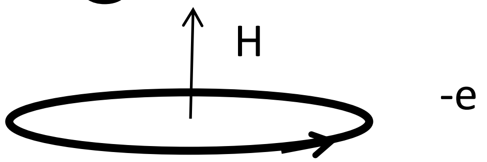
Classical Approach to Diamagnetism



$$2\pi r E = -\frac{d\Phi}{dt}$$ Change in H induces emf

$$\frac{dL}{dt} = Fr = -eEr = \frac{e}{2\pi}\frac{d\Phi}{dt} = \frac{e\pi r^2 \mu_0}{2\pi}\frac{dH}{dt}$$ Electron speeds up increasing angular momentum

$$\Delta L = \frac{er^2 \mu_0}{2}\Delta H$$ Net change in angular momentum

$$\Delta\mu_m = \gamma\Delta L = \frac{-e}{2m_e}\Delta L = -\frac{e^2 r^2 \mu_0}{4m_e}\Delta H$$ Net change in magnetic moment

$$\Delta m = -\frac{NZe^2 \frac{2}{3}\left\langle r^2 \right\rangle \mu_0}{4m_e}\Delta H$$ Net change in total moment of NZ electrons

note: $\left\langle x^2 + y^2 \right\rangle = \frac{2}{3}\left\langle \mathrm{r}^2 \right\rangle$

$$\chi = \frac{\Delta m}{\Delta H} = -\frac{NZe^2 \left\langle r^2 \right\rangle \mu_0}{6m_e}$$ Diamagnetic Susceptibility

Diamagnetism is change in orbital moment due to applied magnetic field

Diamagnetic Susceptibility

- Classical Argument gives correct formula
- All materials show diamagnetism, but it is often masked by other magnetic responses
- Diamagnetic Susceptibility is always negative and small ~ 10^{-5}
- T independent

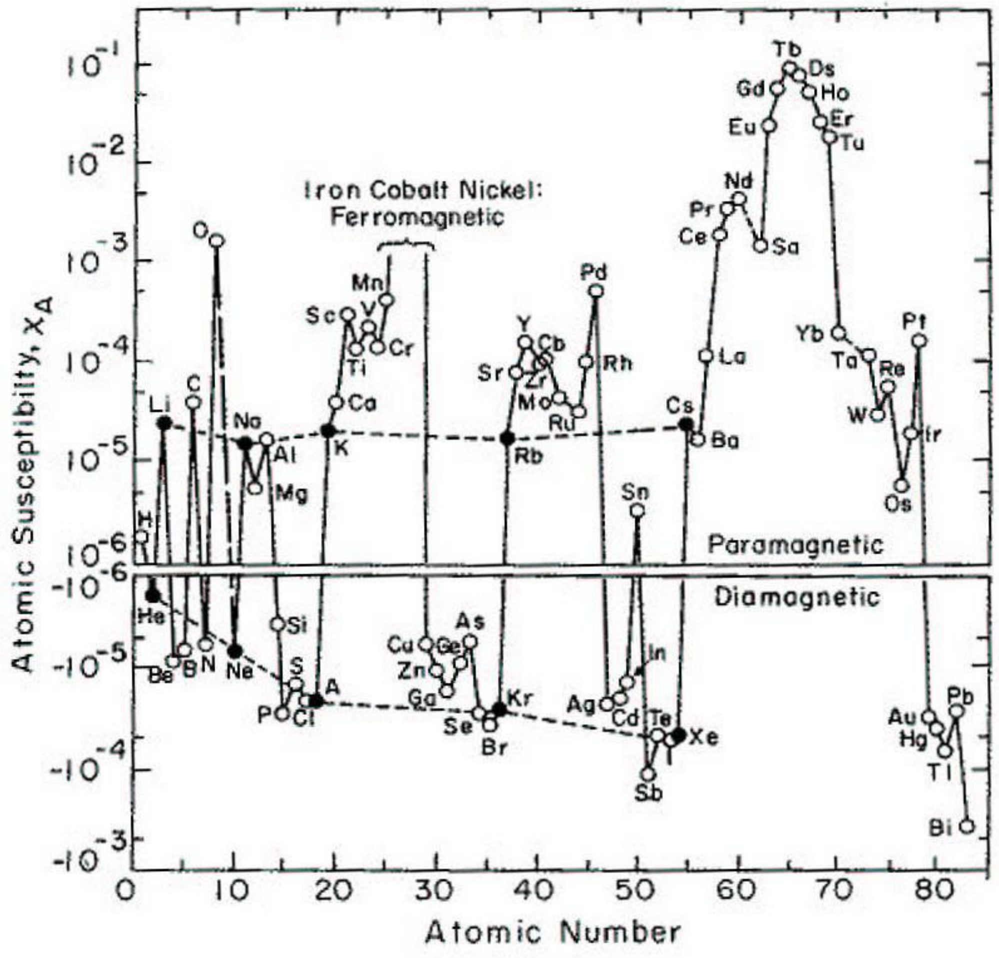


Figure 3.4 Magnetic susceptibilities of the elements in atomic units. Negative values of χ indicate that the diamagnetic part of the susceptibility is greater than the paramagnetic part [After Bozorth, copyright IEEE Press (1993)].

Quantum Diamagnetism

$$H = \frac{\left(p + e\mathbf{A}\right)^2}{2m} + V(r)$$

$\mathbf{B} = \nabla \times \mathbf{A}$ (Because $\nabla \cdot \mathbf{B} = \mathbf{0}$)

$\mathbf{A}$ from current distribution: $\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \iiint \frac{j(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dv'$

Compare to scalar potential from charge distribution: $V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \iiint \frac{\rho(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} dv$

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{ie\hbar}{2m}\left(\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla\right) + \frac{e^2}{2m} A^2 + V(r)$$

$\mathbf{A} = (\frac{-yB}{2}, \frac{xB}{2}, 0)$ for uniform B in z direction (check by taking $\nabla \times \mathbf{A}$)

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{ie\hbar}{2m}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2 B^2}{8m}\left(x^2 + y^2\right) + V(r)$$

Quantum Diamagnetism

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V(r) + \frac{ie\hbar}{2m}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2B^2}{8m}\left(x^2 + y^2\right)$$

Kinetic Energy | Potential Energy | Orbital Paramagnetism | **Diamagnetism**

$$E = \frac{e^2B^2}{12m}\left\langle r^2\right\rangle$$

$$E = -\int \mu_m \cdot \mathbf{B}\, dV$$

$$\mu_m = -\frac{\partial E}{\partial B} = -\frac{e^2\left\langle r^2\right\rangle}{6m}B$$

$$\chi_d = \frac{\mu_m}{H} = -\frac{\mu_0 e^2\left\langle r^2\right\rangle}{6m}$$ (same as classical result)