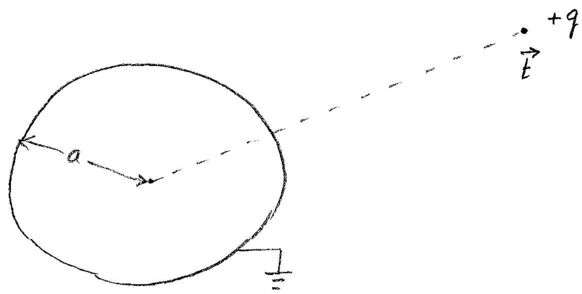
Electromagnetism H.W # ⑥

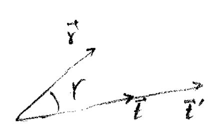
① Conducting sphere – method of images



(a) By symmetry arguements conclude that the image charge has to be on the ray from the origin to charge 'q'.

(b) Determine the charge 'q'' and position '$\vec{t}'$' of the image charge by invoking the boundary condition

$$\phi(|\vec{r}|=a) = \left\{ \frac{q}{|\vec{r}-\vec{t}|} + \frac{q'}{|\vec{r}-\vec{t}'|} \right\}_{|\vec{r}|=a} = 0$$



(Hint : the above eqn. is true for any angle γ and thus the coeficient of $\cos\gamma$ has to be $= 0$.)

(Solution : $q' = -\frac{a}{t}\, q$; $t' = \frac{a}{t}\, a$)

(c) Show that the surface charge density on the conducting sphere is

$$\sigma(\underbrace{\theta, \phi}_{\gamma}, r = a) = \frac{1}{4\pi} (-1) \left\{ \frac{\partial \phi}{\partial r} \right\}_{r=a}$$

$$= -\frac{1}{4\pi} \frac{q}{a^2} \frac{\frac{a}{t}\left(1 - \frac{a^2}{t^2}\right)}{\left(1 + \frac{a^2}{t^2} - 2\frac{a}{t}\cos\gamma\right)^{3/2}}$$

(d) Show that the force on the conducting sphere is

$$\vec{F} = \hat{t} \frac{q^2}{a^2} \frac{\left(\frac{a}{t}\right)^3}{\left(1 - \frac{a^2}{t^2}\right)^2}$$

(2) Two long, cylindrical conductors of radii 'a_1' and 'a_2' are parallel and separated by a distance 'd', which is large compared with either radius. Show that the capacitance per unit length is given approximately by

$$C = \frac{1}{4 \ln\left(\frac{d}{a}\right)} \longrightarrow \text{check the factor!}$$

where 'a' is the geometrical mean of 'a_1' and 'a_2'. $\left(a = \sqrt{a_1 a_2}\right)$.

(taken from prob 1.7, Jackson).