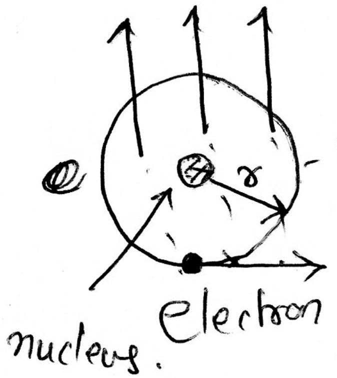
Diamagnetism

1 Classical Approach:

Let us applied the magnetic field in the vicinity of an atom. The magnetic field changes an electric field is generated by magnetic induction. According to Faraday's law, The line integral of E around any closed path is rate of change of the magnetic flux through the path.

$$\int \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\phi \quad \ldots (1.1)$$



$$\vec{E} \cdot 2\pi r = -\frac{d}{dt}(B\pi r^2), \quad \left(B = \frac{\phi}{A}\right)$$

So, The circulating electric field

$$\vec{E} = -\frac{r}{2}\frac{d\vec{B}}{dt} \quad (1.2)$$

The induced electric field action on an electron in the atom produces a torque $\tau = r \times F = -q_e E r$, q_e = electron charge.

The torque is also define the rate of change of angular momentum

$$\frac{dL}{dt} = -q_e E r \quad \text{(using eq}^n \text{ 1.2)}.$$

$$\frac{d\vec{L}}{dt} = \frac{q_e r^2}{2}\frac{d\vec{B}}{dt} \quad \ldots (1.3)$$

Integrating eqn (1.3) at $t=0$, $B=0$, $t=t$, $B=B$

The change in angular momentum due to turning of the field is $\Delta L = \frac{q_e r^2 B}{2}$ --- (1.4)

This is the extra angular momentum which is changed due to the turning of the field.

Now, The extra angular momentum produces the extra magnetic moment. So

Induced diamagnetic moment due to turning of the field is

$$\Delta \mu = -\frac{q_e}{2m} \Delta L$$

$$\Delta \mu = -\frac{q_e^2 r^2}{4m} B \quad \text{-- (1.5)}$$

we know

$L = mvr$, $T = \frac{2\pi r}{v}$

$vr = L/m$ $\quad v = \frac{2\pi r}{T}$

$I = \frac{ev}{2\pi r}$

$\mu = I \times A = \frac{ev}{2\pi r} \times \pi r^2$

$\mu = \frac{qvr}{2} = \frac{qL}{2m}$

Negative sign is According to Len's law, The moment is opposite to the magnetic field. produced

$$\Delta \mu = -\frac{q_e^2}{6m} \langle r^2_{av} \rangle B \quad \text{--- (1.6)}$$

If B is along Z-direction the average $\langle r^2 \rangle_{av} = \frac{2}{3}(x^2+y^2)$

~~This is~~

$\langle r^2 \rangle_{av} = \frac{2}{3} r^2$

$(x^2 = y^2 = z^2)$.

For the Higher Atomic number solid, It will be

$$\Delta \mu = -\frac{e^2 NZ}{6m} \langle r^2 \rangle_{av} B \quad \text{-(1.7)}$$

where N = Number of atom per unit volume

Z = Atomic number

Quantum Theory of Diamagnetism.

The Hamiltonian for single electron system of the Solid.

$$H = \frac{(\vec{P} - e/c\,\vec{A})^2}{2m} + V(r) \quad \cdots (2.1)$$

$$H = p^2/2m - e/2mc\,(\vec{P}\cdot\vec{A} + \vec{A}\cdot\vec{P}) + \frac{e^2A^2}{2mc^2} + V(r)$$

$$H = p^2/2m + V(r) - e/2mc\,(\vec{P}\cdot\vec{A} + \vec{A}\vec{P}) + \frac{e^2A^2}{2mc^2}$$

The Hamiltonian for an electron in crystalline Solid at zero fi

$$H_0 = p^2/2m + V(r)$$

so, The perturbation in Hamiltonian for an electron in solid d

to the field

$$\Delta H = H - H_0$$

$$\Delta H = -e/2mc\,(\vec{P}\cdot\vec{A} + \vec{A}\cdot P) + \frac{e^2A^2}{2mc} \quad \cdots (2.2)$$

we know, ~~the~~ vector potential $A = \frac{1}{2}\vec{B}\times\vec{r}$

The applied field is $B = B\hat{z}$

$$A_x = -\tfrac{1}{2}yB,\ A_y = \tfrac{1}{2}xB,\ A_z = 0 \qquad ,\ P = -i\hbar\nabla$$

Than

$$\Delta H = +\frac{ie\hbar}{2mc}\left(\frac{\partial}{\partial x}A_x + \partial/\partial y\,A_y + \partial/\partial z\,A_z + A_x\partial/\partial x + A_y\partial/\partial y + A_z\partial/\partial z\right)$$

$$+ \frac{e^2}{2mc^2}(A_x^2 + A_y^2 + A_z^2)$$

$$\Delta H = \frac{ie\hbar B}{2mc}\left(x\,\partial/\partial y - y\,\partial/\partial x\right) + \frac{e^2B^2}{8mc^2}(x^2+y^2)$$

$$\Delta H = \underbrace{\frac{ie\hbar B}{2mc} L_z}_{} + \frac{e^2B^2}{8mc^2}(x^2+y^2) \} \text{ diamagnetism}$$

This term ~~ ~~ produe paramagnetism

so, Diamagnetic case.

$$\Delta H = \frac{e^2 B^2}{8mc^2}(x^2+y^2) \quad \cdots (2.4)$$

For the spherical symmetry

$\langle \rho^2 \rangle_{ave} = \frac{2}{3} r^2$

$$\cancel{\Delta H} \quad E = \frac{e^2 B^2}{12mc^2} \langle r^2 \rangle_{ave} \quad (2.5)$$

By using the 1st order perturbation theory the associate magnetic moments for diamagnetism is

$$\Delta \mu = -\partial E' / \partial B = -\frac{e^2 \langle r^2 \rangle_{ave} B}{6mc^2} \quad \cdots (2.6)$$

~~The classical approach is successed only to calculate the magnetic moment even though it does not explain the diamagnetic behavior correctly.~~

The diamagnetic magnetic moment of bound electron by the quantum to the classical approach. ~~So~~ The classical approach is equal to the classical approach. So, The classical approach was successed to calculate the diamagnetic moment even though it does not explain the diamagnetic phenomenon correctly.