PHYS 2414 HWK #6 SOLUTION

$g = 9.8\ m/s^2$

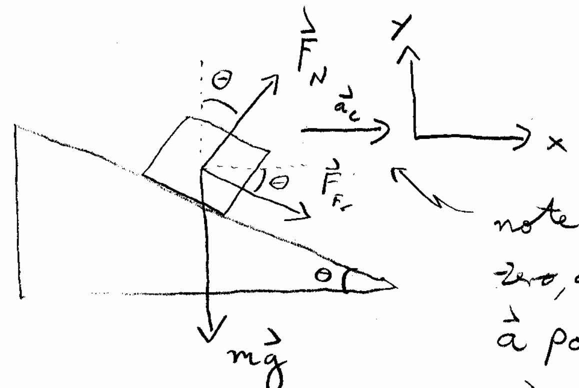
① Car on a banked curve:

$m = \boxed{1600\ kg}$

$R = \boxed{420\ m}$

$\theta = \boxed{5.4^\circ}$

$v = \boxed{38\ m/s}$



note that the acceleration is <u>not</u> zero, as it moves on a circle ⇒ $\vec{a}$ points towards the center of the circle ⇒ centripetal acceleration!

There are three forces acting on the car and we know that the car is accelerating in the $+x$ direction with an acceleration of magnitude $a_c = \frac{v^2}{r}$. We know that because the problem says that the car moves on a <u>circular</u> curve. Also we know that the car is not accelerating in the y-direction. Otherwise it would move up or down through the ground or in the air or down the incline!

This problem is 2-dimensional:

- choose coordinates: $+x$ is to the right (horizontal)
  $+y$ is up (vertical)

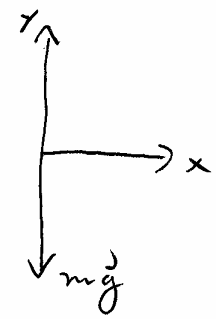
- use Newton and split it up in x and y

$$\left(\vec{F}_{net} = \sum \vec{F} = \right) m\vec{g} + \vec{F}_{fr} + \vec{F}_N = m\vec{a}$$

$$\boxed{X:} \quad (m\vec{g})_x + (\vec{F}_{fr})_x + (\vec{F}_N)_x = m(\vec{a})_x$$

$$\boxed{Y:} \quad (m\vec{g})_y + (\vec{F}_{fr})_y + (\vec{F}_N)_y = m(\vec{a})_y$$

components of $m\vec{g}$:

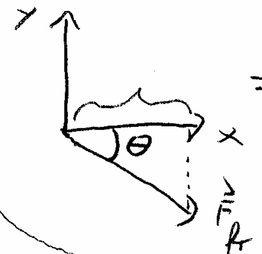


$$\Rightarrow \boxed{(m\vec{g})_x = 0}$$

and also

$$\boxed{(m\vec{g})_y = -mg}$$

components of $\vec{F}_{fr}$:

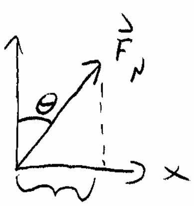


$$\Rightarrow \boxed{(\vec{F}_{fr})_x = |\vec{F}_{F_r}| \cos\theta}$$

and also

$$\boxed{(\vec{F}_{fr})_y = -|\vec{F}_{F_r}| \sin\theta}$$

components of $\vec{F}_N$:



$$\Rightarrow \boxed{(\vec{F}_N)_x = |\vec{F}_N| \sin\theta}$$

and also

$$\boxed{(\vec{F}_N)_y = |\vec{F}_N| \cos\theta}$$

components of $\vec{a}$:

The x-component of $\vec{a}$ is $\boxed{(\vec{a})_x = a_c = \frac{V^2}{R}}$, as the car moves in a horizontal circle. The y-component of $\vec{a}$ is zero $\boxed{(\vec{a})_y = 0}$.

So I get:

$$\boxed{X:} \quad 0 + |\vec{F}_{F_r}| \cos\theta + |\vec{F}_N| \sin\theta = m\frac{V^2}{R} \qquad (1.1)$$

and similarly for Y (using $g = 9.8\ m/s^2$)

$$\boxed{Y:} \quad -mg - |\vec{F}_{fr}| \sin\theta + |\vec{F}_N| \cos\theta = 0 \qquad (1.2)$$

a) $|\vec{F}_{fr}| = ?$

- We do not know $|\vec{F}_N|$, but we have two equations. Use (1.2) to solve for $|\vec{F}_N|$, then plug that into (1.1) and solve for $|\vec{F}_{fr}|$.

⇒ Using the y-eqn.:

$\boxed{y:}$

$$|\vec{F}_N| \cos\theta = mg + |\vec{F}_{fr}| \sin\theta$$

$$\Rightarrow \boxed{|\vec{F}_N| = \frac{mg + |\vec{F}_{fr}| \sin\theta}{\cos\theta}}$$

plug this $|\vec{F}_N|$ into the x-eqn and I get

$\boxed{x:}$

$$|\vec{F}_{fr}| \cos\theta + \overbrace{\left(\frac{mg + |\vec{F}_{fr}| \sin\theta}{\cos\theta} \right)}^{=|\vec{F}_N|} \sin\theta = m\frac{v^2}{R}$$

$$\Rightarrow |\vec{F}_{fr}| \cos\theta + mg\frac{\sin\theta}{\cos\theta} + |\vec{F}_{Fr}| \frac{\sin\theta}{\cos\theta} \sin\theta = m\frac{v^2}{R}$$

$$\Rightarrow |\vec{F}_{fr}| \cos\theta + mg\tan\theta + |\vec{F}_{fr}| \tan\theta \sin\theta = m\frac{v^2}{R}$$

$$\Rightarrow |\vec{F}_{fr}| \left(\cos\theta + \tan\theta \sin\theta \right) + mg\tan\theta = m\frac{v^2}{R}$$

$$\Rightarrow |\vec{F}_{fr}| \left(\cos\theta + \tan\theta \sin\theta \right) = m\frac{v^2}{R} - mg\tan\theta$$

$$\Rightarrow |\vec{F}_{fr}| = \frac{m\frac{v^2}{R} - mg\tan\theta}{\cos\theta + \tan\theta \sin\theta}$$

$$\left|\vec{F}_{fr}\right| = \frac{m\frac{v^2}{R} - mg\tan\theta}{\cos\theta + \tan\theta\sin\theta}$$

using numbers:

$$\left|\vec{F}_{fr}\right| = \frac{\boxed{1600\,kg}\cdot\frac{\boxed{38\,m/s}^2}{\boxed{420\,m}} - \boxed{1600\,kg}\cdot 9.8\,m/s^2\tan\boxed{5.4^\circ}}{\cos\boxed{5.4^\circ} + \tan\boxed{5.4^\circ}\sin\boxed{5.4^\circ}}$$

$$\Rightarrow \left|\vec{F}_{fr}\right| = \boxed{4000.9\,N}$$

b) What speed can you have if $|\vec{F}_{fr}| = 0$?

We have from a):

$$\left|\vec{F}_{fr}\right| = \frac{m\frac{v^2}{R} - mg\tan\theta}{\cos\theta + \tan\theta\sin\theta}$$

now $|\vec{F}_{fr}|$ is zero and we find V!

$$0 = \frac{m\frac{v^2}{R} - mg\tan\theta}{\cos\theta + \tan\theta\sin\theta} \quad \Big| \cdot(\cos\theta + \tan\theta\sin\theta)$$

$$0 = \frac{mv^2}{R} - mg\tan\theta \quad \Big| + mg\tan\theta$$

$\div\, mg$

$$\tan\theta = \frac{v^2}{gR} \quad \Big| \cdot gR \quad \sqrt{}$$

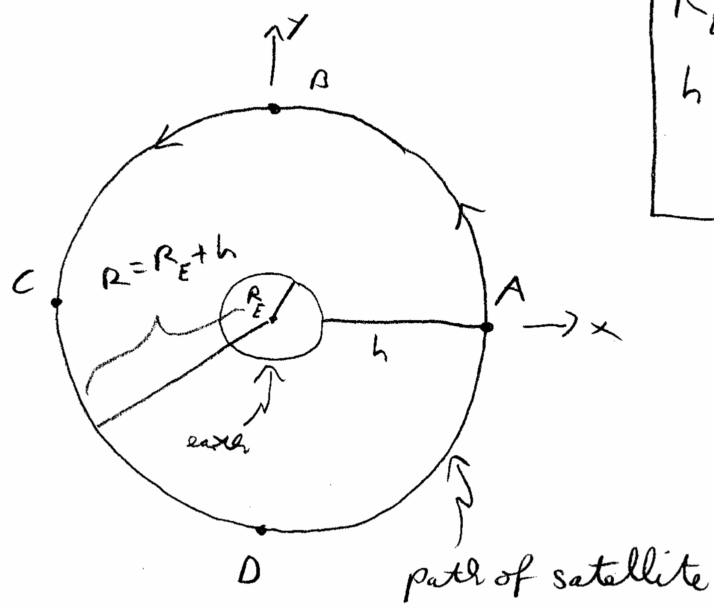
$$\Rightarrow \sqrt{gR\tan\theta} = v$$

using numbers

$$v = \sqrt{9.8\,\frac{m}{s^2}\cdot \boxed{420\,m}\,\tan\boxed{5.4^\circ}}$$

$$\Rightarrow v = \boxed{19.7\,\frac{m}{s}}$$

② uniform circular motion:



R_E : earth radius
h : height of satellite

$R_E = 6378$ km

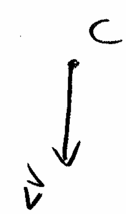
$h = \boxed{35878\text{ km}}$

The satellite moves on a circle of radius

$$R = R_E + h = 6378\text{ km} + \boxed{35878\text{ km}} = \boxed{42256\text{ km}}$$

a) What is it's instantaneous velocity at point C?

The velocity is tangential to the circle, so it points $\boxed{\underline{\text{down}}}$



or in the $-y$-direction. $|\vec{v}|$ is constant as motion is uniform. It takes 1 day for it to go once round as it is in a geosynchronous orbit. So

$$\Rightarrow |\vec{v}| = \frac{\text{distance}}{\text{time}} = \frac{\text{circumference}}{\text{day}} = \frac{2\pi R}{24\text{ hrs}} = \frac{265502\text{ km}}{86400\text{ s}}$$

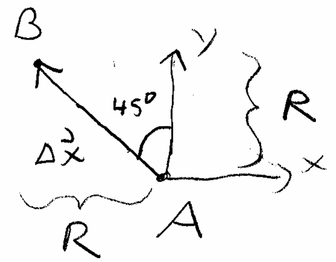
$$\Rightarrow |\vec{V}| = \frac{265502\ \text{km}}{86400\ \text{s}} = \boxed{3.07\ \text{km/s}}$$

b) $\vec{V}_{avg} = \dfrac{\Delta \vec{x}}{\Delta t}$

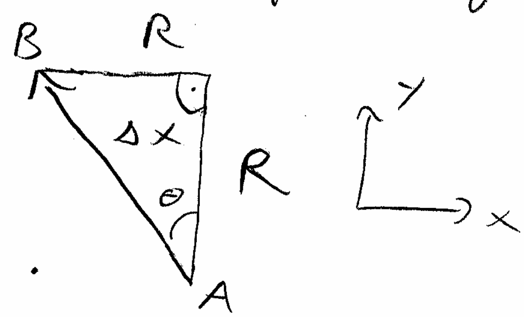
where $\Delta \vec{x}$ is a vector pointing from A to B

and $\Delta t = \frac{1}{4}$ day

$\Delta \vec{x}$:



as A and B are rightmost and upmost points on the circle we have the following triangle



it is a right triangle and $\tan\Theta = \dfrac{R}{R} = 1$

$\Rightarrow$ $\Theta = \tan^{-1} 1 = 45^\circ$ this is Θ for $\Delta\vec{x}$ and is also Θ for $\vec{V}$!

also $|\Delta\vec{x}| = \sqrt{R^2+R^2} = \sqrt{2}\,R$

So $|\vec{V}_{avg}| = \dfrac{\sqrt{2}\,R}{\frac{1}{4}\text{day}} = \dfrac{\sqrt{2}\,\boxed{42256\ \text{km}}}{6\ \text{hrs}} = \boxed{2.77}\ \text{km/s}$

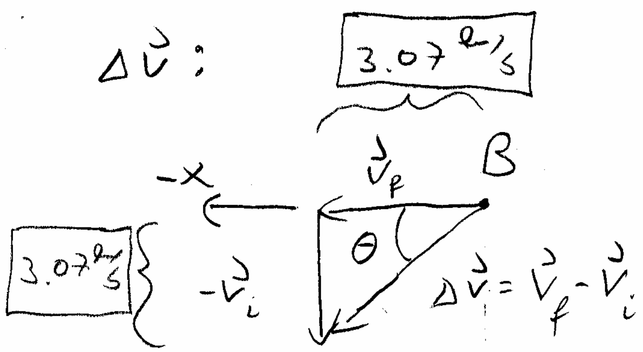
c) $\vec{a}_{avg} = ?$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

where $\Delta t = \frac{1}{4}$ day

and $\Delta \vec{v} = \vec{v}_f - \vec{v}_i$

$\Delta \vec{v}$:



$\theta = 45°$

from this picture, I get

$$|\Delta \vec{v}| = \sqrt{(\Delta v)_x^2 + (\Delta v)_y^2} = \sqrt{\boxed{3.07\ \tfrac{km}{s}}^2 + \boxed{-3.07\ \tfrac{km}{s}}^2}$$

$$= \boxed{4.34\ \tfrac{km}{s}}$$

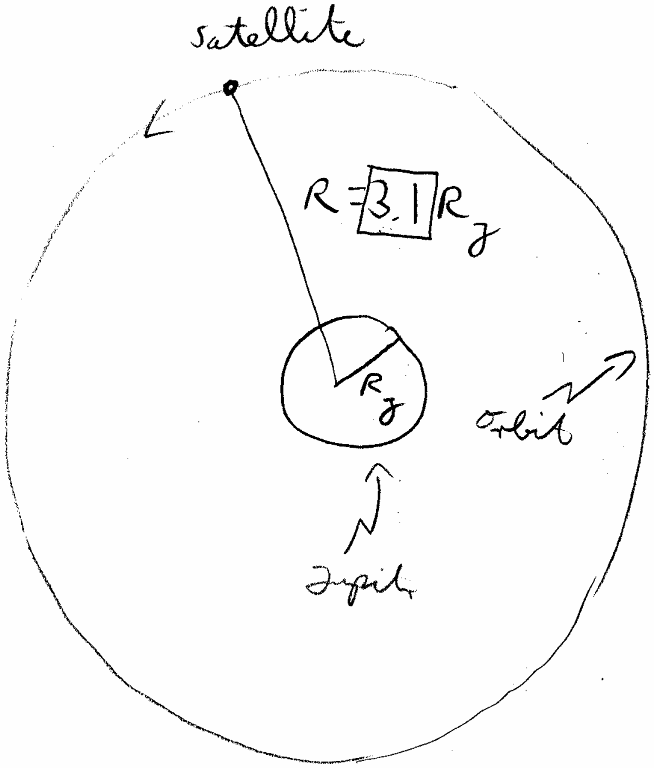
and from the picture

$\theta = 45°$ with $-x$-direction. This is angle of $\Delta\vec{v}$ but is the same for $\vec{a}_{avg}$

then

$$|\vec{a}_{avg}| = \frac{\boxed{4.34\ km/s}}{6\ hrs} = \boxed{2.01 \cdot 10^{-4}}\ km/s = \boxed{0.201\ m/s}$$

3



$R_J = \boxed{71505 \text{ km}}$

a at surface of Jupiter $= a_0 = \boxed{25 \text{ N/kg}}$

$T = ?$

We need to first find a at R, where the satellite is. We know a at the surface of Jupiter is $a_0 = 25$ N/kg. We also know that the gravitational field decreases with distance as $\propto \frac{1}{R^2}$, so we know the ratio

$$\frac{a_0}{a} = \frac{\frac{1}{R_0^2}}{\frac{1}{R^2}} = \frac{R^2}{R_0^2}$$ but $R_0 = R_J$

the radius of Jupiter, so

$$\frac{a_0}{a} = \frac{R^2}{R_J^2} \Rightarrow a = a_0 \frac{R_J^2}{R^2}$$

But $R = \boxed{3.1} R_J$, so

$$a = a_0 \frac{1}{\boxed{3.1}^2} \frac{R_J^2}{R_J^2} = a_0 \frac{1}{\boxed{3.1}^2} = \frac{\boxed{25}\ N/kg}{\boxed{3.1}^2}$$

$$\Rightarrow \quad a = \boxed{2.60}\ N/kg$$

The satellite is moving on a circle and nothing else, so

$$a = a_c = \frac{v^2}{R} \Rightarrow v = \sqrt{aR}$$

But also

$$\omega = \frac{v}{R} \quad \text{and} \quad \omega = 2\pi f \quad \text{and} \quad f = \frac{1}{T}$$

so

$$v = \frac{2\pi R}{T} \Rightarrow \boxed{T = \frac{2\pi R}{v}} \left(= \frac{\text{total distance}}{\text{time}} \right)$$

plug in $v = \sqrt{aR}$ $\Rightarrow$ $T = \dfrac{2\pi R}{\sqrt{aR}}$ and $R = \boxed{3.1} R_J$

So

$$T = \frac{2\pi \boxed{3.1} R_J}{\sqrt{\boxed{2.6}\ N/kg\ \boxed{3.1} R_J}} = \frac{2\pi \boxed{3.1} \boxed{71505 \cdot 10^3 m}}{\sqrt{\boxed{2.6}\ N/kg\ \boxed{3.1} \boxed{71505 \cdot 10^3 m}}}$$

so $\boxed{T = 58015.3 \text{ s} = 16.12 \text{ hrs}}$

(4) $v = \frac{2\pi R}{\Delta t}$

for two satellites around the same planet, we have from the gravitational field

$$\frac{a_1}{a_2} = \frac{R_2^2}{R_1^2}$$

But also $a_1 = \frac{v_1^2}{R_1}$ and $a_2 = \frac{v_2^2}{R_2}$

so also

$$\frac{v_1^2}{R_1} \cdot \frac{R_2}{v_2^2} = \frac{R_2^2}{R_1^2}$$

so $\boxed{v_2^2 = v_1^2 \frac{R_1}{R_2}} \Rightarrow v_2 = v_1 \sqrt{\frac{R_1}{R_2}}$

satellite 1: $v_1 = \frac{2\pi R_1}{\Delta t_1}$ $\quad R_1 = r$, $\boxed{\Delta t_1 = 15\,h}$

satellite 2: $v_2 = \frac{2\pi R_2}{\Delta t_2}$ $\quad R_2 = \boxed{4.1 R_1}$, $\Delta t_2 = ?$

also from above $v_2 = v_1 \sqrt{\frac{R_1}{R_2}}$

so $$\frac{2\pi R_2}{\Delta t_2} = v_1 \sqrt{\frac{R_1}{R_2}} = \frac{2\pi R_1}{\Delta t_1} \sqrt{\frac{R_1}{R_2}}$$

and thus

$$\frac{2\pi R_2}{\Delta t_2} = \frac{2\pi R_1}{\Delta t_1}\sqrt{\frac{R_1}{R_2}}$$

solve for Δt_2

$$\Delta t_2 = \Delta t_1 \frac{R_2}{R_1}\sqrt{\frac{R_2}{R_1}}$$

$$\boxed{\Delta t_2 = \Delta t_1 \left(\frac{R_2}{R_1}\right)^{3/2}}$$

with $\Delta t_1 = \boxed{15\,h}$ and $R_2 = \boxed{4.1\,R_1}$

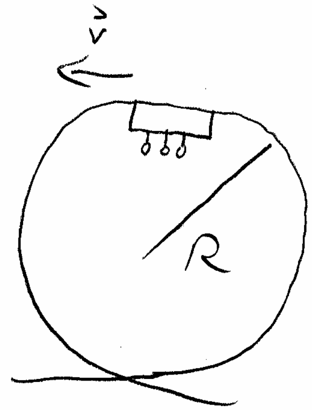
I get

$$\Delta t_2 = \boxed{15\,h}\left(\boxed{4.1}\right)^{3/2}$$

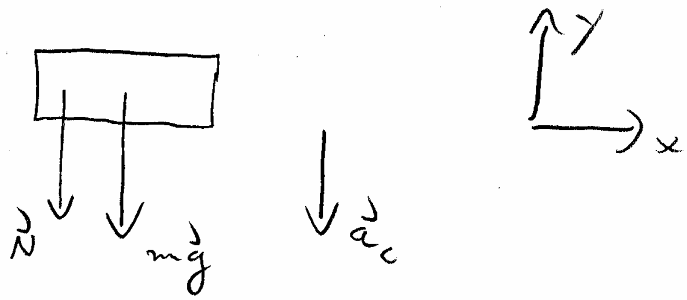
$$\Delta t_2 = \boxed{15\,h} \cdot \boxed{8.3}$$

$$\Delta t_2 = \boxed{124.5\,h}$$

5



$R = \boxed{24.4\,m}$



The normal force must be greater or equal zero so that the passengers are in contact with their seats. They just barely make it, when $\vec{N} = 0$, so use newton

$$m\vec{g} + \vec{N} = m\vec{a}_c$$

and set $N = 0$ to find the minimum

$$m\vec{g} = m\vec{a}_c$$

$\vec{a}_c$ and $m\vec{g}$ point downwards at the highest point, so only one equation (for y)

$\boxed{y:}$ $$-mg = -m\frac{v^2}{R}$$

solve for v:

$$\boxed{V = \sqrt{gR}}$$

$$V = \sqrt{9.8\,\tfrac{m}{s^2} \cdot \boxed{24.4\,m}}$$

$$V = \boxed{15.5}\,\tfrac{m}{s}$$

⑥ fastest way to make U-turn at constant speed?

use $a = \frac{v^2}{R}$. What is the fastest way?

To find the fastest way, we need to find the <u>time</u> it takes. Constant speed, so

$$V = \frac{d}{\Delta t} \leftarrow \text{distance}$$

But $d = \frac{1}{2} 2\pi R$ as we go on a U-turn, we go half-way round a circle!

so $V = \frac{\pi R}{\Delta t} \Rightarrow \Delta t = \frac{\pi R}{V}$

also $V = \sqrt{aR}$ so $\Delta t = \frac{\pi R}{\sqrt{aR}}$

$$\Delta t = \frac{\pi R}{\sqrt{aR}} = \frac{\pi \sqrt{R}\sqrt{R}}{\sqrt{aR}} = \frac{\pi\sqrt{R}}{\sqrt{a}}$$

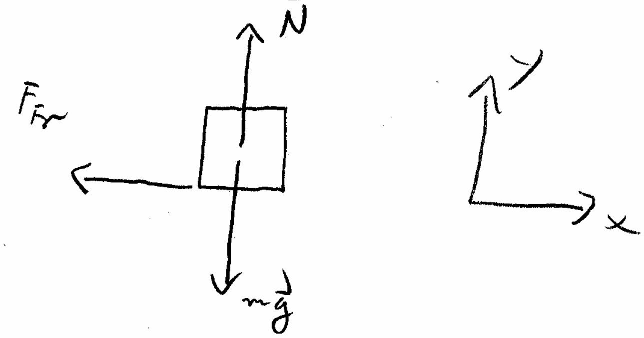
This is it, we have the total time in terms of radius and acceleration. To go fastest we want Δt to be smallest, so we immediately see that we want a small R and a large a! This can be seen from $\Delta t = \pi\sqrt{\frac{R}{a}}$, the smaller R and the larger a, the smaller Δt.

a) From $\Delta t = \pi \frac{\sqrt{R}}{\sqrt{a}}$ we want SMALLEST R

b) Take SMALLEST R and LARGEST a from problem:

$$\Delta t_{MIN} = \pi\sqrt{\frac{\boxed{5.1\,m}}{\boxed{3.1\,m/s^2}}} = \boxed{4.03\,s}$$

(7) COIN on record:



It must move on a circle, so the force needed to keep this motion going is the <u>static friction</u> $\vec{F}_{Fr}$ with the ground. Nothing else can give the needed $\vec{a}_c$, as both normal force $\vec{N}$ and gravity $m\vec{g}$ are only <u>vertical</u>! (But $\vec{a}_c$ is horizontal of course as the record is not banked). The y-direction gives $\boxed{N = mg}$

<u>Note</u> $f = \boxed{33.3\,\text{rpm}} = \frac{33.3}{\text{min}} \Rightarrow \boxed{V = 2\pi R f = 2\pi R \frac{33.3}{60\,s}}$

$\boxed{x:}$ $\quad \sum(\vec{F})_x = m(\vec{a})_x$

circle

$$(\vec{F}_{Fr})_x = m(\vec{a})_x = -m\,a_c\,,$$

$\boxed{F_{Fr} \text{ MAXXES OUT at } F_{Fr} = \mu_s N}$

$F_{Fr\,MAX}$

$$-\mu_s N = -m\frac{V^2}{R_{MAX}}$$

So

$$\mu_s N = m \frac{v^2}{R_{MAX}}$$

replace $N = mg$ and $V = 2\pi R_{max} \frac{\boxed{33.3}}{60s}$

$$\mu_s m g = m \frac{(2\pi)^2 R_{MAX}^2 \left(\frac{33.3}{60s}\right)^2}{R_{MAX}}$$

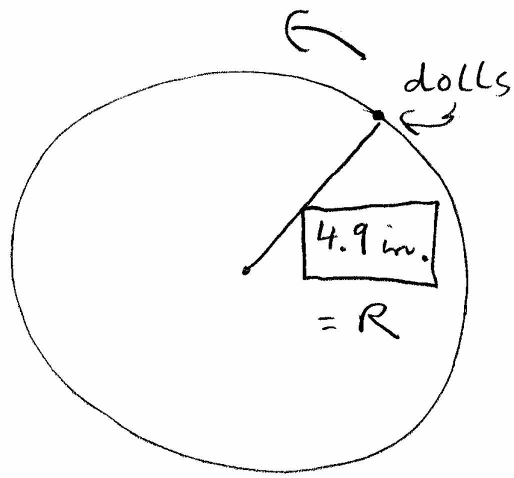
solve for R_{MAX}

$$\boxed{R_{MAX} = \frac{\mu_s g}{(2\pi)^2 \left(\frac{33.3}{60s}\right)^2}}$$

use $\mu_s = \boxed{0.1}$, $g = 9.8$ to get

$$R_{MAX} = \boxed{0.0805\ m} = \boxed{8.05\ cm}$$

(8)



$f = \boxed{45\ rpm}$

a) $V = 2\pi R f = 2\pi \boxed{4.9\ in.} \boxed{45 \frac{1}{min}}$

But 1 inch = 2.54 cm = 0.0254 m

and 1 min = 60 s

So

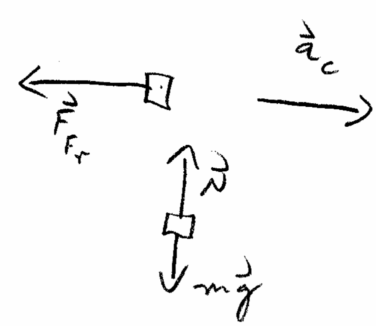
$V = 2\pi \boxed{4.9} \cdot 0.0254\ m \cdot \frac{45}{60\ s}$

$V = \boxed{0.587\ m/s}$

b) Same as problem 7:

in y-direction:

in x-direction:



from x I get

$$N = mg$$

from y I get $|\vec{F}_{F_r}| = m|\vec{a}_c| = m\frac{v^2}{R} = m\omega^2 R$

where this time I used $\frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$

and, as $\omega = 2\pi f$, I get

for y direction $|\vec{F}_{F_r}| = m(2\pi f)^2 R$

r.p.m. no.

but f is given, for me it is $f = \frac{\boxed{45}}{min} = \frac{\boxed{45}}{60s}$

so the friction is in magnitude

$$|\vec{F}_{F_r}| = m\left(2\pi\frac{\boxed{45}}{60s}\right)^2 \boxed{4.9}\ in.$$

$$= m\left(2\pi\frac{\boxed{45}}{60s}\right)^2 \cdot \boxed{4.9} \cdot 0.0254\ m$$

but $|\vec{F}_{F_r}| \leq \mu_s mg = \boxed{0.14}\ mg$

So the question is if

$$\underbrace{\boxed{0.14} \cdot 9.8\ m/s^2}_{\text{MAX FRICTION}} \overset{?}{\geq} \underbrace{\left(2\pi\frac{\boxed{45}}{60s}\right)^2 \boxed{4.9} \cdot 0.0254\ m}_{\text{rhs}}$$

evaluating both sides:

$$\boxed{1.32}\ \frac{m}{s^2} \overset{(?)}{\geq} \boxed{2.76}\ \frac{m}{s^2}$$

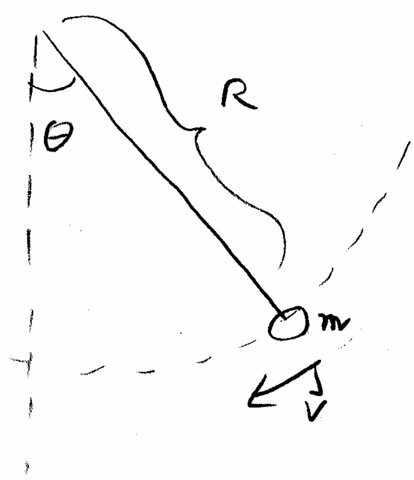
The MAX FRICTION is $\boxed{1.32}\ \frac{m}{s^2} \cdot$ mass

force needed to keep them on their circle at $\boxed{45}$ r.p.m.

is $\boxed{2.76}\ \frac{m}{s^2} \cdot$ mass $\Rightarrow$ FRICTION not strong

enough $\Rightarrow$ they fall

9



$\theta = \boxed{15.0°}$

$g = \boxed{9.8\ m/s^2}$

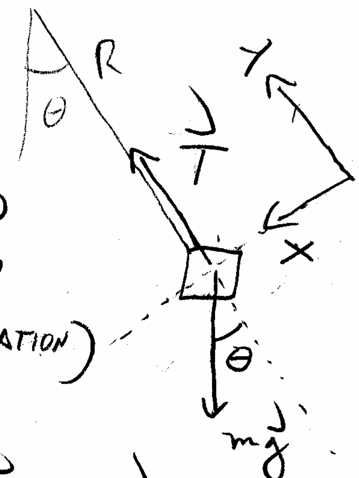
$m = \boxed{1\ kg}$

$R = \boxed{0.8\ m}$

$V = \boxed{1.4\ m/s}$

Do FBD

(ONLY GRAVITY AND TENSION BUT ALSO NONZERO ACCELERATION)



$\sum \vec{F} = m\vec{a} \Rightarrow \vec{T} + m\vec{g} = m\vec{a}$

x(tangent): $+mg\sin\theta + 0 = m\,a_t$

y(centrip): $T - mg\cos\theta = m\,a_c$

From x: $a_t = +g\sin\theta$

$a_t = 9.8\ m/s^2 \sin \boxed{15.0°} = \boxed{2.54\ m/s^2}$

From y: $T = m a_c + m g \cos\theta$

But $a_c = \frac{v^2}{R} = \frac{\boxed{1.4}^2 (m/s)^2}{\boxed{0.8} m} = \boxed{2.45 \; m/s^2}$

and with this

$$T = m \boxed{2.45} \, m/s^2 + m g \cos\theta$$

$$= \boxed{1 \, kg} \cdot \underbrace{\boxed{2.45} \, m/s^2}_{= a_c} + \boxed{1 \, kg} \cdot \boxed{9.8 \, m/s^2} \cos \boxed{15.0°}$$

$T = 11.9 \, N$