Physics 2414, Spring 2005
Group Exercise 6, Mar 24, 2005

Name 1: __________ OUID 1: __________

Name 2: __________ OUID 2: __________

Name 3: __________ OUID 3: __________

Name 4: __________ OUID 4: __________

Solutions **Solutions**

Section Number: ____

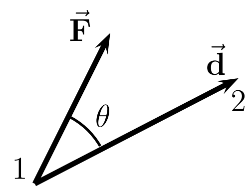
Work and Energy

A mass m moves from point '1' to point '2' under the action of the force $\vec{\mathbf{F}}$. Kinetic energy of the mass is given by the expression

$$K = \frac{1}{2}mv^2 \tag{1}$$

where v is the velocity of the particle. The change in kinetic energy of the mass is

$$\Delta K = K_f - K_i \tag{2}$$
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \tag{3}$$



Work done *by* the force $\vec{\mathbf{F}}$ *on* the mass m is given by the expression

$$W = |\vec{\mathbf{F}}|d\cos\theta \tag{4}$$

where, $\vec{\mathbf{d}}$ is the displacement of the mass m, and θ is the angle between the force vector and the displacement vector (see figure).

The work done on the mass m equals the change in kinetic energy. The expression relating this is given by

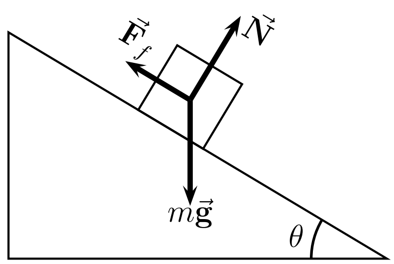
$$\Delta K = W. \tag{5}$$

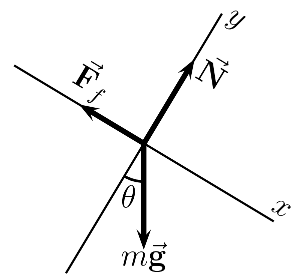
If there is more than one force acting on the mass, the change in kinetic energy equals the total work done by all the forces, and is given by the expression

$$\Delta K = \sum_i W_i. \tag{6}$$

Problems

A block of mass $M = 100$ kg slides on a frictional incline plane under gravity. The incline makes an angle $\theta = 30^o$ with the horizontal. The coefficient of kinetic friction between the mass and the surface of the incline is $\mu_k = 0.25$. The mass starts from the highest point on the incline plane and reaches the lowest point on the plane. The free body force diagram is provided to you.



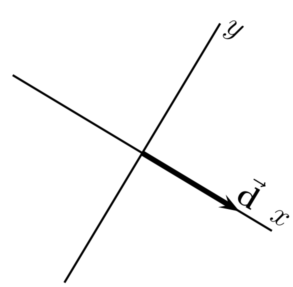


1. *Displacement vector:*

(a) If the base of the incline plane measures 5 meters, what is the magnitude of the displacement of the block.

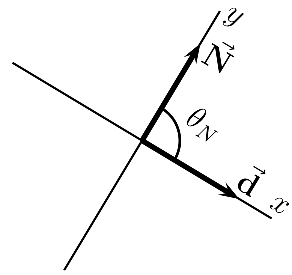
$$|\vec{\mathbf{d}}| = \frac{5}{\cos\theta} = \frac{5}{\cos 30} = 5.75\text{m} \tag{7}$$

(b) Show the displacement vector $\vec{\mathbf{d}}$ on the graph.



2. *Work done by the normal force* $\vec{\mathbf{N}}$*:*

(a) Show the normal force $\vec{\mathbf{N}}$ and the displacement $\vec{\mathbf{d}}$ on the graph.



(b) What is the angle between the Normal force $\vec{\mathbf{N}}$ and the displacement vector $\vec{\mathbf{d}}$?

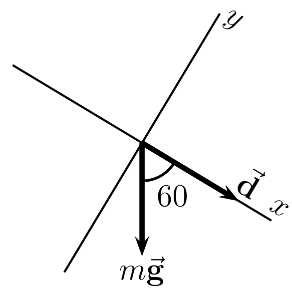
$$\theta_N = 90^o \tag{8}$$

(c) Calculate the work done by the Normal force on the mass.

$$W_N = |\vec{\mathbf{N}}| d \cos\theta_N \tag{9}$$
$$= 0 \qquad (\cos 90 = 0) \tag{10}$$

3. *Work done by gravity* $m\vec{\mathbf{g}}$*:*

(a) Show the force $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ on the graph.



(b) What is the angle between the force $m\vec{\mathbf{g}}$ and the displacement vector $\vec{\mathbf{d}}$?

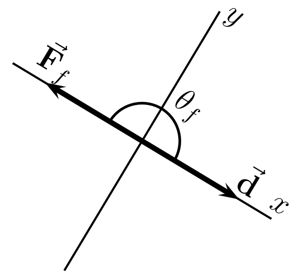
$$\theta_{mg} = 90 - \theta = 60 \tag{11}$$

(c) Calculate the work done by the force $m\vec{\mathbf{g}}$ on the mass.

$$W_{mg} = mgd\cos(90 - \theta) \tag{12}$$
$$= 100 \times 9.8 \times 5.75 \times \cos 60 = 2818\text{J} \tag{13}$$

4. *Work done by the frictional force* $\vec{\mathbf{F}}_f$*:*

(a) Show the frictional force $\vec{\mathbf{F}}_f$ and the displacement $\vec{\mathbf{d}}$ on the graph.



(b) What is the angle between the frictional force $\vec{\mathbf{F}}_f$ and the displacement vector $\vec{\mathbf{d}}$?

$$\theta_f = 180^o \tag{14}$$

(c) Calculate the work done by the frictional force on the mass. (Hint: $|\vec{\mathbf{F}}_f| = \mu_k |\vec{\mathbf{N}}| = \mu_k mg \cos\theta$)

$$\begin{aligned} W_f &= \mu_k mg \cos\theta d \cos\theta_f & (15)\\ &= 0.25 \times 100 \times 9.8 \times \cos 30 \times 5.75 \times \cos 180 & (16)\\ &= 1225\text{J} & (17) \end{aligned}$$

(d) What is the work done by static frictional force?

Since static fiectional forces do not move the mass, the work done by static frictional force is zero.

5. *Total work done by the forces on the mass:*

The total work done by the three forces on the mass is equal to the sum of the individual work done.

$$W_{\text{tot}} = W_N + W_{mg} + W_f \tag{18}$$

(a) Determine the total work done by the three forces on the mass.

$$W_{\text{tot}} = 0 + 2818\text{J} - 1225\text{J} = +1593\text{J} \tag{19}$$

6. *Change in kinetic energy:*

(a) What is the kinetic energy of the mass just before it starts to slide.

$$K_i = \frac{1}{2}mv_i^2 = 0 \qquad (v_i = 0) \tag{20}$$

(b) The acceleration of the mass is determined by the expression

$$a = g\sin\theta - \mu_k g\cos\theta. \tag{21}$$

Evaluate a.

$$a = 9.8 \times \sin 30 - 0.25 \times 9.8 \times \cos 30 = 2.77\text{m/s}^2 \tag{22}$$

(c) Determine the velocity of the mass just before it reaches the ground. (Hint: $v_f^2 - v_i^2 = 2ad$.)

$$v_f = \sqrt{v_i^2 + 2ad} \tag{23}$$

$$= \sqrt{0 + 2 \times 2.77 \times 5.75} = 5.64\text{m/s} \tag{24}$$

(d) Determine the kinetic energy of the block just before it reaches the ground.

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2} \times 100 \times 5.64^2 = 1590\text{J} \tag{25}$$

(e) Determine the change in the kinetic energy of the mass.

$$\Delta K = 1.6 \times 10^3\text{J} \tag{26}$$

7. *Total work done by the forces on the mass equals the change in kinetic energy of the mass:*

(a) Verify that the total work done by the forces on the mass equals the change in the kinetic energy of the mass.

$$W_{\text{tot}} = 1593\text{J} = 1.6 \times 10^3\text{J} \tag{27}$$

$$\Delta K = 1590\text{J} = 1.6 \times 10^3\text{J} \tag{28}$$