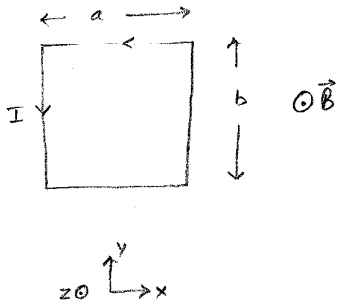
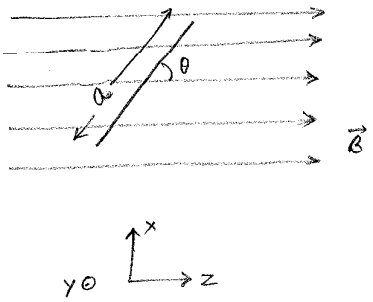
Electromagnetism H·w # (9)

① Consider a loop of wire carrying a constant current I, and kept in a constant magnetic field B.




(a) What is the total force acting on the loop?

(b) What is the total torque acting on the loop?

$(\vec{\tau} = \vec{r} \times \vec{F})$

(c) What is the work done by the resultant torque in moving the loop from $\theta_1$ to $\theta_2$?

$(W = -\int \tau \, d\theta)$

(d) Evaluate the work done in (c) using the expression

$$W = -\frac{1}{c} \int d^3r \; \vec{J}(\vec{r}) \cdot \vec{A}(\vec{r})$$

(Hint: (i) $\int d^3r \; \vec{J}(\vec{r}) = I \oint d\xi \; \hat{n}(\xi)$

(ii) Use curl theorem.

② Calculate the mutual inductance $M = 2L_{12}$ of a pair of parallel wires of length 'L' separated by a distance 'a'.

Answer: $$M = \frac{4}{c^2}\left[ L \ln \frac{L}{a} + L \ln \left\{ 1 + \sqrt{1 + \left(\frac{a}{L}\right)^2} \right\} + L \left\{ \frac{a}{L} - \sqrt{1 + \left(\frac{a}{L}\right)^2} \right\} \right]$$

Also, show that in the limit $L >> a$ it reduces to

$$M \approx \frac{4L}{c^2}\left[ \ln \frac{2L}{a} - 1 \right]$$