Problem 32.1, (Schwinger + Milton's book)

(1) $\frac{mv^2}{r} = q\frac{v}{c}B \qquad v = \omega r$

$\Rightarrow \quad a = \frac{qvB}{cm} \qquad \omega = \frac{qB}{mc}$

$\vec{a} = -a\cos\omega t\ \hat{i} - a\sin\omega t\ \hat{j} \quad ; \quad \vec{B} = B\hat{k}$

(2) $P = \frac{2}{3}\frac{q^2}{c^3}\left[\vec{a}\left(t-\frac{r}{c}\right)\right]^2$

(3) $\left[\vec{a}\left(t-\frac{r}{c}\right)\right]^2 = a^2$

(4) $P = \frac{2}{3}\frac{q^2}{c^3}a^2 = \frac{2}{3}\frac{q^2}{c^3}\left(\frac{qvB}{cm}\right)^2$

$= \frac{4}{3}\frac{q^4B^2}{m^3c^5}\underbrace{\frac{1}{2}mv^2}_{\text{energy of particle}}$

(5) $\frac{dE}{dt} = -\Gamma E \qquad \Gamma = \frac{4}{3}\frac{q^4B^2}{m^3c^5}$

ref: p 558 Milton's book.

(6) $\frac{1}{\Gamma} = \frac{3}{4}\frac{m^3c^5}{q^4B^2}$

$= \frac{3}{4}\frac{(9.1\times10^{-28}\,g)^3\,(3\times10^{10}\,\frac{cm}{s})^5}{(4.8\times10^{-10}\,esu)^4\,(10^4\,gauss)^2} = 2.58\ sec$

Problem 32.2 (Milton's book)

① $\vec{x}(t) = A \cos \omega_0 t \, \hat{k}$

$\vec{a}(t) = -\omega_0^2 A \cos \omega_0 t \, \hat{k}.$

② $V = \frac{1}{2} k x^2 = \frac{1}{2} m \omega_0^2 A^2 \cos^2 \omega_0 t$

$\bar{V} = \frac{1}{4} m \omega_0^2 A^2$

③ $E = 2\bar{V} = \frac{1}{2} m \omega_0^2 A^2$

④ $\vec{a}(t) \cdot \vec{a}(t) = \omega_0^4 A^2 \cos^2 \omega_0 t$

$= \frac{2E}{m} \omega_0^2 \cos^2 \omega_0 t$

⑤ $P = \frac{2}{3} \frac{q^2}{c^3} \left[\vec{a}\left(t - \frac{r}{c}\right)\right]^2$

⑥ $\overline{\left[\vec{a}\left(t - \frac{r}{c}\right)\right]^2} = \frac{2E}{m} \omega_0^2 \int_0^{\frac{2\pi}{\omega_0}} dt \cos^2 \omega_0 t$

$= \frac{E \omega_0^2}{m}$

⑦ $\frac{dE}{dt} = -\frac{2}{3} \frac{q^2}{c^3} \frac{E \omega_0^2}{m} = -\frac{2}{3} \frac{q^2 \omega_0^2}{m c^3} E$

⑧ $\frac{1}{\tau} = \frac{3}{2} \frac{m c^3}{q^2 \omega_0^2}$

$= \frac{3}{2} \frac{(9.1 \times 10^{-28} g) \times (3 \times 10^{10} \frac{cm}{s})^3}{(4.8 \times 10^{-10} esu)^2 \times (10^{15} \frac{1}{sec})^2}$

$\simeq \underline{1.6 \times 10^{-9} \text{ sec}}.$

Problem 32.3 (Milton's book)

① $\dfrac{q^2}{r^2} = \dfrac{mv^2}{r}$

② $E = \frac{1}{2}mv^2 - \dfrac{q^2}{r} = -\frac{1}{2}mv^2 = -\frac{1}{2}\dfrac{q^2}{r}$

③ $a^2 = \left[\dfrac{q^2}{mr^2}\right]^2 = \dfrac{q^4}{m^2r^4}$

$= \dfrac{1}{m^2q^4}\left(\dfrac{q^2}{r}\right)^4 = \dfrac{(2E)^4}{m^2q^4}$

$\vec{a} = a\cos\omega t\,\hat{i} + a\sin\omega t\,\hat{j}$

$a^2 = a^2$ has no time dependence, so we need not worry about retarded time.

④ $P = \dfrac{2}{3}\dfrac{q^2}{c^3}a^2$

$= \dfrac{2}{3}\dfrac{q^2}{c^3}\dfrac{16E^4}{m^2q^4}$

$\alpha = \dfrac{32}{3}\dfrac{1}{m^2q^2c^3}$

⑤ $\dfrac{dE}{dt} = -\dfrac{32}{3}\dfrac{1}{m^2q^2c^3}E^4$

⑥ $\displaystyle\int_{-E_0}^{-\infty}\frac{dE}{E^4} = -\alpha\int_0^T dt$

$-\alpha T = -\dfrac{3}{E^3}\Big|_{-E_0}^{-\infty} = -\dfrac{3}{E_0^3}$

⑦ $T = \dfrac{3}{\alpha E_0^3} = 3 \times \dfrac{3m^2q^2c^3}{32} \times \left(\dfrac{2r_0}{q^2}\right)^3$

$= \dfrac{9}{4}\dfrac{m^2c^3r_0^3}{q^4}$

$= \dfrac{9 \times (9.1\times10^{-28}\,g)^2 \times (3\times10^{10}\,\frac{cm}{s})^3 \times (10^{-8}\,cm)^3}{4 \times (4.8\times10^{-10}\,esu)^4}$

$\simeq 10^{-9}$ sec