}{V}]}$

$$\boxed{\chi_{molar} = \frac{N_A (g_J \mu_B)^2 \mu_0 \; J(J+1)}{3 K_B T}}$$

; where N_A = Avogadro's no

Thus, the susceptibility is inversely proportional to temperature. This variation of χ with T is ~~called~~ known as Curie's Law.

The effective Bohr magneton μ_{eff} is given by,

$$\mu_{eff} = g_J \sqrt{J(J+1)}\,\mu_B$$

$$\Rightarrow \mu_{eff}^2 = g_J^2 J(J+1)\,\mu_B^2$$

$$\because g_J = \frac{3}{2} + \frac{1}{2}\left[\frac{S(S+1) - L(L+1)}{J(J+1)}\right.$$

The Curie's law can be written as,

$$\chi_{molar} = \frac{N_A \mu_0 \mu_{eff}^2}{3 K_B T}$$