Electromagnetism H.W # 10

1 Using the expression for magnetic dipole moment with respect to a stationary origin given by

$$\vec{\mu} = \int d^3r \ \vec{r} \times \vec{J}(\vec{r}) ,$$

calculate the magnetic dipole moment of a circular current carrying wire in the x-y plane centered on the origin to be

$$\vec{\mu} = \pi a^2 I \ \hat{k}$$

where 'a' is the radius of the loop and 'I' is the steady current in the wire.

2 For a macroscopic body we saw,

$$\rho_{eff}(\vec{r},t) = - \vec{\nabla} \cdot \vec{P}(\vec{r},t)$$

$$\vec{J}_{eff}(\vec{r},t) = \frac{\partial \vec{P}(\vec{r},t)}{\partial t} + \vec{\nabla} \times \vec{M}(\vec{r},t) .$$

Show that these effective densities satisfy the equation of charge conservation

$$\frac{\partial}{\partial t} \rho_{eff} + \vec{\nabla} \cdot \vec{J}_{eff} = 0 .$$

(3) Time-independent electric dipole moment:

$$\vec{P}(\vec{r},t) = \vec{P}_0\, \delta^{(3)}(\vec{r}-\vec{r}_p)$$

(a) Show that the potential due to a time-independent electric dipole moment is

$$\phi(\vec{r}) = + \frac{\vec{P}_0 \cdot (\vec{r}-\vec{r}_p)}{|\vec{r}-\vec{r}_p|^3}$$

$$= + \frac{\vec{P}_0 \cdot \hat{r}}{r^3} \quad (\text{if } \vec{r}_p \text{ is the origin})$$

(b) Show that the electric field due to time-independent electric dipole moment is

$$\vec{E}(\vec{r}) = - \frac{\vec{P}_0}{|\vec{r}-\vec{r}_p|^3} + \frac{3\left[\vec{P}_0 \cdot (\vec{r}-\vec{r}_p)\right](\vec{r}-\vec{r}_p)}{|\vec{r}-\vec{r}_p|^5}$$

$$= \frac{3(\vec{P}_0 \cdot \hat{r})\,\hat{r} - \vec{P}_0}{r^3} \quad (\text{if } \vec{r}_p \text{ is the origin})$$

④ Time-dependent (linear) and stationary electric dipole moment.

$$\vec{P}(\vec{r},t) = \vec{P}_0(t) \; \delta^{(3)}\left(\vec{r} - \vec{r}_p(t)\right)$$

Given : (i) $\vec{r}_p(t)$ is constant in time.

(ii) $\vec{P}_0(t) = \vec{\alpha}_0 t + \vec{\beta}_0$

Show that the relevant equations are.

$$\vec{\nabla} \cdot \vec{E} = - \vec{P}_0(t) \cdot \vec{\nabla}_r \; \delta^{(3)}(\vec{r} - \vec{r}_p)$$

$$-\vec{\nabla} \times \vec{E} = \frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c} \vec{\alpha}_0 \; \delta^{(3)}(\vec{r} - \vec{r}_p)$$

Do these equations de-couple into electrostatics and magnetostatics ? (I think they do not.)

5. Magnetic dipole moment (time independent)

$$\vec{M}(\vec{r},t) = \vec{M}_0 \, \delta^{(3)}(\vec{r} - \vec{r}_m)$$

(a) Show that the vector potential for a magnetic dipole moment is

$$\vec{A}(\vec{r},t) = \frac{1}{c} \frac{\vec{M}_0 \times (\vec{r} - \vec{r}_m)}{|\vec{r} - \vec{r}_m|^3}$$

$$= \frac{1}{c} \frac{\vec{M}_0 \times \hat{r}}{r^2} \qquad (\text{if } \vec{r}_m \text{ is at origin})$$

(b) Show that the magnetic field due to a magnetic dipole moment is

$$\vec{B}(\vec{r},t) = \frac{1}{c} \frac{2\vec{M}_0}{|\vec{r} - \vec{r}_m|^3}$$

$$= \frac{1}{c} \frac{2\vec{M}_0}{r^3} \qquad (\text{if } \vec{r}_m \text{ is at origin})$$

6. Work on:

→ Prob 4.5, 4.9

→ Prob 6.1, 6.3, 6.5

(Hint for 6.5 (c) — Use the identity for $\vec{A} \times (\vec{B} \times \vec{C})$ and notice that $\vec{\nabla} \times \vec{E} = 0$ for electrostatics.)