Electromagnetism H·w # ⑦

① For $\vec{A}(\vec{r}) = \frac{1}{c}\int d^3r' \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}$,

verify that $\vec{\nabla}\cdot\vec{A} = 0$.

② Rotating charged spherical shell (a model for an elemeitary particle with mag. dipole moment).

(a) A spherical shell of radius 'a' carrying a charge 'Q' uniformly distributed on its surface is spinning with an angular velocity 'ω'. Verify that the current density on the surface is given by

$$\vec{J}(\vec{r}') = \rho(\vec{r}')\,\vec{v}$$

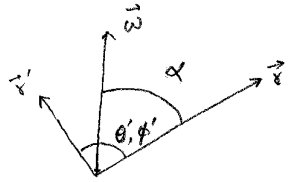
$$= \frac{Q}{4\pi a^2}\,\vec{\omega}\times\vec{r}'$$

(b) Show that the vector potential the rotating charged shell is given by

$$\vec{A}(\vec{r}) = \frac{Q}{4\pi c}\int_0^{\pi}\sin\theta'\,d\theta'\int_0^{2\pi}d\phi'\,\frac{\vec{\omega}\times\vec{r}'}{\sqrt{r^2+a^2-2\,\vec{r}\cdot\vec{r}'}}$$

where $|\vec{r}'| = a$.

(c) Choose the z-axis along $\vec{r}$ and the $\vec{\omega}$ in the x-z plane, such that



$$\vec{\omega} = (\omega \sin\alpha,\ 0,\ \omega \cos\alpha)$$

$$\vec{r} = (0,\ 0,\ r)$$

$$\vec{r}' = (a \sin\theta' \cos\phi',\ a \sin\theta' \sin\phi',\ a \cos\theta')$$

and show that

$$\vec{A}(\vec{r}) = \frac{Q a}{2c} \frac{\vec{\omega} \times \vec{r}}{r} \int_0^{\pi} \sin\theta' d\theta' \frac{\cos\theta'}{\sqrt{r^2 + a^2 - 2ar\cos\theta'}}$$

(d) Show that

$$\int_0^{\pi} \sin\theta' d\theta' \frac{\cos\theta'}{\sqrt{r^2 + a^2 - 2ar\cos\theta'}} = \begin{cases} \frac{2}{3}\frac{r}{a^2} & \text{for } r < a \\ \frac{2}{3}\frac{a}{r^2} & \text{for } r > a \end{cases}$$

(e) Thus conclude

$$A(\vec{r}) = \begin{cases} \frac{Q}{3c} \frac{\vec{\omega} \times \vec{r}}{a} & \text{for } r < a \\ \frac{Q}{3c} \frac{a^2}{r^2} \frac{\vec{\omega} \times \vec{r}}{r} & \text{for } r > a \end{cases}$$

(f) Evaluate the magnetic field for $r < a$ to be

$$\vec{B}(\vec{r}) = \frac{2Q}{3c} \frac{1}{a} \vec{\omega}$$

③ Work on prob. 5.5 and 5.6 in Griffiths.

④ Vector potential:

Griffiths prob. 5.22 – 5.26

→ ignore eqn. 5.64 and eqn. 5.35 in prob. 5.22.

→ Ex. 5.8 mentioned in prob. 5.26 involves a uniform surface current flowing over the x-y plane.

⑤ Biot-Savart law:

Griffiths prob. 5.8, 5.9, 5.11

→ ignore Ex. 5.6 in prob 5.11.