Electromagnetism H.W # 15

① Problem 32.1 in J. Schwinger et. al.

Answer : $\frac{1}{\tau} = \frac{3}{4} \frac{m^3 c^5}{q^4 B^2}$

for $B = 10^4$ gauss :

$\frac{1}{\tau} = 2.58$ sec

The evaluation of the number will be a good exercise in working with Gaussian units.

② Problem 32.2 in J. Schwinger et. al.

Answer : $\frac{1}{\tau} = \frac{3}{2} \frac{m c^3}{q^2 \omega_0^2}$

for $\omega_0 = 10^{15}$ sec^{-1} , $\frac{1}{\tau} \sim 10^{-9}$ sec.

③ Problem 32.3 in J. Schwinger et. al.

Answer : $T = \frac{9}{4} \frac{m^2 c^3 r_0^3}{q^4}$

for $r_0 = 10^{-8}$ cm , $T \sim 10^{-9}$ sec.

(Hint : (i) Integral on E of the form $\int \frac{dE}{E^4}$.

(ii) Total energy of the electron in an atomic orbit is negative.)

$$\phi \qquad (32.45)$$

gradient of this is inversely proportional to

$$\nabla\phi \sim \frac{e}{r^3}\mathbf{r}, \qquad (32.46)$$

l in computing the radiation fields, which e vector potential alone determines the ra-

$$\sim -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \qquad (32.47)$$

$$= \nabla\times\mathbf{A}. \qquad (32.48)$$

n (32.43) enforces the transversality of these $\nabla\cdot\mathbf{A} = 0$, we recover the scalar Maxwell

$$\text{and} \quad \nabla\cdot\mathbf{E} = 0, \qquad (32.49)$$

the relations

$$\text{and} \quad \mathbf{n}\cdot\mathbf{E} = 0, \qquad (32.50)$$

om (32.47) and (32.48). tential, we first write the solution to (32.41)

$$-\frac{1}{\ }4\pi\rho, \qquad (32.51)$$

$$4\pi\frac{\partial}{\partial t}\rho = \frac{1}{\nabla^2}4\pi\nabla\cdot\mathbf{j}. \qquad (32.52)$$

2) as

$$\frac{\pi}{c}\left(\mathbf{1} - \frac{\nabla\nabla}{\nabla^2}\right)\cdot\mathbf{j}, \qquad (32.53)$$

ondition (32.43) transparent. The solution at of (31.20) by applying the operator

$$-\frac{\nabla\nabla}{\nabla^2} \qquad (32.54)$$

$$\int(d\mathbf{r}')\frac{\frac{1}{c}\mathbf{j}(\mathbf{r}', t - \frac{1}{c}|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}. \qquad (32.55)$$

At large distances, by making use of (32.1) and (32.6), we have effectively the replacement

$$\nabla \to -\frac{\mathbf{n}}{c}\frac{\partial}{\partial t} \qquad (32.56)$$

so that the operator $\mathbf{1} - \nabla\nabla/\nabla^2$ can be replaced by

$$\mathbf{1} - \frac{\nabla\nabla}{\nabla^2} \to \mathbf{1} - \frac{\left(-\frac{\mathbf{n}}{c}\frac{\partial}{\partial t}\right)\left(-\frac{\mathbf{n}}{c}\frac{\partial}{\partial t}\right)}{\left(-\frac{\mathbf{n}}{c}\frac{\partial}{\partial t}\right)^2} = \mathbf{1} - \mathbf{nn}. \qquad (32.57)$$

Notice that this symbolic notation is convenient when $1/\nabla^2$ can be computed simply. By making use of (32.11) and (32.1), we obtain the asymptotic form of the vector potential, in the radiation gauge, to be

$$\begin{aligned}\mathbf{A}(\mathbf{r},t) &\sim (\mathbf{1}-\mathbf{nn})\cdot\frac{1}{cr}\int(d\mathbf{r}')\mathbf{j}(\mathbf{r}',t_r)\\ &= -\mathbf{n}\times\left[\mathbf{n}\times\frac{1}{cr}\int(d\mathbf{r}')\mathbf{j}(\mathbf{r}',t_r)\right].\end{aligned} \qquad (32.58)$$

The resulting electric and magnetic fields are precisely the same as those found in the Lorentz gauge, (32.7) and (32.12).

32.6 Problems for Chapter 32

1. A particle, of charge e and mass m, moves with speed v, $v/c \ll 1$, in a uniform magnetic field $\mathbf{B}$. Suppose the motion is confined to the plane perpendicular to $\mathbf{B}$. Calculate the power radiated P in terms of B and v, and show that
$$P = -\frac{dE}{dt} = \gamma E,$$
where E is the energy of the particle, and find γ. Since then
$$E(t) = E(0)e^{-\gamma t},$$
$1/\gamma$ is the mean lifetime of the motion. For an electron, find $1/\gamma$ in seconds for a magnetic field of 10^4 gauss.

2. A nonrelativistic particle of charge e and mass m moves in a Hooke's law potential (a linear oscillator) with natural frequency ω_0. Again find P, the power radiated. Recall that for such motion, the time-averaged kinetic and potential energy satisfy
$$\overline{T} = \overline{V} = \frac{1}{2}E.$$
Show then that the power radiated, averaged over one cycle is
$$P = -\frac{dE}{dt} = \gamma E,$$
and find γ. Compute $1/\gamma$ in seconds when ω_0 is 10^{15} sec^{-1} (a characteristic atomic frequency, corresponding to visible light).

3. An electron of charge e and mass m moves in a circular orbit under Coulomb forces produced by a proton. The average potential energy is related to the total energy by

$$E = \frac{1}{2}\overline{V}.$$

Suppose, as it radiates, the electron continues to move on a circle, and calculate the power radiated, and thereby $-dE/dt$, as a function of E (the relation is no longer linear). Integrate this result, and find how long it takes for the energy to change from E_2 to E_1. In a finite time the electron reaches the center, so calculate how long it takes the electron to hit the proton if it starts from an initial radius of

$$r_{\text{initial}} = 10^{-8}\text{cm}.$$

(This instability was one of the reasons for the discovery of quantum mechanics.)

Derive the alternative form for the angular distribution of radiated power, (32.18),

$$\frac{dP}{d\Omega} = \frac{1}{4\pi c}\left(\left[\int (d\mathbf{r}')\frac{1}{c}\frac{\partial}{\partial t}\mathbf{j}(\mathbf{r}', t_r)\right]^2 - \left[\int (d\mathbf{r}')\frac{\partial}{\partial t}\rho(\mathbf{r}', t_r)\right]^2\right).$$

Chapter 33

Radiation—Sou
View

33.1 Conservation of Ene

Having examined the radiation fields, we t
of the source of the radiated energy. Enc
from the charges to the electromagnetic fiel
work on the field is

$$-\mathbf{j}\cdot\mathbf{E} = \frac{\partial}{\partial t}\left(\frac{\mathbf{E}^2 + \mathbf{B}^2}{8\pi}\right)$$

which is the local statement of energy cons
grated over a large volume enclosing the c
conservation of total energy follows:

$$\int (d\mathbf{r})\,(-\mathbf{j}\cdot\mathbf{E}) = \frac{d}{dt}\int (d\mathbf{r})\,\frac{\mathbf{E}^2 + \mathbf{B}^2}{8\pi} + $$

or stated in words, the rate at which the
the electromagnetic field is equal to the su
electromagnetic energy, E, in the volume,
of the surface bounding the volume. Equat
of calculating the radiated power, P, by cc
transferred to the fields,

$$\int (d\mathbf{r})\,(-\mathbf{j}$$

and discarding total time derivative term
diation. From this point of view, we need
current distribution, in contrast to the pr
puted the radiated power by evaluating th