Stoner Instability → Following a problem in Kittel

Exchange interaction → Lets assume that electrons favor to align themselves, and foreach pair of parallel alignment there is a gain of potential energy $-|V|$, → detailed reason later (Ouston)

Electrons with anti parallel spins do not interact (Assumption)

Lets work out the energetics of this problem similar to Pauli paramagnetism calculation.

There are N_+ electrons with spin up.

$$N_+ = \frac{1}{2} N (1+\Delta) \qquad \Delta << 1$$

And, N_- electrons with spin down

$$N_- = \frac{1}{2} N (1-\Delta)$$

$$\left.\begin{aligned} & N_+ + N_- = N \\ & N_+ - N_- = N\Delta \\ & M = \mu (N_+ - N_-) = \mu N \Delta \end{aligned}\right\}$$

Lets calculate the energy of the spin up band

How many pairs can be chosen with N_+ electrons

$$= {}^{N_+}C_2 = \frac{N_+!}{2!\,(N_+-2)!} \simeq \frac{1}{2} N_+(N_+-1) \approx \frac{1}{2} N_+^2 = \frac{1}{8} N^2 (1+\Delta)^2$$

$$\text{Potential energy} = -\frac{V}{8} N^2 (1+\Delta)^2$$

$$\text{Kinetic energy} = E_0 (1+\Delta)^{5/3}$$ → From Pauli paramagnetism discussion.

$$\text{Magnetic energy} = -\mu B N_+ = -\frac{\mu B N}{2}(1+\Delta)$$

$$\text{Total energy} = E_0 (1+\Delta)^{5/3} - \mu B N_+ - \frac{V}{8} N^2 (1+\Delta)^2$$

$$= E_0 (1+\Delta)^{5/3} - \frac{V}{8} N^2 (1+\Delta)^2 - \frac{\mu B N}{2} (1+\Delta)$$

Energy of down spins :- $E_0(1-\Delta)^{5/3} - \frac{1}{8} V N^2 (1-\Delta)^2 + \frac{N}{2}\mu B(1-\Delta)$

Total energy :-

$$E = E_0\left[(1+\Delta)^{5/3} + (1-\Delta)^{5/3}\right] - \frac{VN^2}{8}\left[(1+\Delta)^2 + (1-\Delta)^2\right] - \mu BN\Delta$$

Minimum energy is obtained when $\frac{\partial E}{\partial \Delta} = 0$

$$\Rightarrow E_0\left[\frac{5}{3}(1+\Delta)^{2/3} - \frac{5}{3}(1-\Delta)^{2/3}\right] - \frac{VN^2}{8}\left[2(1+\Delta) - 2(1-\Delta)\right] - \mu BN = 0$$

$$\Rightarrow E_0\left[\frac{5}{3}\left(1+\frac{2}{3}\Delta\right) - \frac{5}{3}\left(1-\frac{2}{3}\Delta\right)\right] - \frac{VN^2}{8}\left[\cancel{2}+2\Delta-\cancel{2}+2\Delta\right] - \mu BN = 0$$

$$\Rightarrow E_0\left[\cancel{\frac{5}{3}} + \frac{10}{9}\Delta - \cancel{\frac{5}{3}} + \frac{10}{9}\Delta\right] - \frac{VN^2\Delta}{2} - \mu BN = 0$$

$$\Rightarrow \frac{20E_0}{9}\Delta - \frac{VN^2\Delta}{2} - \mu BN = 0$$

$$\frac{20E_0}{9} = \frac{20}{9}\left(\frac{3}{10}\right)N\epsilon_F = \frac{2}{3}N\epsilon_F$$

$$\Rightarrow \Delta\left[\frac{20E_0}{9} - \frac{VN^2}{2}\right] = \mu BN$$

$$\Rightarrow \Delta = \frac{\mu N}{\frac{20E_0}{9} - \frac{VN^2}{2}}B = \frac{\mu N B}{\frac{2}{3}N\epsilon_F - \frac{VN^2}{2}} = \frac{\mu B}{\frac{2}{3}\epsilon_F - \frac{VN}{2}} = \frac{3\mu B}{2\epsilon_F - \frac{3}{2}VN}$$

$$M = \mu(N_+ - N_-) = \mu N\Delta = \left(\frac{3\mu^2 N}{2\epsilon_F - \frac{3}{2}VN}\right)B$$

When $V=0$ $M \to M_{pauli}$. So exchange interaction enhances M since $2\epsilon_F - \frac{3}{2}VN < 2\epsilon_F$

what happens when $B=0$?

It is easy to show that the solution is unstable at all B (including zero) when $V > \frac{4\epsilon_F}{3N}$

$$\frac{\partial^2 E}{\partial \Delta^2} = E_0\left[\frac{5}{3}\frac{2}{3}(1+\Delta)^{-1/3} + \frac{2}{3}\frac{5}{3}(1-\Delta)^{-1/3}\right] - \frac{VN^2}{8}[2+2]$$

$$= \frac{10E_0}{9}\left[1 - \frac{\Delta}{3} + 1 + \frac{\Delta}{3}\right] - \frac{VN^2}{2}$$

$$= \frac{20E_0}{9} - \frac{VN^2}{2} = \frac{2}{3}N\epsilon_F - \frac{VN^2}{2}$$

$\frac{\partial^2 E}{\partial \Delta^2} > 0$ when $V < \frac{4\epsilon_F}{3N}$

But when $V > \frac{4\epsilon_F}{3N}$ $\frac{\partial^2 E}{\partial \Delta^2} < 0 \rightarrow$ unstable. even at $B=0$

Therefore the paramagnetic state is no longer stable for $V < \frac{4\epsilon_F}{3N}$

$\Delta = \left(\frac{3\mu^2 N}{2\epsilon_F - \frac{3}{2}VN}\right)B$ at $V = \frac{4\epsilon_F}{3N}$ and $B=0$, $\Delta \neq 0$ solution is possible

There is another state called the ferromagnetic state that has lower energy than the paramagnetic state

$V \geq \frac{4\epsilon_F}{3N}$ is called the stoner criterion. $\frac{4}{3}\frac{\epsilon_F}{N} = 2\left(\frac{2\epsilon_F}{3N}\right) = \frac{2}{D(\epsilon_F)}$

There is a <u>sponteneous</u> splitting of the bands.

This is a phase transition at zero-temperature that occurs by changing a control parameter (exchange interaction)

$V \geq 2/D(\epsilon)$ high $D(\epsilon)$ stabilizes ferromagnetism Observed in 3d atoms.