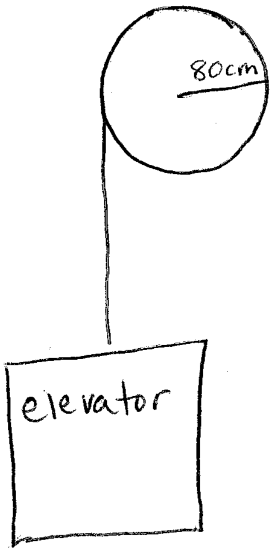
# Homework # 5



1) An elevator cable winds on a drum of radius 80 cm that is connected to a motor.

a) If the elevator is moving down at 0.60 m/s, what is the angular speed of the drum?

The drum will be moving at 0.60 m/s also so we need to convert this to rad/s.




$$\omega = \frac{v}{r} = \frac{0.60\ m/s}{.80\ m} = .75\ \frac{rad}{s}$$

b) If the elevator moves down 6.0 m, how many revolutions has the drum made?

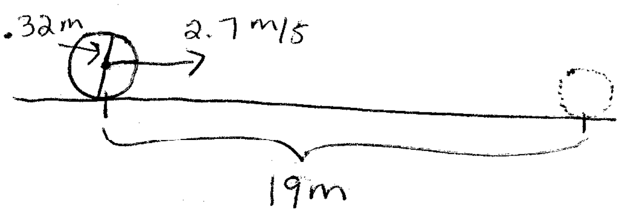
There will be 6.0 m of rope let out, So lets figure out how much rope each revolution lets out.

The circumference of the drum is $2\pi r = 2\pi(.80\ m) = 5.03\ m$.

This means for each revolution 5.03 m is let out.

So the number of revolutions $= \dfrac{6.0\ m}{5.03\ m/rev.} = 1.19$ revolutions

2) A soccer ball of diameter 32 cm rolls without slipping at a linear speed of 2.7 m/s. Through how many revolutions has the soccer ball turned as it moves a linear distance of 19 m?



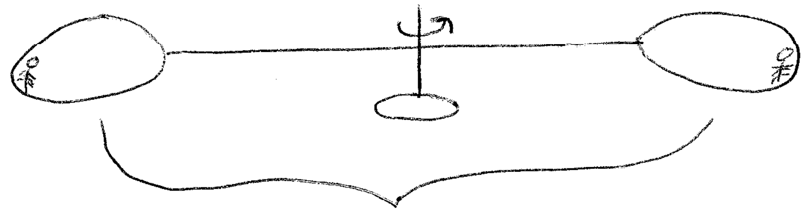

One revolution will move the distance of the circumference.

$\Rightarrow$ The ball moves $\dfrac{2\pi r}{\text{revolution}} = \dfrac{2\pi(.16m)}{\text{revolution}}$

The ball will move $\dfrac{1.0\ m}{\text{revolution}}$ $\Rightarrow$ It will make $\dfrac{19\ m}{\frac{1.0\ m}{\text{revolution}}}$

$= 19$ revolutions

3) The apparatus of the figure below is designed to study insects at an acceleration of magnitude 980 $m/s^2$ (= 100g). The apparatus consists of a 2.0 m rod with insect containers at either end. The rod rotates about an axis perpendicular to the rod at its center.




a) How fast does an insect move when it experiences a centripetal acceleration of 980 $m/s^2$?

$$\text{Centripetal acceleration} = \frac{(\text{linear velocity})^2}{\text{radius}} \Rightarrow$$

$$\text{linear velocity} = \sqrt{(\text{c. acceleration})(\text{radius})} = \sqrt{(980\ m/s^2)(1.0\ m)} = 31.3\ m/s$$

b) What is the angular speed of the insect?

$$\omega = \frac{v}{r} = \frac{31.3\ m/s}{1.0\ m} = 31.3\ rad/s$$

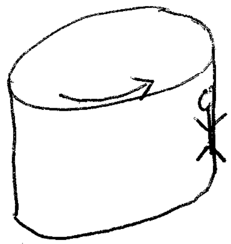
4) The rotor is an amusement park ride where people stand against the inside of a cylinder. Once the cylinder is spinning fast enough, the floor drops out.

a) What force keeps the people from falling out the bottom of the cylinder? Static friction.

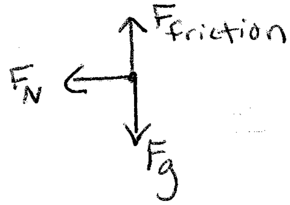
(the people aren't moving against the cylinder so there isnt any kinetic friction, gravity is pulling down, so its not helping them stay up, there is no such thing as centrifugal force, and centripetal force pushes you in towards the center of the cylinder so its not helping you stay up).

4) continued.

part b) If the coefficient of ★ Static ★ friction is 0.41 and the cylinder has a radius of 2.2 m, what is the minimum angular speed of the cylinder so that people don't fall out?



Free body diagram




$\Sigma F = ma$

$\Sigma F_{vertical} = F_{fr} + F_g = ma_{vert} = 0$

$a_{vert} = 0$ because we dont want to be moving in the vertical direction!

$F_{fr} - mg = 0 \Rightarrow F_{fr} = mg$

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$\Sigma F_{radial} = ma$

$\Sigma F_{radial} = F_N = ma_c = m\frac{v^2}{r}$

(The normal force is the only force acting in the radial direction)

So we know $F_N = \frac{mv^2}{r}$ and $F_{fr} = mg$

But $F_{fr} = \mu_s F_N \Rightarrow F_{fr} = \mu_s F_N = mg$

$$F_N = \frac{mg}{\mu_s}$$

So now we have two equations for $F_N$, they have to equal eachother, so

$\frac{mv^2}{r} = \frac{mg}{\mu_s}$ (we want to know v)

divide each side by m and multiply by r to get to get $v^2 = \frac{gr}{\mu_s} \Rightarrow v = \sqrt{\frac{gr}{\mu_s}}$

(This doesn't have m in it which is good since we weren't told m, and it means no matter how much you weigh, you won't fall out!)

$$V = \sqrt{\frac{(9.8\,m/s^2)(2.2\,m)}{(.41)}} = 7.25\,\frac{m}{s}$$

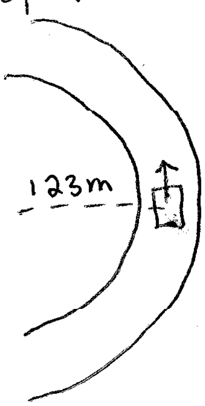
4) b) continued

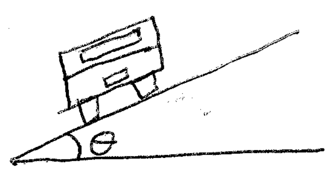
$$\omega = \frac{v}{r} = \frac{7.25\ m/s}{2.2\ m} = 3.3\ \frac{rad}{s}$$

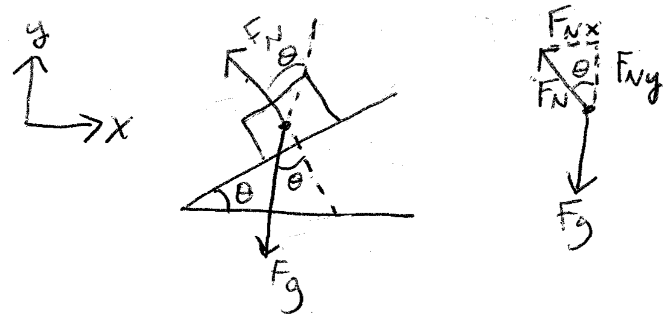
They asked for angular speed so we have to use $\omega = \frac{v}{r}$ to get the answer they are looking for.

5) A highway curve has a radius of 123m. At what angle should the road be banked so that a car traveling at 25.9 m/s has no tendency to skid sideways on the road? [Hint: No tendency to skid means the frictional force is zero]

top view: back view: free body diagram from back view

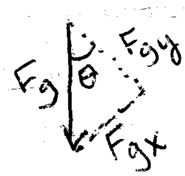



$\Sigma F = ma$

$F_{net,y} = F_{Ny} - F_g = ma_y = 0$ (car isn't moving in the y direction)




$\sin\theta = \frac{F_{Nx}}{F_N}$ $\quad F_{Nx} = F_N \sin\theta$

$\cos\theta = \frac{F_{Ny}}{F_N}$ $\quad F_{Ny} = F_N \cos\theta$

$F_{net,y} = F_{Ny} - F_g = 0 \Rightarrow F_{Ny} = F_g$ $\quad F_N \cos\theta = F_g$

$$F_N = \frac{F_g}{\cos\theta}$$

5) continued

now lets look at the x direction

$$\sum F_x = ma_x$$

$$\sum F_x = F_{Nx} = ma_c = \frac{mv^2}{r}$$ (in the x direction the acceleration of the car is centripetal acceleration)

$$F_{Nx} = F_N \sin\theta \Rightarrow F_N \sin\theta = \frac{mv^2}{r}$$ divide each side by $\sin\theta$

$$F_N = \frac{mv^2}{r\sin\theta}$$

We have 2 equations for $F_N$ now

$F_N = \frac{F_g}{\cos\theta}$ and $F_N = \frac{mv^2}{r\sin\theta}$ and they must equal eachother

So $\frac{F_g}{\cos\theta} = \frac{mv^2}{r\sin\theta}$ divide each side by $F_g$ and multiply by $\sin\theta$ to get...

$$\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{rF_g} \qquad F_g = mg \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{\cancel{m}v^2}{r\cancel{m}g} = \frac{v^2}{rg}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \text{so} \quad \tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{(25.9\,m/s)^2}{(123m)(9.8\,m/s^2)}\right) =$$

$$\theta = \tan^{-1}(.56) = 29.5°$$

6) A roller coaster of mass 320 Kg (including passengers) travels around a horizontal curve of radius 35m. It's speed is 18 m/s. What is the magnitude and direction of the total force exerted on the car by the track?

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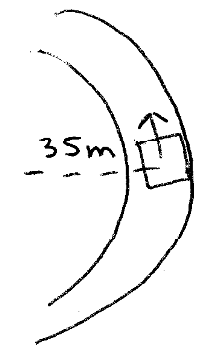
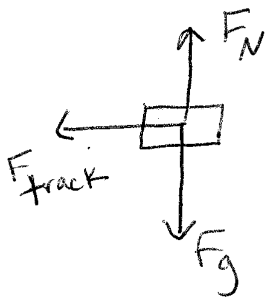
top view:




Free body diagram of side view.




$\Sigma F_y = m a_y$ ($a_y = 0$ since RC car isn't moving in the y direction.)

$\Sigma F_y = F_N - F_g = m a_y = 0$

$F_N - F_g = 0 \qquad F_N = mg$

$\Sigma F_x = m a_x = m a_c$ (in the x direction our acceleration is centripetal acceleration.)

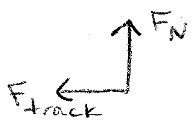
$\Sigma F_x = F_{track} = m a_c = \frac{mv^2}{r} \qquad F_{track} = \frac{mv^2}{r}$

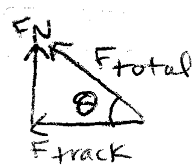
The total force exerted on the car by the track is $= F_{track} + F_N$

Remember the Normal Force is the amount the track pushes on the car because the car is pushing down on the track.

$$|F_{total}| = \sqrt{|F_{track}|^2 + |F_N|^2} = \sqrt{\left(\frac{mv^2}{r}\right)^2 + (mg)^2} =$$

$$\sqrt{\left(\frac{(320\,Kg)(18\,m/s)^2}{(35m)}\right)^2 + \left((320\,Kg)(9.8\,m/s^2)\right)^2} = 4313.8\,N$$



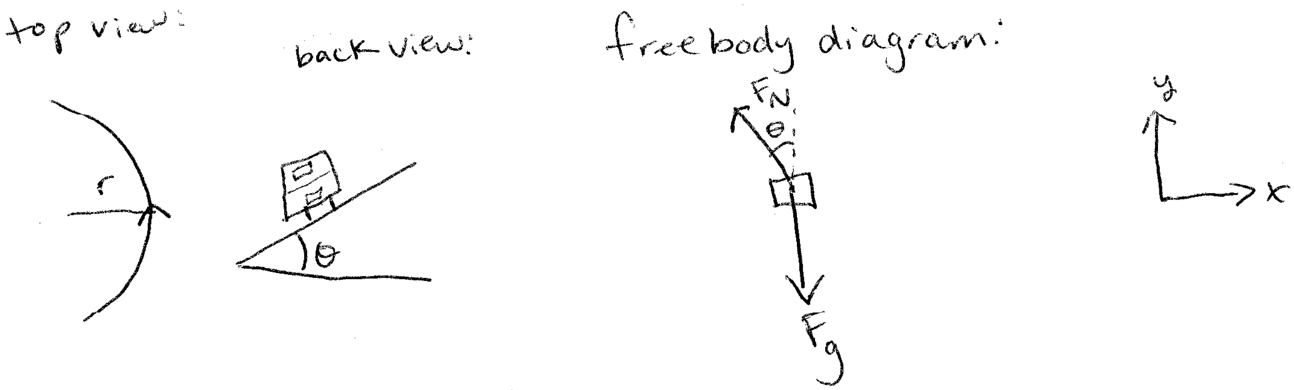




$\tan\theta = \frac{F_N}{F_{track}}$

$$\theta = \tan^{-1}\left(\frac{F_N}{F_{track}}\right) = \tan^{-1}\left(\frac{mg}{\frac{mv^2}{r}}\right) = 46.6^\circ$$

7) A car drives around a curve with radius 410 m at a speed of 32 m/s. The road is banked at 5.1°. The mass of the car is 1600 kg.

a) What is the frictional force on the car?




$\Sigma F = ma$

$\Sigma F_y = ma_y$

The fact that I am adding these forces means that when I plug in for g, I will use $-9.8\,m/s^2$.

$\Sigma F_y = \underbrace{F_{Ny} + F_g + F_{fy}} = ma_y = 0$ (the car isn't moving in the y-direction)

$F_{Ny} + F_g + F_{fy} = 0$

$F_{Ny} = F_N \cos\theta$

$F_{fy} = F_f \sin\theta$

$F_N \cos\theta + F_g + F_f \sin\theta = 0$

$F_N \cos\theta = -F_g - F_f \sin\theta = -mg - F_{fr} \sin\theta$ divide by $\cos\theta$

$$F_N = \frac{(-mg - F_{fr}\sin\theta)}{\cos\theta}$$

Now lets look at $\Sigma F_x$

$\Sigma F_x = F_{Nx} + F_{fx} = ma_x = ma_c$ (acceleration in the x direction is centripetal acceleration)

$F_{Nx} = F_N \sin\theta$ $F_{fx} = F_f \cos\theta$

$$F_N \sin\theta + F_f \cos\theta = ma_c = \frac{mv^2}{r}$$

$$F_N \sin\theta + F_f \cos\theta = \frac{mv^2}{r}$$

$$F_N \sin\theta = \frac{mv^2}{r} - F_f \cos\theta$$

$$F_N = \frac{\left(\frac{mv^2}{r} - F_f \cos\theta\right)}{\sin\theta}$$

We have 2 equations for $F_N$ so they must equal eachother.

$$\frac{(-mg - F_{fr}\sin\theta)}{\cos\theta} = \frac{\left(\frac{mv^2}{r} - F_{fr}\cos\theta\right)}{\sin\theta}$$

multiply both sides by $\sin\theta$ and $\cos\theta$ to get

$$(-mg - F_{fr}\sin\theta)\sin\theta = \left(\frac{mv^2}{r} - F_{fr}\cos\theta\right)\cos\theta$$

$$-mg\sin\theta - F_{fr}\sin^2\theta = \frac{mv^2}{r}\cos\theta - F_{fr}\cos^2\theta$$

add $mg\sin\theta + F_{fr}\cos^2\theta$ to each side to get

$$F_{fr}\cos^2\theta - F_{fr}\sin^2\theta = \frac{mv^2}{r}\cos\theta + mg\sin\theta$$

$$F_{fr}(\cos^2\theta - \sin^2\theta) = \frac{mv^2}{r}\cos\theta + mg\sin\theta$$

$$F_{fr} = \frac{\left(\frac{mv^2}{r}\cos\theta + mg\sin\theta\right)}{(\cos^2\theta - \sin^2\theta)} =$$

$$\frac{\left(\frac{1600\,kg\,(32\,m/s)^2}{410m}\cos(5.1^\circ) + (1600\,kg)(-9.8\,m/s^2)(\sin 5.1^\circ)\right)}{(\cos^2 5.1^\circ - \sin^2 5.1^\circ)} =$$

$$F_{fr} = 2627\,N$$

7) continued

part b) At what speed could you drive around this curve so that the force of friction is zero?

We will take our last equation for $F_{fr}$ and now say $F_{fr} = 0$

$$F_{fr} = \frac{\left(\frac{mv^2}{r}\cos\theta + mg\sin\theta\right)}{(\cos^2\theta - \sin^2\theta)} = 0$$

If we multiply both sides by $\cos^2\theta - \sin^2\theta$ we get

$$\frac{mv^2}{r}\cos\theta + mg\sin\theta = 0$$

Subtract $mg\sin\theta$ from each side of the equation

$$\frac{mv^2}{r}\cos\theta = -mg\sin\theta$$

divide each side by m to get

$$\frac{v^2}{r}\cos\theta = -g\sin\theta$$

multiply by r and divide by $\cos\theta$ to get

$$v^2 = -\frac{gr\sin\theta}{\cos\theta}$$

$$V = \sqrt{\frac{-gr\sin\theta}{\cos\theta}} = \sqrt{\frac{-(-9.8\,m/s^2)(410\,m)\sin(5.1^\circ)}{\cos(5.1^\circ)}}$$

$$= 18.94\ m/s$$