note: boxed #'s are #'s in red given in the problem set.

Problem set #7

1) <u>Given</u>: sled dragged along horizontal path @ const. speed 1.5 m/s by rope inclined @ ∡ 30.°. Weight of sled: **470N**

Tension in rope: **270 N**

<u>Question</u>: How much work done by rope on sled in **15s**?

<u>solution</u>: Know $W = F \Delta r \cos\theta$ $V = \frac{\Delta r}{\Delta t} \Rightarrow \Delta r = V\Delta t$

$$= T\, v \Delta t \cos\theta$$

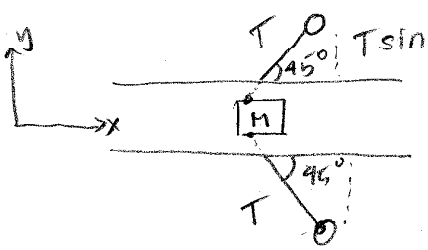
$$= (270N)\left(1.5\tfrac{m}{s}\right)(15s)\cos 30.0^\circ = \boxed{5.26kJ}$$

2) <u>Given</u>: barge mass 5.0×10^4 kg pulled along canal by 2 mules walking along towpaths parallel to canal on either side of it. Ropes make 45° ∡ to canal. Each mule pulling on its rope w/ Force of 1.0 kN.

<u>Questn</u>: How much work done on barge by both mules as they pull barge **120m** along canal?

<u>solution</u>: x-axis // to canal; y-axis ⊥.

using $\Sigma F = ma$: $\Sigma F_y = T\sin\theta - T\sin\theta = 0$



$$\Sigma F_x = T\cos\theta + T\cos\theta = F_x$$

$$W = F_x \Delta X = 2T\cos\theta\, \Delta X$$

$$= 2(1.0kN)\cos 45\,(120m) = \boxed{170\,kJ}$$

3) <u>Given</u>: plane weighs: **220kN**; veloc. horizontally: **63 m/s (141 mi/h)**

cables bring plane to stop in distance: **87m**

a) <u>Question</u>: work done on plane by cables?

<u>Solution</u>: $W = \Delta K = \frac{1}{2}m(v^2 - v_0^2) = \frac{mg}{2g}(0 - v_0^2)$

$$= \frac{220 \times 10^3 N}{2(9.8 m/s^2)}(63 m/s)^2 = \boxed{-44.5\,MJ}$$

b) <u>Question</u>: Force exerted on plane by cables?

$$W = F\Delta r\cos\theta = F\Delta r\underbrace{\cos 0^\circ}_{1} = F\Delta r$$

$$F = \frac{W}{\Delta r} = \frac{4.45 10^7 J}{87 J} = \boxed{512\,kN}$$

4) Given: desk moves ~~5.0~~ 4.11 m w/ horiz. force: 360N. Then he moves it back to starting pnt w/ constant force: 360N

a) Question: change in desk's gravitational pot. energy during round trip?

solution: **0 J** because the floor is level.

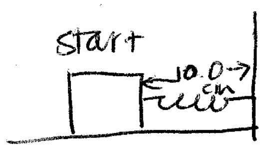
b) Question: work done to desk?

sol: know: $W = F_x \Delta x = (360N)(2 \cdot 4.1m) =$ **2.95 kJ**

c) Question: if work done ≠ to change in gravitational pot. energy of desk, then where has energy gone?

solution: the energy has been dissipated as heat by friction between the bottom of the desk & the floor.

5) Given:



orig. length **2.0cm** → total length **6.0cm**

mass pulled so spring stretches to ~~total~~ length of 10 cm.

Question: max. speed of mass as oscillates?

solution: use $\Sigma F = ma \Rightarrow \Sigma F_y = kx_1 - mg = 0$

so $kx_1 = mg \Rightarrow k = \frac{mg}{x_1}$

$V_{max} = ?$ Know $K_i + K_f = U_i + U_f$ [pot energy U for spring $= \frac{1}{2}kx^2$]

$\Rightarrow 0 + \frac{1}{2}mV_{max}^2 = \frac{1}{2}kX_{max}^2 + 0$

$\Rightarrow V_{max} = X_{max}\sqrt{\frac{k}{m}}$ ✓ sub in for k

$= X_{max}\sqrt{\frac{mg}{x_1} \cdot \frac{1}{m}}$

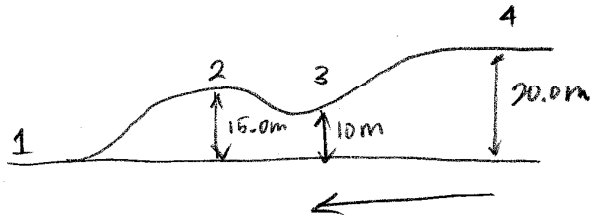
$= (.100m - .020m)\sqrt{\frac{9.8m/s^2}{(.060m - .020m)}}$

$=$ **1.25 m/s**

6) Given: cart moving to ~~right~~ left ~~passes~~ from pnt. 4 with veloc: 12 m/s to left.

Question: Find speed @ pnts 1, 2, 3

Solution:



$E_4 = K_4 + U_4 = \frac{1}{2}mv_4^2 + mgy_4 = E_i$

$E_n = K_n + U_n = \frac{1}{2}mv_n^2 + mgy_n$

where $n = 1, 2, 3$

Know $E_f = E_i$

So $K_n + U_n = K_4 + U_4$

$\Rightarrow \frac{1}{2}mv_n^2 + mgy_n = \frac{1}{2}mv_4^2 + mgy_4$

$v_n^2 + 2gy_n = v_4^2 + mgy_4$

$v_n = \sqrt{v_4^2 + 2g(y_4 - y_n)}$

(use this to calc. v for n = 1, 2, 3)

$n=1 \quad v_1 = \sqrt{(12\,m/s)^2 + 2(9.8\,m/s^2)(20m - 0m)}$

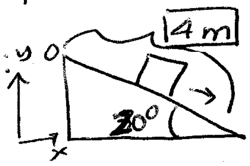
$= 23.2\,m/s$

$n=2 \quad v_2 = \sqrt{(12\frac{m}{s})^2 + 2(9.8\frac{m}{s^2})(20m - 15m)}$

$= 15.6\,m/s$

$n=3 \quad v_3 = \sqrt{(12\frac{m}{s})^2 + 2(9.8\frac{m}{s^2})(20m - 10m)} = 18.4\,m/s$

7) Given:



20.0°

$v_i = 5\,m/s$

Question: what is speed @ bottom of incline?

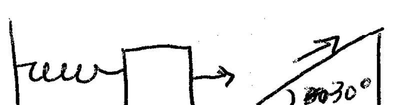
Solution: Know $\Delta K = -\Delta U \Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mg\Delta y$

$v_f = \sqrt{(5\frac{m}{s})^2 - 2(9.8\frac{m}{s^2})(0 - (4m)\sin 20°)} \Rightarrow v_f^2 - v_i^2 = -2g\Delta y$

$= 7.2\,m/s$

$v_f = \sqrt{v_i^2 - 2g\Delta y}$

8) Given:



k = ~~40.0N/m~~ 20N/m

.50kg ~~spr.~~ object compressed .2m & released

Question: if object not attached to spring, how far up incline does it travel before coming to rest & then sliding back down?

solution: elastic pot. energy of spring is converted to gravitational pot. energy, so $\frac{1}{2}kx^2 = mgh = mg(l\sin\theta)$
where l = distance traveled up incline

$$\text{so } l = \frac{kx^2}{2mg\sin\theta} = \frac{(20.0\,N/m)(.20m)^2}{2(.50kg)(9.8\,\frac{N}{kg})(\sin 30°)} = \boxed{.163m}$$

9) Given: spring gun ($\boxed{k = 22\,N/m}$) used to shoot 56-g ball horizontally. Initially, spring compressed by $\boxed{19cm}$ ball loses contact w/ spring & leaves gun when spring still compressed by 12cm.

Question: what is speed of ball when it hits ~~[illegible]~~ ground $\boxed{1.2m}$ below gun.

Solution: $\boxed{V_y}$ found from K due to gravity

$$\rightarrow \frac{1}{2}mv_y^2 = mgh \Rightarrow V_y^2 = 2gh$$

$\boxed{V_x}$ found from K from spring

$$\rightarrow \frac{1}{2}mv_x^2 = \frac{1}{2}k(x_i^2 - x_f^2) \Rightarrow V_x^2 = \frac{k(x_i^2 - x_f^2)}{m}$$

know $V = \sqrt{V_x^2 + V_y^2} = \sqrt{\frac{k(x_i^2 - x_f^2)}{m} + 2gh}$

$$= \sqrt{\frac{(22\frac{N}{m})(.19m^2 - .12m^2)}{.056\,kg} + 2(9.8)(1.2m)} = \boxed{5.66m/s}$$

10) Given: power output @ const. speed of $\boxed{6.0m/s}$ on level road is $\boxed{110W}$

a) Question: ~~How much power~~ what is force exerted on cyclist & bicycle by air? (note: $\vec{V}$ is anti-// to F_a)

solution:

know $P_a + P_c = 0$ since $\frac{\Delta U}{\Delta t} = 0$

know instantaneous power: $P = Fv\cos\theta$

so $P_a = F_a v\cos\theta$

$F_a = \frac{P_a}{V\cos\theta}$

$$F_a = \frac{-110W}{(6.0\frac{m}{s})\cos(180°)} = \boxed{18.3N}$$

(− b/c anti-parallel)

b) Question: By bending low over handlebars, cyclist reduces air resistance to $\boxed{15N}$. If she ~~retains~~ maintains a power output $\boxed{110W}$, what is speed?

solution: $V = P_c / F_c\cos\theta = 110W / (-15N)\cos 180° = \boxed{7.33N}$

11) Given:

roller coaster car mass: 988kg
starts from rest
ignore friction & air resistance

a) Question: what speed does car reach top of loop?

Solution: use conservation of energy: $\Delta K = -\Delta U$

$\Rightarrow \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mg(h_f - h_i)$

$\frac{1}{2}mv_f^2 - 0 = mg(h_i - h_f) \Rightarrow v_f = \sqrt{2g(h_i - h_f)}$

$v_f = \sqrt{2(9.8)(40m - 20m)} = \boxed{19.8 m/s}$

b) Question: force exerted on car by track @ top of loop?

Solution:
- Need to find Normal force on car.
- use Newton's 2nd Law: $\Sigma F_c = ma_c \Rightarrow N + mg = ma_c$

so $N = m(a_c - g) = m(\frac{v^2}{r} - g)$

sub in for v from part a:

$N = m\left(\frac{2g(h_i - h_f)}{r} - g\right) = 988kg\left(\frac{2(9.8)(20m)}{10m} - 9.8\right)$

r → rad of loop

$= \boxed{29.1 kN}$

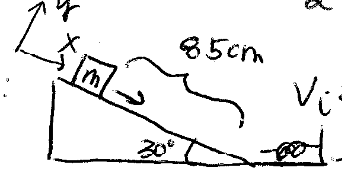
c) Question: from what min h above bottom of loop can car be released so that it does not lose contact w/ track @ top of loop?

Solutn: Set $F = 0 \Rightarrow 0 = m(\frac{v^2}{r} - g) \Rightarrow \frac{v^2}{r} - g = 0 \Rightarrow gr = v^2$

$\rightarrow gr = 2g(h_i - h_f) \Rightarrow \frac{r}{2} = h_i - h_f \Rightarrow h_i = \frac{r}{2} + h_f = \frac{10m}{2} + 20m$

$= \boxed{25.0 m}$

12) Given:



$v_i = 0$ $m = .50 kg$

$\mu_k = .25$

(block slides 85cm & encounters a spring $k = 35\frac{N}{m}$)

a) Question: what is max compression of spring?

Solution: First, find speed @ bottom of incline

know: $\Delta K = W_c + W_{nc} = W_{total}$

(conservative force → W_c; nonconservative force → W_{nc})

$\Rightarrow \frac{1}{2}mv^2 - 0 = mgh - Fd \rightarrow v = \sqrt{2gh - \frac{2Fd}{m}}$

use Newton's 2^{nd} Law:

$\Sigma F_y = N - mg\cos\theta = 0$ so $N = mg\cos\theta$

Know $d = .85m$ $h = d\sin\theta$, $F = \mu_k N = \mu_k mg\cos\theta$

sub in:

$$v = \sqrt{2gd\sin\theta - \frac{2\mu mgd\cos\theta}{m}} = \sqrt{2gd(\sin\theta - \mu\cos\theta)}$$

2nd

Find max. compression using conservation of energy:

$K_f + U_f = K_i + U_i \Rightarrow 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$

$$\Rightarrow x = v\sqrt{\frac{m}{k}} = \sqrt{\frac{2mgd}{k}(\sin\theta - \mu\cos\theta)}$$

$$= \sqrt{\frac{2(.50kg)(9.8\frac{m}{s^2})(.85m)}{35N/m}(\sin 30 - .25\cos 30°)}$$

$= \boxed{26\ cm.}$

Question:

b) After compression of part (a) spring rebounds & shoots block back up incline. How far along incline does block travel before coming to rest?

Solution: note: when block accel. by spring, it attains its previous kinetic energy & speed.

∴ Need to find distance along incline d'

Know $\Delta K = W_c + W_{nc} \Rightarrow 0 - \frac{1}{2}mv^2 = -mgh - Fd'$

$\Rightarrow \frac{1}{2}mv^2 = mgd'\sin\theta + \mu mgd'\cos\theta$ ← from part a

$$d' = \frac{v^2}{2g(\sin\theta + \mu\cos\theta)} = \frac{2gd(\sin\theta - \mu\cos\theta)}{2g(\sin\theta + \mu\cos\theta)}$$

$$= (85cm)\frac{\sin 30 - .25\cos 30}{\sin 30 + .25\cos 30} = \boxed{34cm}$$