Electromagnetism H·W # ⑭

① Show that

$$\vec{\nabla}_r \left(\hat{r} \cdot \vec{r}\,' \right) = - \frac{1}{r} \, \hat{r} \times \left(\hat{r} \times \vec{r}\,' \right)$$

② A non-relativistic particle of charge '$+q$' oscillates on the z-axis with frequency ω_0 and amplitude of oscillation 'A' given by

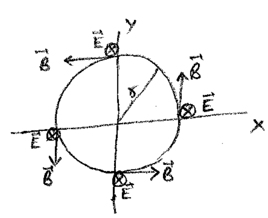
$$\vec{x}(t) = \hat{z} \; A \cos \omega_0 t \, .$$

(a) Show that the radiation fields are

$$\vec{B}(\vec{r},t) = + \frac{q}{c^2} \frac{(\hat{r} \times \hat{z})}{r} \, \omega^2 A \cos\left[\omega_0 \left(t - \frac{r}{c} \right) \right] + O\left(\frac{1}{r^2}, \frac{v}{c} \right)$$

$$\vec{E}(\vec{r},t) = - \hat{r} \times \vec{B}(\vec{r},t) + O\left(\frac{1}{r^2}, \frac{v}{c} \right) .$$

(b) Verify that at '$t = \frac{r}{c}$' the direction of the radiation fields in the x-y plane are



Draw similar diagrams for x-z plane & y-z plane.

(3) A non-relativistic particle of charge '$+q$' moves in a circle of radius 'a' with an angular velocity 'ω_0'. Write down the expressions for the radiation fields $\vec{B}$ & $\vec{E}$.

(4) A non-relativistic particle of charge '$+q$' moves along the z-axis with uniform acceleration 'a'

$$\vec{r}(t) = \hat{z}\,\tfrac{1}{2}at^2 .$$

Write down the expressions for the radiation fields $\vec{B}$ & $\vec{E}$. Draw diagrams illustrating the directions of $\vec{B}$ & $\vec{E}$ at a distance 'r' from the source. (In the book by J. Schwinger et. al. they persuade you to think and conclude that a uniformly accelerated charge particle does not radiate!)