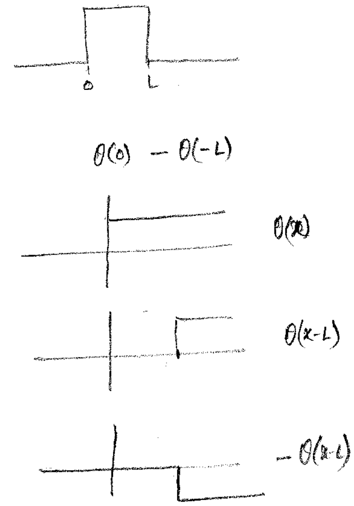
Problem 4.11 (Griffiths) {Also see the magnetic analog in prob. 6.9 Griffiths}

① $$\vec{P}(s,\theta,z) = \begin{cases} P_0\,\hat{z}\;\theta(a-s) & 0 < z\ L \\ 0 & \text{otherwise} \end{cases}$$



$$= P_0\,\hat{z}\;\theta(a-s)\left[\theta(z) - \theta(x-L)\right]$$

② $$\vec{\nabla}\cdot\vec{P} = P_0\,\hat{z}\;\vec{\nabla}\left[\theta(a-\sqrt{x^2+y^2})\,\theta(z) - \theta(a-\sqrt{x^2+y^2})\,\theta(z-L)\right]$$

$$= P_0\,\hat{z}\Big[\delta(a-s)(-1)\,\hat{s}\;\theta(z)$$

$$+\ \theta(a-s)\;\hat{z}\;\delta(z)$$

$$-\ \delta(a-s)\,(-1)\;\hat{s}\;\theta(z-L)$$

$$-\ \theta(a-s)\;\hat{z}\;\delta(z-L)$$

$$-\frac{1}{2}\frac{2\vec{s}}{\sqrt{x^2+y^2}}$$

$$= P_0\;\theta(a-s)\;\delta(z) - P_0\;\theta(a-s)\;\delta(z-L)$$

③ $$\rho(s,\theta,z) = -P_0\;\theta(a-s)\;\delta\left(z+\tfrac{L}{2}\right) + P_0\;\theta(a-s)\;\delta\left(z-\tfrac{L}{2}\right)$$

④ $$\phi(s,\theta,z) = \int_0^{\infty} s'ds'\int_0^{2\pi}d\theta'\int_{-\infty}^{+\infty}dz'\;\frac{1}{|\vec{r}-\vec{r}'|}\left[-P_0\;\theta(a-s)\;\delta\left(z'+\tfrac{L}{2}\right) + P_0\;\theta(a-s)\;\delta\left(z'-\tfrac{L}{2}\right)\right]$$

$$= \int_0^{a} s'ds'\int_0^{2\pi}d\theta'\int_{-\infty}^{+\infty}dz'\left\{\frac{-P_0\,\delta\left(z'+\frac{L}{2}\right)}{\left[(\vec{s}-\vec{s}')^2+(z-z')^2\right]^{1/2}} + \frac{P_0\,\delta\left(z'-\frac{L}{2}\right)}{\left[(\vec{s}-\vec{s}')^2+(z-z')^2\right]^{1/2}}\right\}$$

$$= \int_0^{a} s'ds'\int_0^{2\pi}d\theta'\;\frac{-P_0}{\left[(\vec{s}-\vec{s}')^2+\left(z+\frac{L}{2}\right)^2\right]^{1/2}} + \int_0^{a} s'ds'\int_0^{2\pi}d\theta'\;\frac{+P_0}{\left[(\vec{s}-\vec{s}')^2+\left(z-\frac{L}{2}\right)^2\right]^{1/2}}$$