I. Elementary model to understand Ohm's law

① A free electric charge (electron) moving under the influence of an external electric field, and subject to collisions with atoms in the material.

$$m \frac{d\vec{v}(t)}{dt} = - m \gamma \vec{v}(t) + q \vec{E}(t)$$

γ → parameter introducing effects of collision.

$$\frac{d\vec{v}(t)}{dt} + \gamma \vec{v}(t) = \frac{q}{m} \vec{E}(t)$$

$$\frac{d}{dt}\left[e^{\gamma t} \vec{v}(t) \right] = \frac{q}{m} \vec{E}(t) e^{\gamma t}$$

$$\vec{v}(t) = \frac{q}{m} e^{-\gamma t} \int_{-\infty}^{t} dt' \, \vec{E}(t') e^{\gamma t'}$$

② If $\vec{E}(t)$ assumed constant in time

$$\vec{v}(t) = \frac{q}{m} \frac{\vec{E}}{\gamma}$$

$n_f \equiv$ number density of free charges.

③
$$\vec{J} = n_f q \vec{v}$$
$$= n_f q \frac{q}{m} \frac{\vec{E}}{\gamma}$$
$$= \left(n_f \frac{q^2}{m\gamma} \right) \vec{E}$$

→ define $\sigma \equiv n_f \dfrac{q^2}{m\gamma}$

$$\sigma = \frac{1}{\rho}$$

σ → conductivity

ρ → resistivity

(4) Thus in this over-simplified model

$$\vec{J} = \frac{1}{\rho} \vec{E}$$

$$\vec{E} = \rho \vec{J}$$

(5) For a cylindrical wire

$$J = \frac{I}{A}$$

$$E l = V$$

$$E = \rho J$$

$$E A l = \rho l J A$$

$$V = \frac{\rho l}{A} I$$

$$R = \frac{\rho l}{A}$$

resistance

$$\underline{\underline{V = I R}}$$

II Faraday's law

① CAUTION :

We shall mainly concentrate on

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

separately.

② $$\int_S d\vec{a} \cdot (\vec{\nabla} \times \vec{E}) = -\frac{1}{c}\frac{\partial}{\partial t}\int_S \vec{B} \cdot d\vec{a}$$

$$\phi_B = \int_S \vec{B} \cdot d\vec{a}$$

$$\oint d\vec{l} \cdot \vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\phi_B$$

③ In electrostatics we had $\oint \vec{E} \cdot d\vec{l} = 0$, but we are not in electrostatics anymore.

④ $$V_{in} \equiv \oint d\vec{l} \cdot \vec{E}$$

$$V_{in} = -\frac{1}{c}\frac{\partial}{\partial t}\phi_B.$$

induced emf.

III Induced current

① $$V_{in} = -\frac{1}{c}\frac{\partial}{\partial t}\phi_B$$

$$I_{in} R = -\frac{1}{c}\frac{\partial}{\partial t}(N B A \cos\theta)$$

R → resistance of the loop in $\oint \vec{E}\cdot d\vec{l}$.

② Case

(i) $I_{in} R = -\frac{1}{c} N A \cos\theta \frac{\partial B}{\partial t}$ ⟶ B

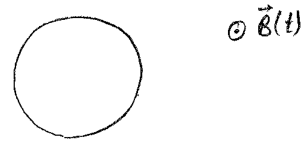
(ii) $I_{in} R = -\frac{1}{c} N B \cos\theta \frac{\partial A}{\partial t}$ ⟶ A

(iii) $I_{in} R = -\frac{1}{c} N B A \frac{\partial}{\partial t}\cos\theta$ ⟶ θ

(iv) $I_{in} R = -\frac{1}{c} B A \cos\theta \frac{\partial N}{\partial t}$ ⟶ N

IV Examples

① Variatn in B



$N = 0$
$\theta = 0$

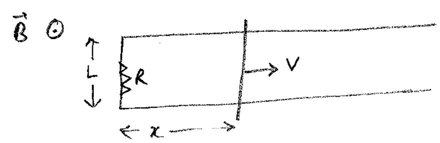
$$I_{in} R = - A \frac{\partial B}{\partial t}$$

$$I_{in} = - \frac{A}{R} \frac{\partial B}{\partial t}$$

→ direction of I_{in} is decided by Lenz's law.

→ the induced current generates its own magnetic field.

② Variation in area A



$$I_{in} R = - B \frac{\partial}{\partial t} (L x)$$

$$= - B L V$$

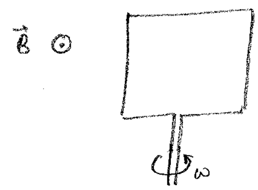
→ $\vec{B}$ exerts a force on the induced current

$$F_B = I_{in} L B$$

$$= \frac{BLV}{R} L B$$

$$= \frac{B^2 L^2 V}{R}$$

③ Variation in θ



$$I_{in} R = -BA \frac{\partial}{\partial t} \cos\theta$$

$$= BA \sin\theta \frac{\partial \theta}{\partial t}$$

$$= BA\omega \sin\theta$$