Electromagnetism ⑧

I Ampere's law

① Magnetostatics involves

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B}(\vec{r}) = \frac{4\pi}{c} \vec{J}(\vec{r})$$

{note that as a consequence $\vec{\nabla} \cdot \vec{J} = 0$}

② $$\int_S d\vec{a} \cdot (\vec{\nabla} \times \vec{B}) = \frac{4\pi}{c} \int_S d\vec{a} \cdot \vec{J}(\vec{r})$$

$$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} \int_S d\vec{a} \cdot \vec{J}(\vec{r})$$ (using curl theorem)

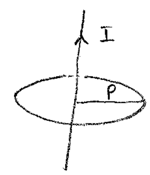
$$= \frac{4\pi}{c} I$$ (using definition of $J \to \frac{I}{A} = J$)

③ Thus we have the Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I$$

II. Application of Ampere's law

1. Long wire:



$$\oint \vec{B}\cdot d\vec{l} = \frac{4\pi}{c} I$$

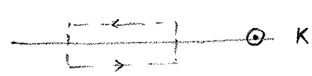
$$B\, 2\pi \rho = \frac{4\pi}{c} I$$

(conclude separately that $\vec{B}$ has only the $\hat{\phi}$ component.)

$$\vec{B} = \frac{2I}{c\rho}\,\hat{\phi}$$

$$\vec{B}(\rho, \phi, z) = \frac{2I}{c}\,\frac{\hat{z}\times\hat{\rho}}{\rho}$$

2. Long sheet:



$$K = \frac{I}{L} = \frac{\text{current}}{\text{length}}$$

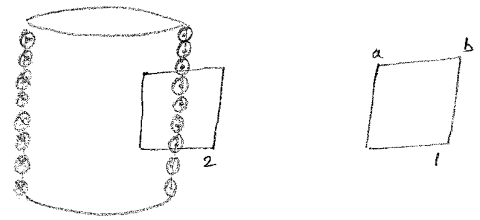
$$\oint \vec{B}\cdot d\vec{l} = \frac{4\pi}{c} I$$

$$B\, 2L = \frac{4\pi}{c} K L$$

(conclude separately that $\vec{B}$ has $\hat{j}$ only.)

$$\vec{B} = \begin{cases} \frac{2\pi}{c} K \; -\hat{j} & z > 0 \\ \frac{2\pi}{c} K \; +\hat{j} & z < 0 \end{cases}$$

③ <u>Infinite solenoid</u> :



conclude separately that $\vec{B}$ has only $\hat{z}$ component.

<u>loop 1</u> :

$$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I$$

$$(B_a - B_b) L = 0$$

$$\Rightarrow \quad B_a = B_b$$

→ At $\rho = \infty$ the solenoid looks like a straight wire carrying a current I. $\vec{B}$ due to a straight wire at infinity is zero.

Thus conclude $\vec{B} = 0$ outside the solenoid.

<u>loop 2</u> :

$$n = \frac{\text{no. of wires}}{\text{length}}$$

$$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I$$

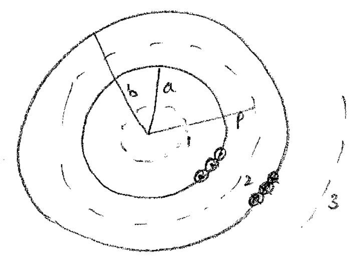
$$B L = \frac{4\pi}{c} I n L$$

$$B = \frac{4\pi}{c} I n$$

<u>Conclusion</u> :

$$\vec{B} = \begin{cases} -\frac{4\pi}{c} I n \, \hat{z} & \rho < a \\ 0 & \rho > a \end{cases}$$

4 Toroid :



conclude separately that $\vec{B}$ has only $\hat{\phi}$ component.

loop 1 & loop 3

$\Rightarrow \quad \vec{B} = 0 \quad$ for $\quad \rho < a \;\; \& \;\; \rho > b$.

loop 2 :

$$\oint \vec{B} \cdot d\vec{l} = \frac{4\pi}{c} I \qquad N = \text{total number of turns}$$

$$B \, 2\pi \rho = \frac{4\pi}{c} I N$$

$$B = \frac{2I}{c} \frac{N}{\rho}$$

Conclusion

$$\vec{B} = \begin{cases} \dfrac{2I}{c} \dfrac{N}{\rho} \, \hat{\phi} & a < \rho < b \\ 0 & \text{otherwise} \end{cases}$$

III Force :

① $$\vec{F} = \frac{q}{c}\,\vec{v}\times\vec{B}$$

② Noting that $\vec{J}(\vec{r}) = \rho(\vec{r})\,\vec{v}$ the above expression generalises to for a distribution of current

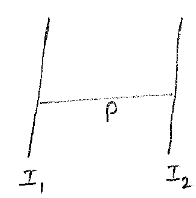
$$\vec{F} = \frac{1}{c}\int d^3r\;\vec{J}(\vec{r})\times\vec{B}(\vec{r})$$

③ For a wire carrying a current I we have.

$$\int d^3r\;\vec{J}(\vec{r}) \;\rightarrow\; \int dl\int d^2r_\perp\;\vec{J}(r_\perp, l) \;\rightarrow\; I\int d\vec{l}$$

$$\vec{F} = \frac{1}{c}\int I\,d\vec{l}\times\vec{B}(l)$$

④ Force between parallel wires.

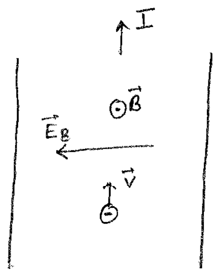


$$\vec{F} = \frac{1}{c}\int dz\;I_1\;\hat{n}_1\times\vec{B}(z)$$
$$= \frac{1}{c}\int dz\;I_1\;\hat{n}_1\times\left(\frac{\hat{n}_2\times\hat{\rho}}{\rho}\;\frac{2I_2}{c}\right)$$

$$\frac{\text{Force}}{\text{length}} = \frac{1}{c^2}\,\frac{2I_1I_2}{\rho}\;\hat{n}_1\times(\hat{n}_2\times\hat{\rho})$$
$$= -\frac{1}{c^2}\,\frac{2I_1I_2}{\rho}\,(\hat{n}_1\cdot\hat{n}_2)\;\hat{\rho}.$$

→ if $\hat{n}_1 = \hat{n}_2$ the force is attractive and vice versa.

IV. Hall effect



$F_B \longleftarrow \longrightarrow F_E$

$$F_B = e v B$$

$$F_E = e E_B$$

$$e E_B = e v B$$

$$E_B = v B$$