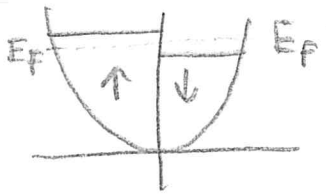
($B=0$) Stoner Model (Alternate derivation)

Again lets assume there is an unequal population of spin up & spin down electrons ($n_\uparrow > n_\downarrow$) due to exchange interactions.



spin up electrons are pushed up ~~to~~ from E_F to $E_F + \Delta E$ $\quad \Delta E \ll E_f$

and spin down electrons are pushed down ~~for~~ from $E_f - \Delta E$

Essentially, some electrons are transferred from dn to up.

This will cost KE. lets calculate that

Give $g(E) =$ total Density of states, # of electrons involved in the

transfer $= g\left(\frac{E_F}{2}\right)\Delta E \quad \frac{1}{2} \Rightarrow$ spin dn electrons $D_\downarrow(E_F) = \frac{1}{2} D(E_F)$

$n_\uparrow$ after the transfer $= \frac{1}{2}\left[n + g(E_F)\Delta E\right] \qquad \forall\ (n_\uparrow + n_\downarrow) = N$

$n_\downarrow$ " " " $= \frac{1}{2}\left[n - g(E_F)\Delta E\right]$

$$n_\uparrow - n_\downarrow = g(E_F)\Delta E$$

ΔKE involved in ~~moving~~ moving $\frac{g(E_F)\Delta E}{2}$ by ΔE is $\frac{1}{2} g(E_F)(\Delta E)^2$

Now we will assume that there is an internal molecular field λM (curie weiss type) that favor alignment of spins. ext field $= 0$

M = Magnetization $\quad \mu_B(n_\uparrow - n_\downarrow) = \mu_B g(E_F)\Delta E$ $\quad \Big| \begin{array}{l} B = \mu_0(H + \lambda M) \\ B = \mu_0 \lambda M \end{array}$

Energy gain $= -\int \mu \cdot dB = -\int V M \mu_0 \lambda\, dM = -\mu_0 V \frac{\lambda^2 M^2}{2}$

$$\Rightarrow \Delta PE = -\frac{U\mu_B^2}{2}(n_\uparrow - n_\downarrow)^2 = -\frac{U\mu_B^2}{2} g^2(E_F)(\Delta E)^2 \quad \Big| U = \mu_0 \mu_B^2 \lambda$$

Total Energy $= \Delta KE + \Delta PE = \frac{1}{2} g(E_F)(\Delta E)^2\left[1 - U g(E_F)\right]$

If $\Delta E < 0$, Ferromagnetism occurs spontaneously

$$1 - U g(E_F) \leq 0 \Rightarrow \boxed{U g(E_F) \geq 1}$$ Stoner criterion

$U = \mu_0 \mu_B^2 \lambda \Rightarrow \lambda$ indicates the strength of the internal field which is related to exchange interactions

when $B_{ext} \neq 0$ $\quad \Delta E = \frac{1}{2} g(E_F)(\Delta E)^2 [1 - U g(E_F)] - M B$

$$M = \mu_B (n_\uparrow - n_\downarrow) = \mu_B g(E) \Delta E$$

$$g(E)(\Delta E)^2 = \frac{M^2}{\mu_B^2} \Rightarrow g(E_F)(\Delta E)^2 = \frac{M^2}{\mu_B^2 g(E_F)}$$

$$\Delta E = \frac{1}{2} \frac{M^2}{\mu_B^2 g(E_F)} [1 - U g(E_F)] - M B$$

This is minimized when $\frac{\Delta E}{\Delta M} = 0$

$$\Rightarrow \frac{M}{\mu_B^2 g(E_F)} [1 - U g(E_F)] - B = 0$$

$$\Rightarrow M = \frac{\mu_B^2 B g(E_F)}{[1 - U g(E_F)]} \Rightarrow \chi = \frac{\partial M}{\partial B} = \frac{\mu_B^2 g(E_F)}{[1 - U g(E_F)]}$$

χ is enhanced due to exchange interaction by a factor $[1 - U g(E_F)]^{-1}$, called Stoner enhancement. It is responsible for the enhanced Pauli susceptibility of Pt and Pd, which are on the verge of ferromagnetism

$U \sim 1eV \Rightarrow g(E_F) \sim 1$.

Fe, Co, Ni have a large $g(E_F)$ (3d)

Pt, Pd $U g(E_F) \simeq 0.9$ called incipient ferromagnets (4d, 5d)