Electromagnetism HW #3

1. Evaluate the electric potential due to a rod of length $2L$ on the z-axis carrying a uniform charge density to be

(a) $$\phi(\vec{r}) = \lambda\left[\sinh^{-1}\left(\frac{L-z}{\sqrt{x^2+y^2}}\right) + \sinh^{-1}\left(\frac{L+z}{\sqrt{x^2+y^2}}\right)\right]$$

(b) Show that

$$\sinh^{-1} x = \ln\left\{x + \sqrt{x^2+1}\right\}.$$

(Hint: Use $\sinh y = \frac{e^y - e^{-y}}{2}$ and solve for e^y.)

(c) Thus show that

$$\phi(\vec{r}) = -2\lambda \ln R + F\left(\frac{z}{L}, \frac{R}{L}\right)$$

where,

$$F\left(\frac{z}{L}, \frac{R}{L}\right) = \lambda \ln\left\{1 - \frac{z}{L} + \sqrt{1 + \frac{R^2+z^2}{L^2} - 2\frac{z}{L}}\right\} + \lambda \ln\left\{1 + \frac{z}{L} + \sqrt{1 + \frac{R^2+z^2}{L^2} + 2\frac{z}{L}}\right\}$$

(d) Show that

$$\underset{l \to \infty}{Lt} \; \phi(\vec{r}) = -2\lambda \ln R$$

(e) Show that

$$\underset{L \to \infty}{Lt} \; \vec{E}(\vec{r}) = \frac{2\lambda}{R} \hat{R}$$

(f) What will be the electric field due to a point charge in 2-D ?

2 → Work on prob 2.3 to prob 2.8 in Griffiths.

→ Work on prob 2.27 to 2.29 in Griffiths

eqn. 2.27 → $V(\vec{r}) = \sum_{i=1}^{n} \frac{q_i}{r_i}$

eqn. 2.28 → $V(\vec{r}) = \int \frac{dq}{r}$

eqn. 2.29 → $V(\vec{r}) = \int \frac{\rho(\vec{r}')\, d^3r'}{|\vec{r}-\vec{r}'|}$