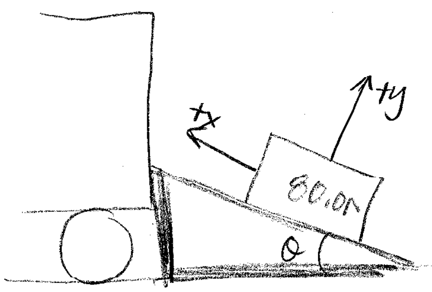
note: boxed #'s are #'s in red given in the problem set.

Problem set #

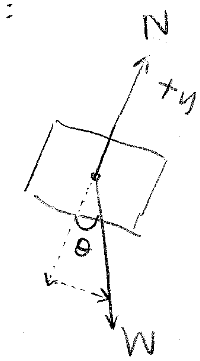
1) Given: An $\boxed{80.0N}$ crate of apples sits at rest on a ramp that runs from the ground to the bed of the truck. The ramp is inclined at $\boxed{18.4^\circ}$ to the ground.

a) what is normal force exerted on crate by ramp?

solution:



FBD:



want y-comp of weight Force vector

$W_y = |\vec{W}| \cos\theta$

use Newton's 2nd Law:

$\Sigma F_y = N - |\vec{W}| \cos\theta = 0$

Answer:

$N = |\vec{W}| \cos\theta = (80.0N)\cos(18.4^\circ) = 75.9\ N$

b) what is static frictional force exerted on the crate by ramp?

solution: want x-components:

use Newtons 2nd Law:

$\Sigma F_x = f_s - |\vec{W}| \sin\theta = 0$

Answer:

$f_s = |\vec{W}| \sin\theta = (80.0N)\sin(18.4^\circ) = 25.3N$

c) what is minimum poss. value of coeff of static friction?

Solution:

- Know: $f_s = \mu_s N$
- use Force equation from part b.

$f_s = \mu_s N = |\vec{W}| \sin\theta$ and solve for μ_s

$\rightarrow \mu_s = \frac{|\vec{W}| \sin\theta}{N}$

But from part a, $N = |W| \cos\theta = 75.9N$

& from part b, $W \sin\theta = 25.3N$

& $\mu_s = \dfrac{25.3N}{75.9N} = .333$ Answer

d) The normal and frictional forces are perpendicular comp. of the contact force exerted on the crate by the ramp. Find mag & direction of contact force.

Solution:

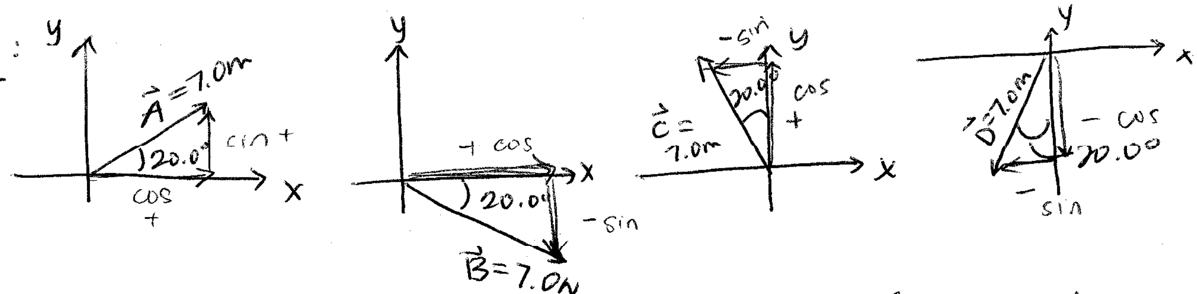
Know magnitude $\Rightarrow$ $F = \sqrt{f^2 + N^2}$

so $F = \sqrt{(25.3N)^2 + (75.9N)^2} = 80N$ Answer

Direction:

$\theta = \tan^{-1} \dfrac{N}{f} = \tan^{-1}\left(\dfrac{75.9N}{25.3N}\right) = 71^\circ$ a upward

2) Given:



Question: Find x & y- components of these four vectors.

Solution:

$A_x = |\vec{A}| \cos\theta$
$= 7.0m \cos 20^\circ$
$= 6.6m$

$A_y = |\vec{A}| \sin\theta$
$= 7.0m \sin 20^\circ$
$= 2.4m$

$B_x = |\vec{B}| \cos\theta$
$= 7.0m \cos 20^\circ$
$= 6.6m$

$B_y = -|\vec{B}| \sin\theta$
$= -7.0m \sin 20^\circ$
$= -2.4m$

$C_x = -|\vec{C}| \sin\theta$
$= -7.0 \sin 20$
$= -2.4m$

$C_y = |\vec{C}| \cos\theta$
$= 7.0 \cos 20$
$= 6.6m$

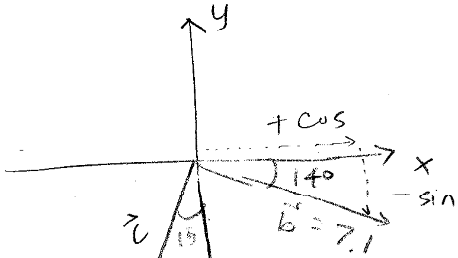
$D_x = -|\vec{D}| \sin\theta$
$= -7.0 \sin 20$
$= -2.4m$

$D_y = -|\vec{D}| \cos\theta$
$= -7.0 \cos 20$
$= -6.6m$

3) Given: vector $\vec{b}$ has mag 7.1 & dir 14° below the +x-axis. vector $\vec{c}$ has x-comp $c_x = -1.8$ & $c_y = -6.7$

Question:

a) Find the x & y comp of $\vec{b}$.



$b_x = 7.1\cos(-14°) = 6.9$

$b_y = -7.1\sin(-14°) = -1.7$

b) Find mag & dir. of $\vec{c}$

know $|\vec{c}| = \sqrt{c_x^2 + c_y^2} = \sqrt{(-1.8)^2 + (-6.7)^2} = 6.9$

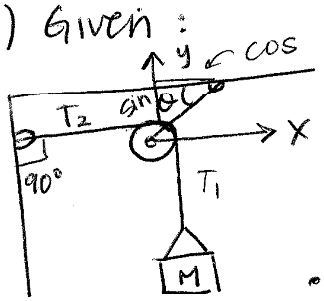
$\theta = \tan^{-1}\frac{c_y}{c_x} = \tan^{-1}\left(\frac{-6.7}{-1.8}\right) = \boxed{15° \text{ clockwise from } -y\text{-axis}}$

c) Find mag & dir of $\vec{c} + \vec{b}$

$|\vec{c} + \vec{b}| = \sqrt{(c_x + b_x)^2 + (c_y + b_y)^2} = \sqrt{(-1.8 + 6.9)^2 + (-6.7 + -1.7)^2}$
$= 9.8$

$\theta = \tan^{-1}\left(\frac{c_y + b_y}{c_x + b_x}\right) = \tan^{-1}\left(\frac{-8.4}{5.1}\right) = 31° \text{ CCW from } -y \text{ axis}$

4) Given:



Question:

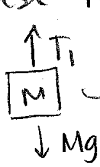
a) Find tension in rope from which the pulley hangs.

Solution:

- 1st designate a +x & +y dir
- Let T_1 = tension in rope from which pulley hangs

  T_2 = tension in rope fastened to wall

- use Newton's 2nd Law:



$\Sigma F_y = T_1 \sin\theta - Mg = 0$

$\Sigma F_x = T_1 \cos\theta - T_2 = 0$

Know: Tension in T_2 is due to Mass M, so $T_2 = Mg$

so using x comp of Force eqtn: $T_1 \cos\theta - Mg = 0$

$\rightarrow T_1 \cos\theta = Mg$

using y comp of Force eqtn: $T_1 \sin\theta = Mg$

setting these equal:

$$\cancel{T_1}\cos\theta = \cancel{T_1}\sin\theta \Rightarrow \cos\theta = \sin\theta$$

only true when $\theta = 45°$

so $T_1 = \dfrac{Mg}{\cos\theta} = \dfrac{Mg}{\cos 45} = \sqrt{2}\,Mg$ Answer

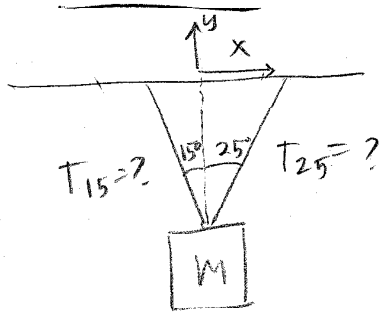
b) Find θ that rope makes with the ceiling.

as seen in part a: $\theta = 45°$

5) Given: A 45 N lithograph is supported by 2 wires. One wire makes 25° angle w/ the vertical and the other makes a 15° angle.

Question: Find tension in each wire.

solution:



- designate a coordinate axes.
- use Newton's 2nd Law

$$\Sigma F_x = T_{25}\sin 25 - T_{15}\sin 15 = 0$$

$$T_{25}\sin 25 = T_{15}\sin 15$$

$$T_{25} = \frac{\sin 15}{\sin 25} T_{15}$$

$$\Sigma F_y = T_{25}\cos 25 + T_{15}\cos 215 - W = 0$$

$$T_{15}\cos 15 = W - T_{25}\cos 25$$

$$= W - \left(\frac{\sin 15}{\sin 25} T_{15}\right)\cos 25$$

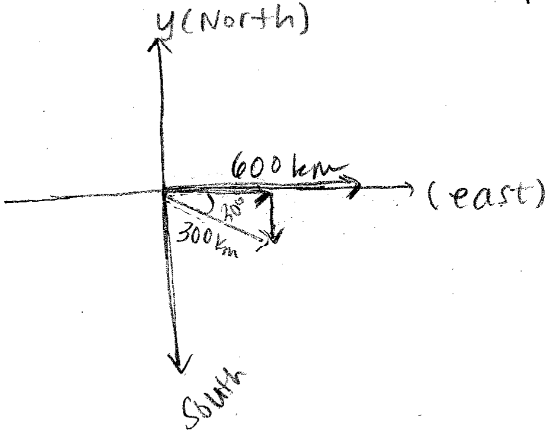
Answer

$$.97\,T_{15} = 45 - .56\,T_{15} \rightarrow T_{15} = 30\text{N}$$

back to $T_{25} = \frac{\sin 15}{\sin 25} T_{15} = \frac{\sin 15}{\sin 25}(30N) = 18N$ Answer

6) Given: ~~Show that for a projectile~~
A jetliner flies east for 600. km, then turns 30.0° toward the south & flies another 300.0 km

Question:
a) How far is the plane from its starting point?



Find mag of displacement:

$$|\Delta\vec{r}| = \sqrt{[600\,km + (300\,km)\cos(-30°)]^2 + [300\,km \sin(-30)]^2}$$

$= 873\,km$ Answer

b) In what dir could the jetliner have flown directly to the same destination (in a straight-line path)?

Find dir. of displacement:

$$\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = \tan^{-1}\left(\frac{(300\,km)\sin(-30)}{[600\,km + (300\,km)\cos(-30)]}\right)$$

$= 9.90°$ south of east Answer

c) If jetliner flew at const. speed of 400 km/h, how long did trip take?

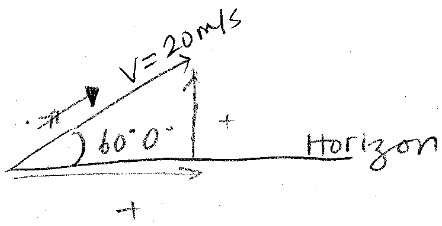
Know $v = \frac{x}{t}$; $t = \frac{x}{v} = \frac{600\,km + 300\,km}{400\,km/h} = 2.250\,h$ Answer

7) Given: an arrow is shot into the air at an $\measuredangle$ of $60.0°$ above the horizontal with a speed of $20.0 m/s$

Question:

a) what are the x & y comp of the velocity of the arrow 3.0s after it leaves the bowstring? Answer:

$$V_x = V_0 \cos\theta = (20 m/s) \cos 60 = 10 m/s$$



$$V_y = V_0 \sin\theta - gt = (20) \sin 60 - 9.8(3s)$$
$$= -12 m/s$$ Answer

b) what are x & y comp of the displacement of the arrow during the 3.0s interval?

1st calculate x-comp. of displacement using

$$\Delta x = V_x t \quad (\text{from } V = \frac{x}{t})$$

$$\Delta x = (10 m/s)(3s) = 30 m$$ Answer

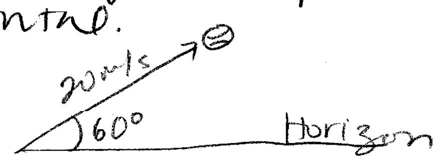
Now calc. y-comp of displacement using

$$\Delta y = V_y t - \frac{1}{2}gt^2$$ (from eqtn of motion)

$$= (-12 m/s)(3s) - \frac{1}{2}(9.8)(3.0s)^2 = 8m$$ Answer

8) Given: A ball is hit w. an initial speed of 20 m/s at an angle of 60° above the horizontal.

Question:



a) How high will ball rise?

know at max. height $V_y = 0$

$$V_y = V_0 \sin\theta - gt = 0$$

Solve for t: $t = \frac{V_0 \sin\theta}{g}$

Max. height $\Rightarrow \Delta y = V_y t - \frac{1}{2}gt^2$

$$\Delta y = (V_0 \sin\theta)t - \tfrac{1}{2} g t^2 = (V_0 \sin\theta)\left(\frac{V_0 \sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{V_0 \sin\theta}{g}\right)^2$$

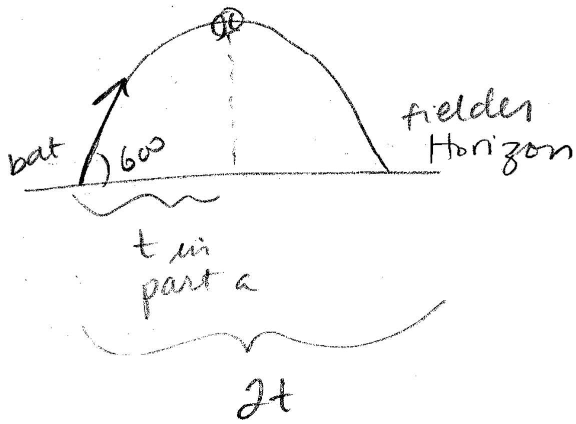
$\uparrow$ plug in for t

$$= \frac{V_0^2 \sin^2\theta}{g} - \frac{V_0^2 \sin^2\theta}{2g} = \frac{V_0^2 \sin^2\theta}{2g}$$

$$= \frac{(22\,\text{m/s})^2 \sin^2(60.0°)}{2(9.8\,\text{m/s}^2)} =$$ 19 m higher than where it was hit

Answer

b) How much time will elapse from the time the ball leaves the bat until it reaches the fielder?



$$t = 2\left(\frac{V_0 \sin\theta}{g}\right)$$

$$= 2\left(\frac{22\,\text{m/s} \sin 60°}{9.8\,\text{m/s}^2}\right)$$

$$= 3.9\,\text{s}$$ Answer

c) At what distance from home plate will the fielder be when he catches the ball?

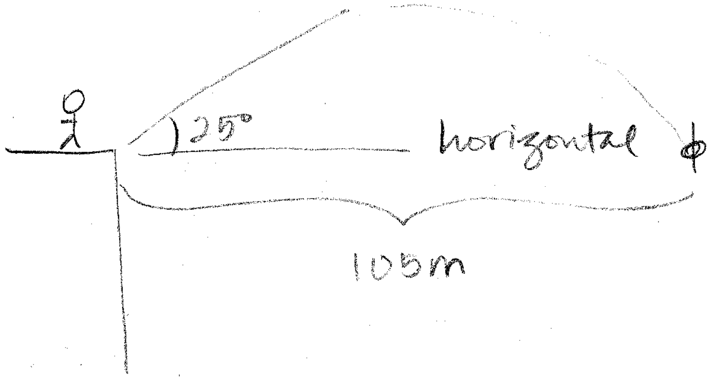
$$X = V_{0x} t = (V_0 \cos\theta)t = (22\,\text{m/s}) \cos 60° (3.9\,\text{s})$$

$$= 43\,\text{m}$$ answer

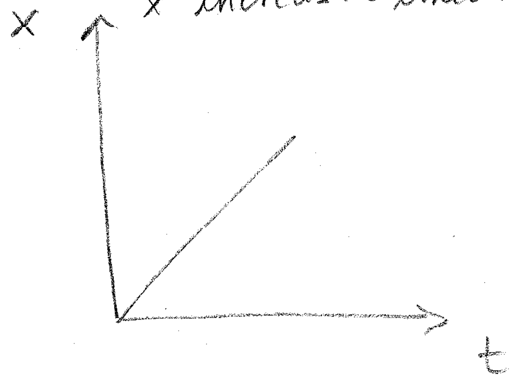
9) **given**: from edge of rooftop of building, a boy throws a stone @ an ∡ of 25° above horizontal. Stone hits 4.20 s later, 105 m away from base of building

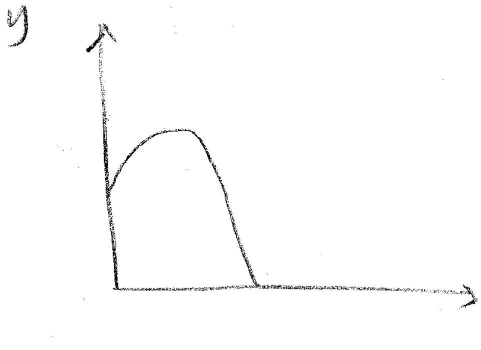
Question:

a) for stone's path in air, sketch graphs of x, y, v_x, v_y as func. of time.

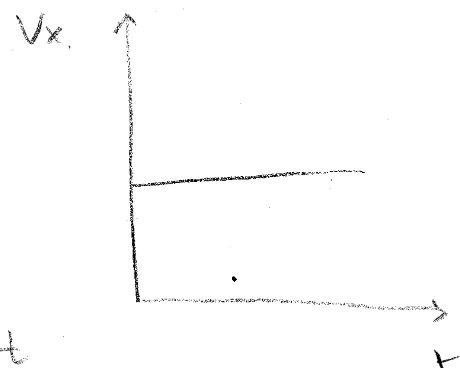


x increases linearly

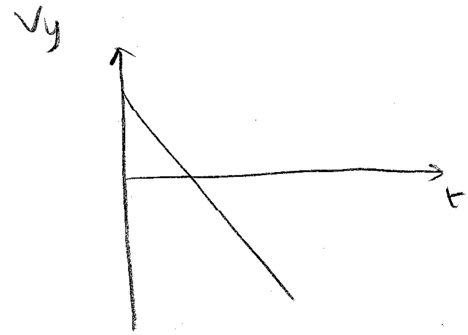




y is parabolic



V_x is constant



V_y starts positive & decreases linearly

b) Find V_0 of stone.

know $x = (v_0 \cos\theta)\, t$; solve for V_0: $V_0 = \dfrac{x}{t\cos\theta}$

$$V_0 = \frac{105\,m}{(4.20\,s)\cos(25°)} = 27.6\,m/s$$ Answer

c) Find h initial from which stone was thrown.

use V_0 from part a.

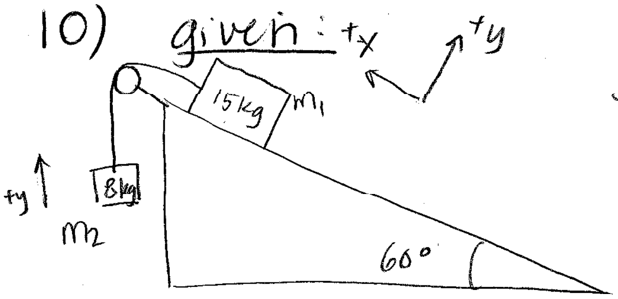
know $\Delta y = (V_0 \sin\theta)t - \frac{1}{2}gt^2$

$$= \left(\frac{x}{\cancel{t}\cos\theta}\sin\theta\right)\cancel{t} - \frac{1}{2}gt^2$$

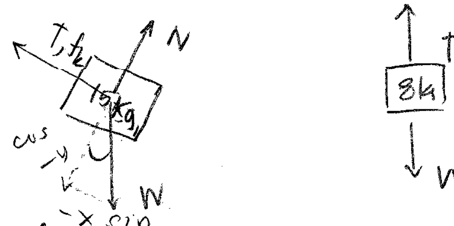
$$= 105\text{m}\left(\frac{\sin 25°}{\cos 25°}\right) - \frac{1}{2}(9.8\,\text{m/s}^2)(4.20\,\text{s})^2$$

$= -37\text{m}$ so $h = 37\text{m}$ answer.

10) given:



$\mu_s = .40$ & $\mu_k = .30$



Question: a) as crate slides down incline, what is T in rope?

Solution:

(m_1) crate: $\Sigma F_x = T + f_k - m_1 g\sin\theta = m_1 a_x$ where $f_k = \mu_k N$

$\Sigma F_y = N - m_1 g\cos\theta = 0$

(m_2) box: $\Sigma F_x = 0$

$\Sigma F_y = T - m_2 g = m_2 a_y$

from way we designated our axes: $a_x = -a_y$

→ solve for a_x: (for crate)

$m_1 a_x = T + \mu_k N - m_1 g\sin\theta = T + \mu_k m_1 g\cos\theta - m_1 g\sin\theta$

$a_x = \frac{T}{m_1} + \mu_k g\cos\theta - g\sin\theta = -a_y$

→ now solve for a_y (for box):

$$m_2 a_y = T - m_2 g \Rightarrow a_y = \frac{T}{m_2} - g$$

→ eliminate a_x & a_y & solve for T

$$\frac{T}{m_1} + \mu_k g\cos\theta - g\sin\theta = g - \frac{T}{m_2}$$

$$T\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = g(1 + \sin\theta - \mu_k \cos\theta)$$

$$T = \frac{m_1 m_2 g}{(m_1 + m_2)}(1 + \sin\theta - \mu_k\cos\theta)$$

$$= \frac{(15kg)(8kg)(9.8 m/s^2)}{(15 kg + 8kg)}(1 + \sin 60 - .30\cos 60°)$$

$$= 88N$$ Answer

b) How long does it take the crate to slide 2.00m down the incline?

from part a: $a_x = -a_y = g - \frac{T}{m_2}$

The crate begins @ rest so $V_0 = 0$

$$\Delta X = -\frac{1}{2} a_x t^2 \rightarrow t^2 = -\frac{2\Delta X}{a_x} \rightarrow t = \sqrt{\frac{-2\Delta X}{a_x}} = \sqrt{\frac{-2\Delta X}{g - T/m^2}}$$

$$= \sqrt{\frac{-2(-2.00m)}{(9.8 m/s^2) - 88N/8.0kg^2}} = 2s$$ Answer

c) To push crate back up incline @ const. speed w/ what fn P should Pauline push on the crate (parallel to incline)?

for crate: $\Sigma F_x = T + P - f_k - m_1 g \sin\theta = 0$

$$\text{so } P = f_k + m_1 g \sin\theta - T$$

fn Box: $\Sigma F_y = T - m_2 g = 0 \Rightarrow T = m_2 g$

substitute fn T & f_k

$$P = f_k + m_1 g \sin\theta - T = \mu_k m_1 g \cos\theta + m_1 g \sin\theta - m_2 g$$

$$= g[m_1(\mu_k \cos\theta + \sin\theta) - m_2]$$

$$= 70N$$ Answer

11) Given:
Boy can swim @ speed .500 M/s relative to water
river is 25.0m wide & boy ends up at 50.0m downstream from starting point.

Question:

a) How fast is current flowing in river?

know $V = \frac{X}{t}$ so $t = \frac{X_{across}}{V_{boy}}$; $V_{water} = \frac{X_{downstream}}{t}$

Answer

$$\text{so } V_{water} = \frac{X_{downstream}}{\frac{X_{across}}{V_{boy}}} = \frac{50m}{\left(\frac{25m}{.500m/s}\right)} = 1.00m/s$$

b) what is speed of boy relative to friend standing on river bank?

→ use pythagorean thm.

Answer

$$V_{bf} = \sqrt{(.500m/s)^2 + (1.00m/s^2)^2} = \boxed{1.12m/s}$$

12) <u>Given</u>: range R of projectile is defined as mag of horiz. displ. of the projectile when it returns to its orig. altitude. A projectile is launched @ $t=0$ w/ V_0 @ and angle θ above horizontal.

<u>Question</u>:

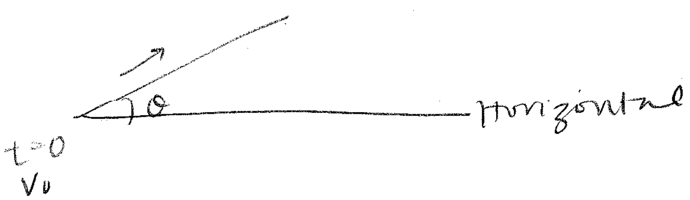
a) Find time t @ which projectile returns to its orig. amplitude.

know $\Delta y = V_{oy}t - \frac{1}{2}gt^2$

$$0 = V_{oy}t - \frac{1}{2}gt^2 \Rightarrow V_{oy}\cancel{t} = \frac{1}{2}gt^{\cancel{2}}$$

Answer

so, $V_{oy} = \frac{1}{2}gt$; $t = \dfrac{2V_{oy}}{g} = \dfrac{2V_0 \sin\theta}{g}$



b) show range is $R = \dfrac{V_0^2 \sin 2\theta}{g}$ (use trig id: $\sin 2\theta = 2\sin\theta\cos\theta$)

know $V = \dfrac{x}{t} \Rightarrow x = V_{ox}t \Rightarrow t = \dfrac{x}{V_{ox}}$

But $x=R$ so $t = \dfrac{R}{V_{ox}}$ (1)

Find t in terms of V_{oy}: from part a: $t = \dfrac{2V_{oy}}{g}$ (2)

Set (1) & (2) equal: $\dfrac{R}{V_{ox}} = \dfrac{2V_{oy}}{g}$; $R = \dfrac{2V_{oy}V_{ox}}{g}$

Answer.

$= \dfrac{2V_0^2 \sin\theta\cos\theta}{g}$, using trig identity: $R = \dfrac{V_0^2 \sin 2\theta}{g}$