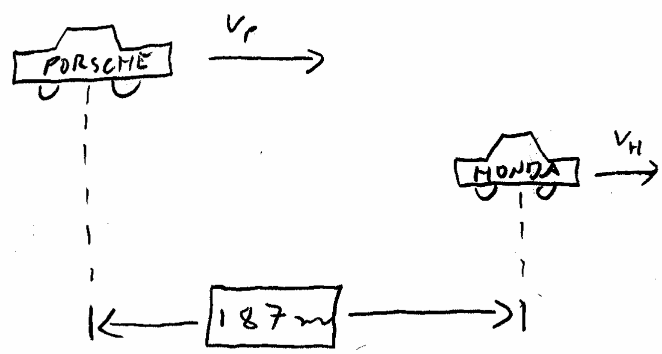
Homework #3

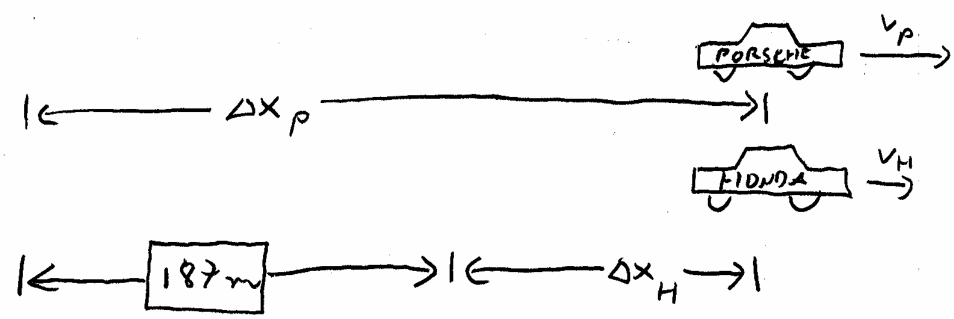
1.) The Porsche starts being 187 m behind the Honda, but is moving faster. Thus at some time he will catch up with the Honda.

Before (at time zero) $\longrightarrow x$



v_p : speed of porsche
v_H : speed of Honda
Δx_p: displacement of Porsche
Δx_H: displacement of Honda

After (after time interval Δt)



When the Porsche catches up (at the time t_f), the displacements satisfy

$$\Delta x_p = \boxed{187\,m} + \Delta x_H$$

But we know for motion along a line with constant velocity we have

$$\Delta x = v \cdot \Delta t \Rightarrow \Delta x_p = v_p \cdot \Delta t\ ,\ \Delta x_H = v_H \cdot \Delta t$$

and thus, plugging in, we get

$$v_p \Delta t = \boxed{187\,m} + v_H \Delta t$$

Now we solve for Δt, the time that passes from when they start to when they catch up:

do this to both sides of equation!

$$v_p \Delta t = \boxed{187\,m} + v_H \Delta t \qquad \Big|\ -v_H \Delta t$$

$$\Leftrightarrow \quad (v_p - v_H)\,\Delta t = \boxed{187\,m} \qquad \Big|\ \div (v_p - v_H)$$

$$\Leftrightarrow \quad \Delta t = \frac{\boxed{187\,m}}{(v_p - v_H)}$$

and with the given numbers $v_p = \boxed{25.1\ m/s}$, $v_H = \boxed{18.9\ m/s}$

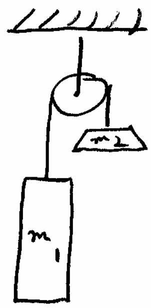
we get

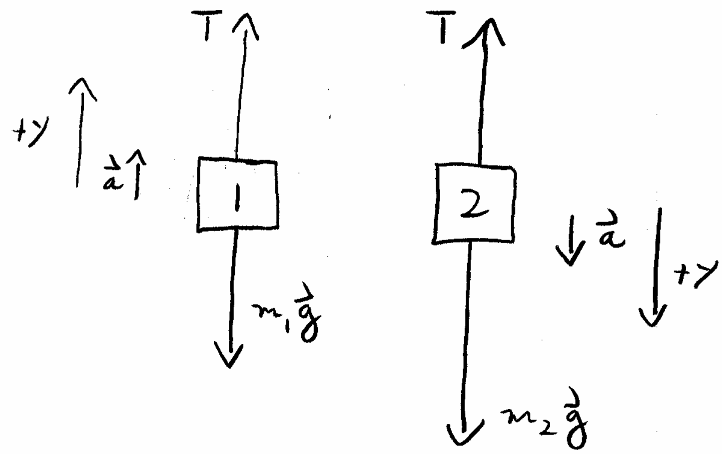
$$\Delta t = \frac{\boxed{187\,m}}{\boxed{25.1\ m/s} - \boxed{18.9\ m/s}}$$

$$\Rightarrow \quad \Delta t = 30.161\ \frac{\cancel{m}}{\cancel{m}/s}$$

$$\Rightarrow \quad \Delta t = 30.161\ s$$

2.)





note how $\vec{a}_1 = \vec{a}_2 = \vec{a}$!

We need to find the accelerations for each block.
The blocks are connected by the rope, so if one moves up, the other one has to move down and vice versa. In the FBD's above we used our intuitive knowledge that the heavier block will accelerate downward, and thus the other block will accelerate upwards. Because of the rope the tension forces a an action-reaction pair and the acceleration will be of same magnitude for both blocks. We know that for any object

$$\sum \vec{F}_{on\ obj} = m_{obj} \cdot \vec{a}_{obj}$$

Sum forces in vertical direction

block 1: $$T - m_1 g = m_1 a \qquad (1)$$

block 2: $$-T + m_2 g = m_2 a \qquad (2)$$

The changed signs in last equation are from the choice of +Y-direction. This choice is needed because the rope goes around the pulley and thus changes direction from "upwards" on the left to "downwards" on the right.

Add eqn ① to eqn. ② to eliminate T, to get

$$\cancel{T} - m_1 g - \cancel{T} + m_2 g = (m_1 + m_2) a$$

(alternatively solve eqn. ① for T and plug that T into eqn. ②)

Solve for a

$$-m_1 g + m_2 g = (m_1 + m_2) a \quad \Big| \cdot \frac{1}{m_1 + m_2}$$

$$\left(\frac{m_2 - m_1}{m_1 + m_2}\right) g = a$$

With $m_1 = \boxed{3.2\ kg}$

$m_2 = \boxed{5.4\ kg}$ and $g = 9.8\ m/s^2$

We get

$$a = \left(\frac{\boxed{5.4\ kg} - \boxed{3.2\ kg}}{\boxed{3.2\ kg} + \boxed{5.4\ kg}}\right) \cdot 9.8\ m/s^2$$

$$a = 2.507 \frac{\cancel{kg}}{\cancel{kg}}\ m/s^2$$

2.) b) Sum of forces on block 1 from before, solve for T

$$T - m_1 g = m_1 a$$

$$\Rightarrow \quad T = m_1 (a+g)$$

Plug in m_1, a, g from part a)

$$T = \boxed{3.2\,kg}\,(2.506 + 9.8)\ m/s^2$$

$$T = 39.38\ kg\ m/s^2 \qquad \left(1N = 1\,\frac{kg\ m}{s^2}\right)$$

$$T = 39.38\ N$$

3.) When the driver starts breaking, the car is accelerated backwards with constant acceleration. It accelerates with negative acceleration from its initial speed V_{0x} to a final speed of $V_x = 0$.

For motion with constant acceleration, we know (book eqn. 3-13)

$$V_x^2 - V_{0x}^2 = 2 a_x \Delta x$$

Δx : displacement
V_{0x} : initial speed
V_x : final speed

Here we have $V_{0x} = 0$, Δx is the stopping distance, and a_x is the same for both breaking processes

$$\Rightarrow \qquad V_x^2 = 2 a_x \Delta X$$

Use index 1 for first breaking process, and index 2 for second one. We need to find ΔX_2. a is the same for both. So

$$V_{x_1}^2 = 2 a_x \Delta X_1 \qquad (1)$$

and $$V_{x_2}^2 = 2 a_x \Delta X_2 \qquad (2)$$

divide eqn. (2) by eqn (1) to get

$$\frac{V_{x_2}^2}{V_{x_1}^2} = \frac{2 a_x \Delta X_2}{\cancel{2 a_x} \Delta X_1}$$

and solve for ΔX_2

$$\Delta X_2 = \frac{V_{x_2}^2}{V_{x_1}^2} \Delta X_1$$

With $\Delta X_1 = \boxed{13m}$, $V_{x_1} = \boxed{39.8 \frac{mi}{h}}$ and $V_{x_2} = \boxed{55.9 \frac{mi}{h}}$

We get

$$\Delta X_2 = \frac{\boxed{55.9 \frac{mi}{h}}^2}{\boxed{39.8 \frac{mi}{h}}^2} \cdot \boxed{13 m}$$

$$\Delta X_2 = 25.64 \, m$$

Note how $\frac{mi}{h}$ cancelled out

$\Rightarrow$ no need for unit conversion when working with ratios!

4.) see problem 2, there we got for the system of 2 masses on a pulley: ($m_2 = \boxed{9.0\,kg}$, $m_1 = \boxed{3.6\,kg}$)

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g = \left(\frac{\boxed{9.0\,kg} - \boxed{3.6\,kg}}{\boxed{9.0\,kg} + \boxed{3.6\,kg}}\right) \cdot 9.8\,m/s^2 = 4.2\,m/s^2$$

The time to reach the floor can be found by using the KINEMATICS for constant acceleration; for mass 2:

$$\Delta Y = V_0 \Delta t + \frac{1}{2} a \Delta t^2$$

here $V_0 = 0$, as it starts from rest

Thus

$$\Delta Y = \frac{1}{2} a \Delta t^2$$

Solve for the time it takes (Δt)

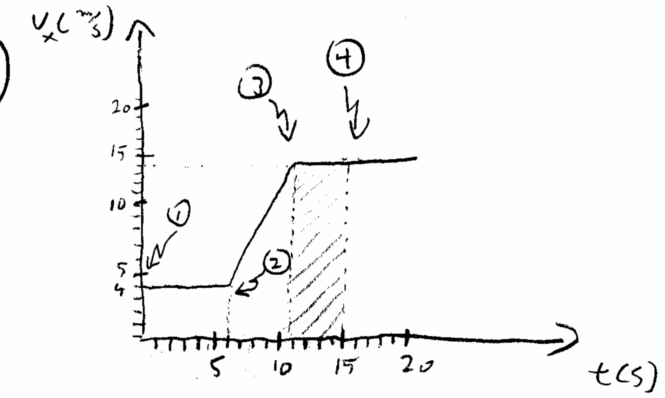
$$\Delta t = \sqrt{\frac{2 \Delta Y}{a}}$$

But we know the displacement ΔY for mass has to be $\boxed{130\,cm}$ as it has to move all the way from it's initial position down to the floor. Plug it in:

$$\Delta t = \sqrt{\frac{2 \cdot \boxed{130\,cm}}{4.2\,m/s^2}}$$, use $\boxed{130\,cm} = 130 \cdot 10^{-2}\,m = \boxed{1.3\,m}$

$$\Delta t = \sqrt{\frac{2 \cdot \boxed{1.3\,m}}{4.2\,m/s^2}} = \sqrt{0.6190 \cdot s^2} = 0.7867\,s$$

5.)



a) What is $a_{av,x}$ from $t=6s$ till $t=11s$?

$a_{av,x}$ is the average slope of the $V_x(t)$ graph

here, from $t=6s$ till $t=11s$, we have, remembering

$$a_{av,x} = \frac{\Delta V}{\Delta t} = \frac{V_{xf} - V_{xi}}{t_f - t_i},$$

f : final
i : initial

from graph:

point ③ : $V_{xf} = V_x(t=11s) = \boxed{14\,m/s}$

$t_f = \boxed{11\,m/s}$

point ② : $V_{xi} = V_x(t=6s) = \boxed{4\,m/s}$

$t_i = \boxed{6s}$

plug those in

$$a_{av,x} = \frac{\boxed{14\,m/s} - \boxed{4\,m/s}}{\boxed{11\,s} - \boxed{6\,s}} = \frac{10\,m/s}{5\,s} = 2\,m/s^2$$

6) To find the average velocity, we note that the velocity is changing <u>linearly</u> in time during the interval from $t=6s$ to $t=11s$ (the graph is a straight line with no curves or kinks)

Then we can use

$$V_{av,x} = \frac{V_{xi} + V_{xf}}{2} \qquad \text{(book, eqn. 3-11, page 79)}$$

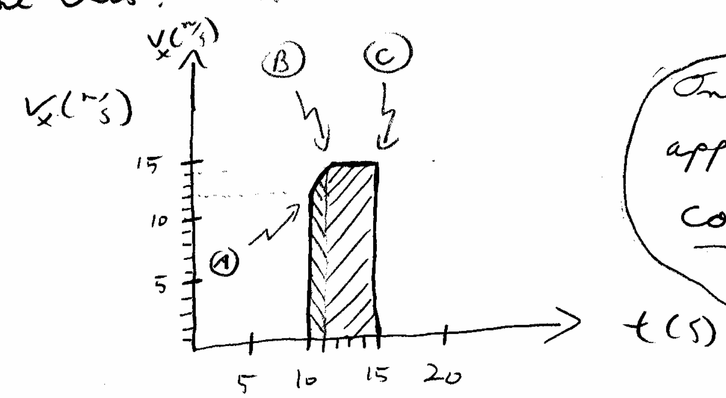
Using same numbers as before, we get

$$V_{av,x} = \frac{\boxed{4\ m/s} + \boxed{14\ m/s}}{2} = \frac{18\ m/s}{2} = 9\ m/s$$

c) How far does the car travel from $t = 10s$ to $t = 15s$

How far it travels is the area under the V_x graph.

Here it is the area under the curve between point $t = 10s$ and (4). The area we are looking at looks like this:

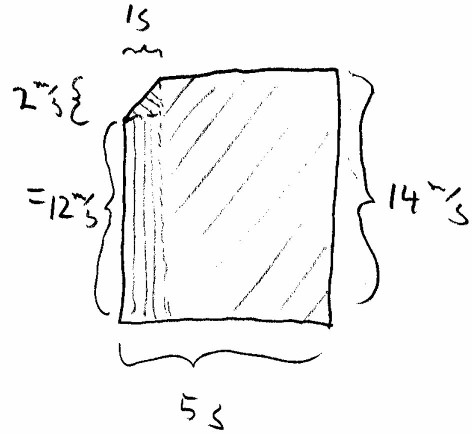


One way to get an approximate answer: count the squares under the curve

where I only drew the interesting part. The area can be divided into a small rectangle, a triangle and a large rectangle as follows:

symbolically

total area =



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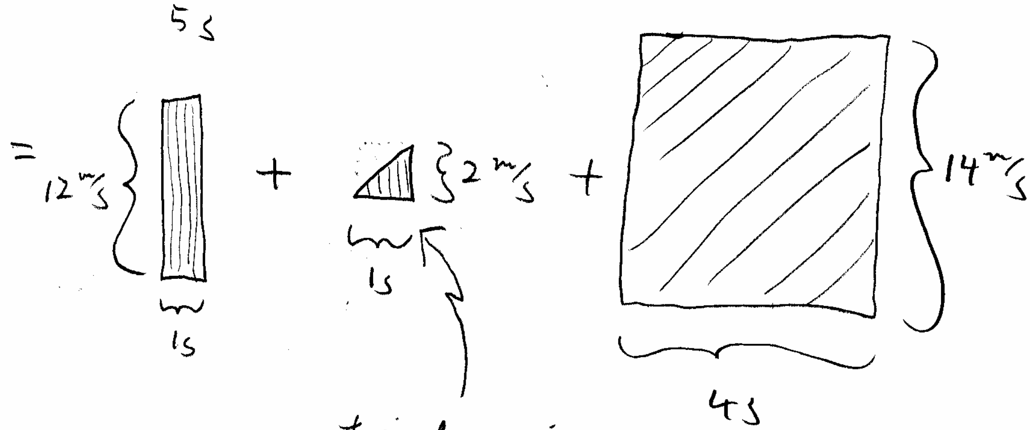


triangle area is half the area of a rectangle

$$= A_1 + A_2 + A_3$$

where $A_1 = 12\,\text{m/s} \cdot 1\,\text{s} = 12\,\text{m}$

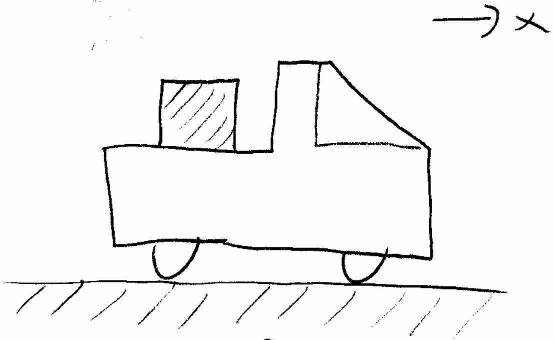
$$A_2 = \frac{1}{2}\left(1\,\text{s} \cdot 2\,\text{m/s}\right) = 1\,\text{m}$$

$$A_3 = 4\,\text{s} \cdot 14\,\text{m/s} = 56\,\text{m}$$

and so the total area is $(12+1+56)\,\text{m} = 69\,\text{m}$

and this total area is <u>equal to how far the car travels</u>!

6.)

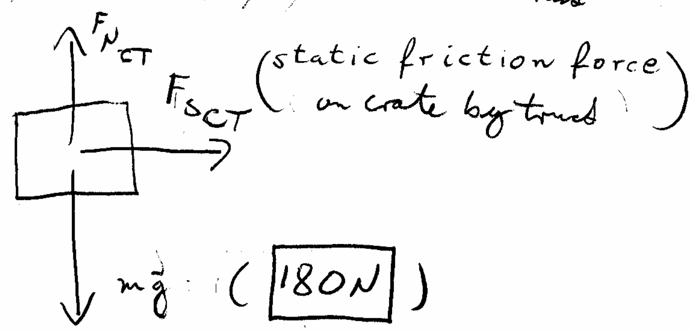


$\mu_s = 0.3$

$\mu_k = 0.2$

$V_{x_{truck}} = 8 \ m/s$

Crate:



F_f : friction force

F_S : static fr. force

F_N : normal force

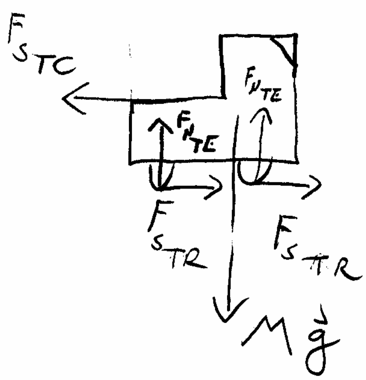
R : road

T : truck

E : earth

C : crate

(Truck (not needed)



)

a) Truck moves at constant velocity $\Rightarrow a = 0 \Rightarrow$ no net force on crate in any direction $\Rightarrow$ no net force in horizontal direction on the crate $\Rightarrow$ $F_{S_{CT}} = 0$

b) Truck has $a = 1.0 \ m/s^2$

$\Rightarrow$ Crate also has $a = 1.0 \ m/s^2 \Rightarrow$ net horizontal force on crate is of magnitude $F^H_{tot} = F_{S_{CT}} = m \cdot 1.0 \ m/s^2$

So what is m?

We know $m\,g = 180\,N$ for the crate and $g = 9.8\ m/s^2$

So $$m = \frac{180N}{g} = 18.4\ N\,\frac{s^2}{m} = 18.4\ kg$$

and with this m we get

$$F_{S\,CT} = m \cdot a = 18.4\ kg \cdot 1.0\ m/s^2 = 18.4\ N \simeq 18N$$

c) What is the max. a that truck can have without the crate sliding?

* The static friction that prevents the crate from sliding can never be larger than $\mu_S F_{NCT}$.
If the truck accelerates too fast, then the crate would need a net force $F^H_{net} = m\,a$ that is larger than the maximum possible $\mu_S F_{NCT}$ in order to keep up with the truck. Thus the crate would slide backwards.
The threshold is when $m\,a = \mu_S F_{NCT}$, because that is the maximum force that static friction can provide!

So $$m\,a_{max} = \mu_S F_{NCT}$$

Solve for a_{max}, plug in numbers $\mu_S = 0.3$, $F_{NCT} = 180N$, $m = 18\,kg$

$$a_{max} = \frac{\mu_S F_{NCT}}{m} = \frac{0.3 \cdot 180N}{18.4\,kg} = 2.93\ m/s^2$$ //

7.) Train is slowing down at constant rate

$\Rightarrow$ a is constant

$\Rightarrow$ we can use the equation for constant acceleration that has the speeds and displacement (book eqn. 3-13)

$$V_x^2 - V_{0_x}^2 = 2 a_x \Delta x$$

V_x : final speed

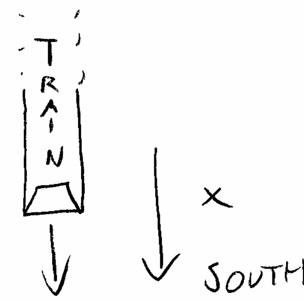
V_{0_x} : initial speed

a_x : acceleration

Δx : displacement

$\Rightarrow$ so, using f and i: $V_{x_f}^2 - V_{x_i}^2 = 2 a_x \Delta x$

a) What is a? Here a is constant, so $a_{av,x} = a_x$



Use $\quad a_{avg,x} = \dfrac{\Delta V}{\Delta t} = \dfrac{V_{x_f} - V_{x_i}}{\Delta t}$

here $V_{x_f} = 6 \text{ m/s}$

$V_{x_i} = 24 \text{ m/s}$

note how positive means going south!

$\Delta t = 9 \text{ s}$

to get $\quad a_{avg,x} = \dfrac{(6-24) \text{ m/s}}{9 \text{ s}} = -2 \text{ m/s}^2$

a_x is negative, negative x-dir means NORTH

(see figure above right)

b) Using (see last page)

$$V_{x_f}^2 - V_{x_i}^2 = 2a_x \Delta x$$

solve for displacement Δx:

$$\Delta x = \frac{V_{x_f}^2 - V_{x_i}^2}{2a_x}$$

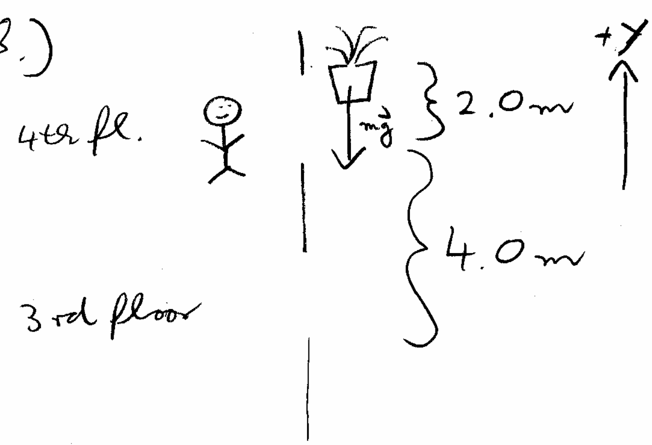
plug in the known numbers

$$\Delta x = \frac{(6\ \tfrac{m}{s})^2 - (24\ \tfrac{m}{s})^2}{2(-2\ \tfrac{m}{s^2})} = 135\ m$$

note how the UNITS come out fine!

keeping the UNITS is an essential double-check to see if your equations are right

8.)



$$F^{V}_{net} = -mg = ma$$

$$\Rightarrow a = -g$$

The flower pot fell

$\Delta x = -2m$ in $\Delta t = 0.093s$

So it's average velocity when zipping by was $V_{avg} = -\frac{2m}{0.093s} = 21.5\ \tfrac{m}{s}$ $\approx V_f$

The flower pot started somewhere higher at $V_i = 0\ \tfrac{m}{s}$

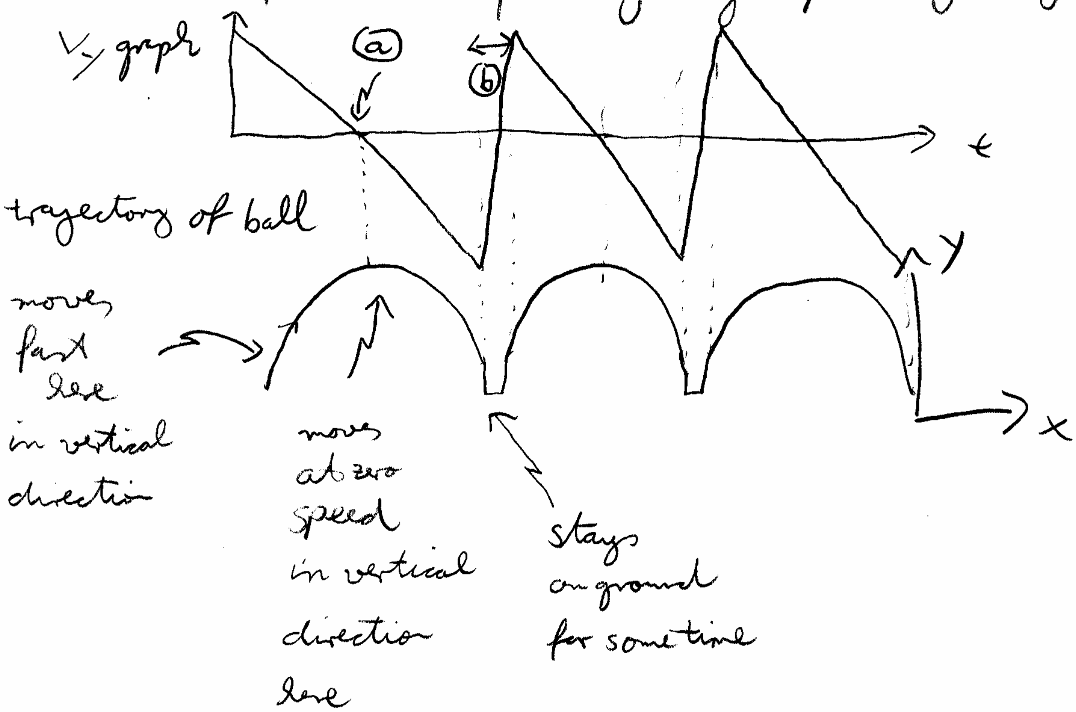
Its acceleration $a = -g$ is constant $\Rightarrow$ we can use

$V_f^2 - V_i^2 = 2a\Delta Y$ to find the displacement, using $V_f \approx V_{ave} = 21.5\ \tfrac{m}{s}$

$$\frac{(21.5\ \tfrac{m}{s})^2 - 0\ \tfrac{m}{s}}{-2 \cdot 9.8\ \tfrac{m}{s^2}} = \Delta Y \Rightarrow \Delta Y = -23.6m$$

it fell 23.6 m down before zipping by $\Rightarrow$ started 6 floors above $\Rightarrow$ started on 10th floor

9.) a) The ball reaches its maximum height whenever $V_y = 0$. This is because the ball is at rest just when it changes its direction of motion from going up to going down.



a) - at what time does the ball reach its maximum height?

→ when V_y is first equal zero, the ball is at its highest point (see figure above) ⇒ from graph: $t \simeq 0.3s$

b) - for how long does the ball stay on the ground?

→ While it is on the ground, it changes its vertical speed from negative to positive. The time it needs for that is the time (b) in above graph: $\Delta t \simeq 0.05 s$

c) - What is the maximum height of the ball?

→ Because of gravity, the ball's acceleration is g (in negative y-direction) during the time interval before it first hits the ground. There is constant acceleration from $t = 0s$ till it is on its highest point at $t = 3s$. Therefore we can use eqn. 3-13 from the book again:

$$V_y^2 - V_{y_0}^2 = 2a\,\Delta x$$

V_{y_0} : initial speed

V_y : final speed

here $V_{y_0} = 3\ m/s$,

$V_y = 0\ m/s$, $a = -g = -9.8\ m/s^2$

Δx = displacement between $t = 0s$ and $t = 3s$

= height h_{MAX}

$$\Rightarrow \quad h_{Max} = \frac{V_y^2 - V_{y_0}^2}{-2g}$$

$$= \frac{(0\ m/s)^2 - (3\ m/s)^2}{-2 \cdot 9.8\ m/s^2}$$

$$= \frac{-9\ m^2/s^2}{-19.6\ m/s^2}$$

$$h_{MAX} = 0.45\ m$$

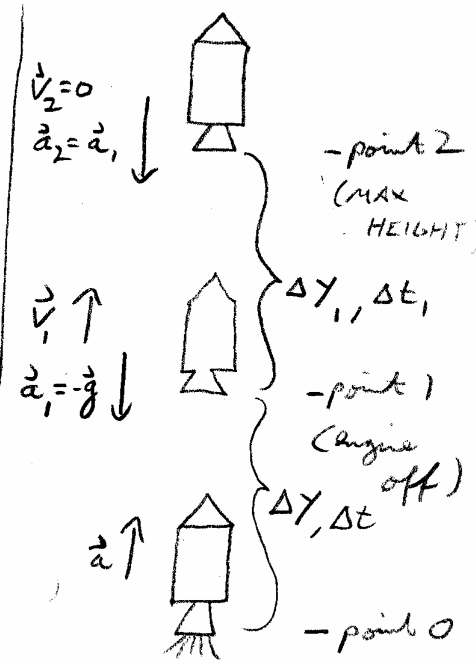
10.) a) Rocket accelerates upward with $a_{rocket} = 20\,m/s^2$ for $50\,s$. The acceleration is constant, so we can use kinematics to find the net displacement

$$\Delta Y = \frac{1}{2} a \Delta t^2 + V_0 \Delta t$$

Here the rocket starts from rest so V_0 is equal to zero.

$$\Delta Y = \frac{1}{2}(20\,m/s^2)(50\,s)^2$$

$$\Delta Y = 25000\,m = 25\,Km$$



b) When does it reach its max. height?

It accelerates up for 50 seconds, then only gravity acts and accelerates it downward. So, after it reached its 25 km, it is going to go up for some time but will slow down until it stops. Then it will start to move downward.

When the engine fails, rocket has been accelerating for $50\,s$ with $20\,m/s^2$ from rest, so its velocity at that time will be

$$V_1 = a\Delta t = 20\,m/s^2\,(50\,s) = 1000\,m/s$$

This velocity has to decrease to zero for the rocket to reach its max height. Gravity accelerates the rocket with $-g$ so that $a = \text{constant} = -g$. Because $a = \text{const}$ we can use kinematics again.

So from $t = 50\,s$ till t it reaches MAX height, we have

$$\Delta V = V_2 - V_1 = -g\,\Delta t_1 \Rightarrow \Delta t_1 = \frac{\Delta V}{-g}$$

we know ΔV must be equal to $-1000\,m/s$. After all it has to slow down to a halt, plug it in:

$$\Delta t_1 = \frac{-1000\,m/s}{-9.8\,m/s^2} = 102\,s$$

The total time that passes from start to reaching max height is then

$$\text{total time} = \Delta t + \Delta t_1 = 50\,s + 102\,s = 152\,s$$

c) The max height is 25 km plus the extra height it gained while slowing down.

It slowed down with constant acceleration $a_1 = -g$ for $\Delta t_1 = 102\,s$,

$V_2 = 1000\,m/s$ is initial velocity of slowing down process

Use kinematics to find ΔY_1, the extra height gained while slowing down

$$\Delta Y_1 = \frac{1}{2} a_1 \Delta t_1^2 + V_2 \Delta t_1$$

$$= -\frac{1}{2} g (102.5\,s)^2 + (1000\,m/s)(102.5\,s)$$

$$= 51019\,m$$

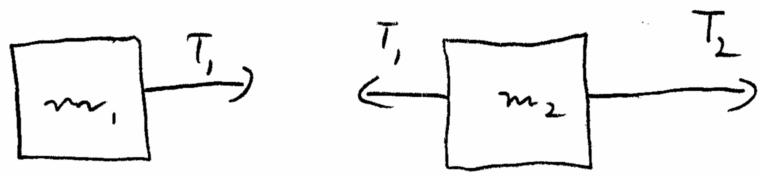
$$\Delta Y_1 = 51\,km$$

Then the total max height is

$$\text{total max height} = \Delta Y + \Delta Y_1 = 25\,km + 51\,k = 71\,k$$

11.)

20/20



The problem states that each block moves with the same acceleration $\Rightarrow$ The net force on block 1 is $m_1 a$ and the net force on block 2 is $m_2 a$

Net forces)

$$T_1 = m_1 a \quad \text{(block 1)} \quad (1)$$

$$T_2 - T_1 = m_2 a \quad \text{(block 2)} \quad (2)$$

from block 1, we know $T_1 = m_1 a$, plug into net force equation of block 2

$$T_1 = m_1 a \quad (1)$$

$$T_2 - m_1 a = m_2 a \quad (2)$$

so we know both T_1 and T_2

$$T_1 = m_1 a \quad (1)$$

$$T_2 = (m_1 + m_2) a \quad (2)$$

divide (1) by (2):

$$\frac{T_1}{T_2} = \frac{m_1}{m_1 + m_2}$$