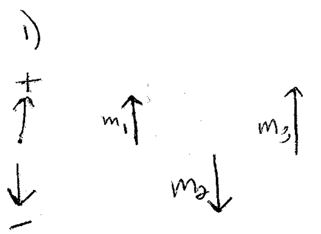
Homework # 8 Solutions

1)



	mass	velocity
m_1	3.1 Kg	3.2 m/s
m_2	3.8 Kg	-4.8 m/s
m_3	6.8 Kg	1.8 m/s

momentum = mass · Velocity

total momentum = $m_1V_1 + m_2V_2 + m_3V_3 =$

$(3.1\,Kg)(3.2\,m/s) + (3.8\,Kg)(-4.8\,m/s) + (6.8\,Kg)(1.8\,m/s) =$

$9.92\,kg\,m/s - 18.24\,Kg\,m/s + 12.24\,Kg\,m/s = 3.92\,Kg\,m/s$

The magnitude is 3.92 Kg m/s and because our answer is positive the direction is north.

2)

20 mi/h ⟶ 60.2 mi/h

a) what is the ratio of the final to the initial magnitude of its momentum?

$$\frac{\text{momentum final}}{\text{momentum inital}} = \frac{m V_{final}}{m V_{initial}} = \frac{V_{final}}{V_{initial}} = \frac{60.2\,mi/h}{20\,mi/h} = 3.01$$

b) What is the ratio of the final to the initial Kinetic energy?

$$\frac{\text{Kinetic energy final}}{\text{Kinetic energy initial}} = \frac{\frac{1}{2} m V_{final}^2}{\frac{1}{2} m V_{initial}^2} = \frac{V_{final}^2}{V_{initial}^2} = \frac{(60.2\,mi/h)^2}{(20\,mi/h)^2} = 9.06$$

3)

m = 3.5 Kg

t = 3.6 s

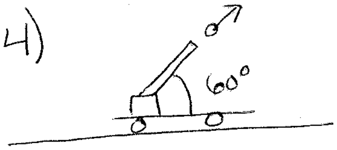
$F = \frac{\Delta p}{\Delta t}$ $\quad \Delta p = F \Delta t$

The change in momentum will be equal to force times change in time.

Force here equals mg so $\Delta p = mg\Delta t = (3.5\,Kg)(9.8\,m/s^2)(3.6\,s)$

$= 123.48\,Kg\,m/s$

4)



$m_{\text{cannon ball}} = 97 \text{ Kg}$ $\qquad$ $m_{\text{cannon + car}} = 4.9 \times 10^4 \text{ Kg}$

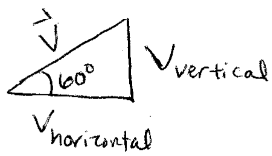
$V_{\text{cannon ball final}} = 105 \text{ m/s}$ $\qquad$ $V_{\text{cannon + car final}} = ?$

Momentum is conserved so

$$m_{CB} V_{CB\,\text{initial}} + m_{c+c} V_{c+c\,\text{initial}} = m_{CB} V_{CB\,\text{final}} + m_{c+c} V_{c+c\,\text{final}}$$

Initially both the cannon ball and cannon plus car are at rest so $V = 0$

We are concerned with the horizontal direction only and we need to find the horizontal velocity of the cannon ball after it is fired



$$V_{\text{horizontal cannon ball final}} = \vec{V} \cos 60° = (105 \text{ m/s})(\cos 60°) = 52.5 \text{ m/s}$$

Plugging in all our known values into the conserved momentum equation we get

$$(97 \text{ Kg})(0 \text{ m/s}) + (4.9 \times 10^4 \text{ Kg})(0 \text{ m/s}) = (97 \text{ Kg})(52.5 \text{ m/s}) + (4.9 \times 10^4 \text{ Kg})(V_{c+c\,\text{final}})$$

$$0 + 0 = (97 \text{ Kg})(52.5 \text{ m/s}) + (4.9 \times 10^4 \text{ Kg})(V_{c+c\,\text{final}})$$

$$0 = 5092.5 \text{ Kg m/s} + (4.9 \times 10^4 \text{ Kg})(V_{c+c\,\text{final}})$$

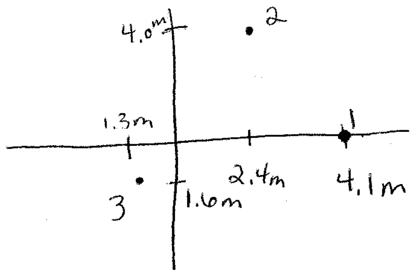
$$-5092.5 \text{ Kg m/s} = (4.9 \times 10^4 \text{ Kg})(V_{c+c\,\text{final}})$$

$$-\frac{5092.5 \text{ Kg m/s}}{4.9 \times 10^4 \text{ Kg}} = V_{c+c\,\text{final}}$$

$$= -.104 \text{ m/s}$$

recoil speed of cannon + car = .104 m/s in the opposite direction

5)



$m_1 = 4.2\,kg$

$m_2 = 5.5\,kg$

$m_3 = 2.7\,kg$

What is the location of their center of mass?

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3} = \frac{(4.2kg)(4.1m)+(5.5kg)(2.4m)+(2.7kg)(-1.3m)}{4.2kg + 5.5kg + 2.7kg}$$

$$= \frac{17.22\,kgm + 13.2\,kgm - 3.51\,kgm}{12.4\,kg} = \frac{26.91\,kgm}{12.4\,kg} = 2.17\,m$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1+m_2+m_3} = \frac{(4.2kg)(0m)+(5.5kg)(4.0m)+(2.7kg)(-1.6m)}{4.2kg+5.5kg+2.7kg}$$

$$= \frac{0\,kgm + 22\,kgm - 4.32\,kgm}{12.4\,kg} = \frac{17.68\,kgm}{12.4\,kg} = 1.42\,m$$

6) $m_{\substack{freight\\car}} = m$ $\quad V_{\substack{freight\\car\\initial}} = 1.0\,m/s$

$m_{Boxcar} = 3.9m$ $\quad V_{\substack{Boxcar\\initial}} = 0\,m/s$

Conservation of momentum says

a) $m_{\substack{freight\\car}} V_{\substack{freight\\car\\initial}} + m_{Boxcar} V_{\substack{Boxcar\\initial}} = (m_{\substack{freight\\car}} + m_{Box\,car}) V_{final}$ (they coupled together so they move together at the same velocity)

Plugging in for our known values

$(m)(1.0\,m/s) + (3.9m)(0\,m/s) = (m + 3.9m)V_{final}$

$(m)(1.0\,m/s) = (4.9\,m)\,V_{final}$ $\Rightarrow$ $V_{final} = \dfrac{\cancel{m}(1.0\,m/s)}{4.9\,\cancel{m}} = .204\,m/s$

6) b) now the two cars are at rest after the collision and we want to know how fast the boxcar was going initially.

$m_{\text{freight car}} = m$ $\quad V_{fc\,\text{initial}} = 1.0\,m/s$ $\quad V_{fc\,\text{final}} = 0\,m/s$

$m_{\text{boxcar}} = 3.9m$ $\quad V_{bc\,\text{initial}} = ?$ $\quad V_{bc\,\text{final}} = 0\,m/s$

Conservation of momentum says

$$(m_{fc} V_{fc\,\text{initial}}) + (m_{BC} V_{BC\,\text{initial}}) = (m_{fc} V_{fc\,\text{final}}) + (m_{BC} V_{BC\,\text{final}})$$

plugging in our known values we get

$$(m)(1.0\,m/s) + (3.9m)(V_{BC\,\text{initial}}) = (m)(0\,m/s) + (3.9m)(0\,m/s)$$

$$(m)(1.0\,m/s) + (3.9m)(V_{BC\,\text{initial}}) = 0 + 0$$

$$(3.9m)(V_{BC\,\text{initial}}) = -(m)(1.0\,m/s)$$

$$V_{BC\,\text{initial}} = \frac{-(m)(1.0\,m/s)}{(3.9m)} = -.256\,m/s$$

(negative means its moving in the opposite direction from the freight car)

7) $m_{\text{Bullet}} = .026\,kg$ $\quad V_{\text{Bullet initial}} = 204.8\,m/s$ $\quad V_{\text{Bullet final}} = -101.7\,m/s$

$m_{\text{block}} = 2.0\,kg$ $\quad V_{\text{Block initial}} = 0\,m/s$ $\quad V_{\text{Block final}} = ?$

(I'm setting east to be positive, west to be negative) $- \leftarrow \quad \rightarrow +$

conservation of momentum means that

$$m_{\text{Bullet}} V_{\text{Bullet initial}} + m_{\text{Block}} V_{\text{Block initial}} = m_{\text{Bullet}} V_{\text{Bullet final}} + m_{\text{Block}} V_{\text{Block final}}$$

Plug in known values

$$(.026\,kg)(204.8\,m/s) + (2.0\,kg)(0\,m/s) = (.026\,kg)(-101.7\,m/s) + (2.0\,kg)(V_{\text{Block final}})$$

7) continued

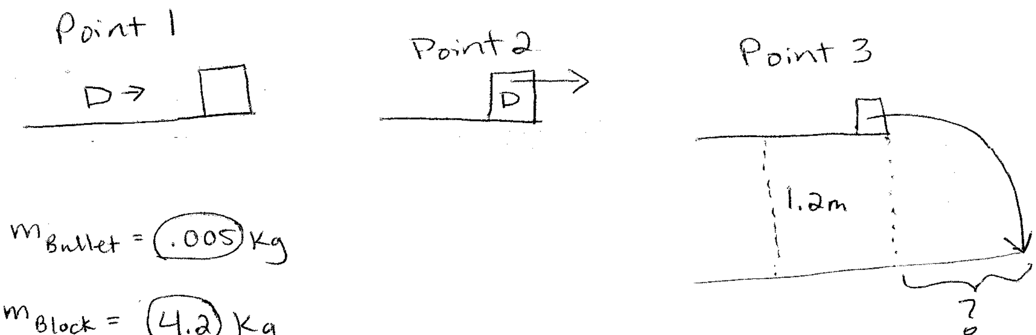
$5.325\ kg\,m/s + 0\ kg\,m/s = -2.644\ kg\,m/s + (2.0\ kg)(V_{Block\ final})$

$5.325\ kg\,m/s = -2.644\ kg\,m/s + (2.0\ kg)(V_{Block\ final})$

$7.969\ kg\,m/s = (2.0\ kg)(V_{Block\ final})$

$V_{Block\ final} = \dfrac{7.969\ kg\,m/s}{2.0\ kg} = 3.985\ m/s$

positive means direction is to the east.

8) 

$m_{Bullet} = .005\ kg$

$m_{Block} = 4.2\ kg$

We will use conservation of momentum between points 1 and 2 to solve for the blocks velocity at point 2, then use kinematics to solve for the distance in the horizontal direction the block goes before hitting the ground.

$V_{Bullet\ 1} = 400.2\ m/s$

$V_{B+B\ 2} = ?$

$V_{Block\ 1} = 0\ m/s$

Conservation of momentum says that

$m_{Bullet} V_{Bullet\ 1} + m_{Block} V_{Block\ 1} = m_{Bullet} V_{Bullet\ 2} + m_{Block} V_{Block\ 2}$

Plugging in known values we get...

$(.005\ kg)(400.2\ m/s) + (4.2\ kg)(0\ m/s) = (.005\ kg + 4.2\ kg)(V_{B+B\ 2})$

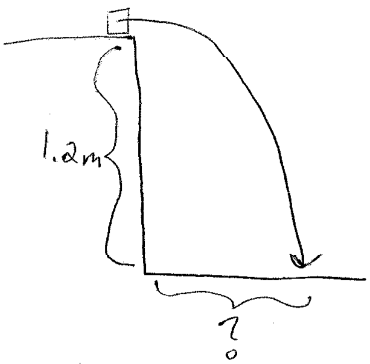
$2.001\ kg\,m/s + 0 = (4.205\ kg)(V_{B+B\ 2})$

$(V_{B+B\ 2}) = \dfrac{2.001\ kg\,m/s}{4.205\ kg} = .476\ m/s$

This is the velocity the block and bullet will have as they fall off the table.

8) continued

now we have



$m = (4.2\,kg + .005\,kg) = 4.205\,kg$

$V_{xo} = .476\,m/s$ $\quad V_{yo} = 0\,m/s$

$X_o = 0\,m$ $\quad Y_o = 1.2\,m$

$X_f = ?$ $\quad Y_f = 0\,m$

$a_x = 0\,m/s^2$ $\quad a_y = -9.8\,m/s^2$

we need to know X_f so lets use the kenematic equations.

$X_f = X_o + V_{xo}t + \frac{1}{2}a_x t^2$

Plugging in known values

$X_f = 0\,m + (.476\,m/s)(t) + \frac{1}{2}(0\,m/s^2)t^2$

$X_f = (.476\,m/s)\,t$

we need to find out how long it took to hit the floor to find the distance so lets analyze the y direction now.

$Y_f = Y_o + V_{oy}t + \frac{1}{2}a_y t^2$

Plugging in known values

$0\,m = 1.2\,m + (0\,m/s)t + \frac{1}{2}(-9.8\,m/s^2)t^2$

$0\,m = 1.2\,m + 0\,m - \frac{1}{2}(9.8\,m/s^2)t^2$

$-1.2\,m = -\frac{1}{2}(9.8\,m/s^2)t^2$ now multiply by -2 and divide by $9.8\,m/s^2$

$\frac{-2(-1.2\,m)}{(9.8\,m/s^2)} = t^2$ $\quad t^2 = \frac{4.2\,m}{9.8\,m/s^2} = .429\,s^2$

$t = .655\,s$

now we can plug this into our Kinematic equation in the x direction.

$X_f = (.476\,m/s)(t) = (.476\,m/s)(.655\,s) = .312\,m$

The block will land .312 m away from the base of the table.

9)

$m_{wood} = 1.04$ Kg $V_{\substack{wood \\ initial}} = 0$ m/s

$m_{bullet} = .053$ kg $V_{\substack{bullet \\ initial}} = 104.0$ m/s $V_{w+b} = ?$

Conservation of momentum means

$$\left(m_{wood} V_{\substack{wood \\ initial}}\right) + \left(m_{Bullet} V_{\substack{bullet \\ initial}}\right) = \left(m_{wood} + m_{bullet}\right) V_{wtb}$$

plugging in for known values

$$(1.04 \text{ Kg})(0 \text{ m/s}) + (.053 \text{ Kg})(104.0 \text{ m/s}) = (1.04 \text{ Kg} + .053 \text{ Kg}) V_{wtb}$$

$$0 \text{ Kg m/s} + 5.512 \text{ Kg m/s} = (1.093 \text{ Kg}) V_{wtb}$$

$$5.512 \text{ kg m/s} = (1.093 \text{ kg}) V_{wtb}$$

$$V_{wtb} = \frac{5.512 \text{ Kg m/s}}{1.093 \text{ Kg}} = 5.043 \text{ m/s}$$

10) $m = 3900$ Kg

$V_i = 2.7 \times 10^4$ m/s

$V_f = 1.0 \times 10^4$ m/s

$F = 1.9 \times 10^5$ N

$$F = \frac{\Delta p}{\Delta t} \Rightarrow \Delta t = \frac{\Delta p}{F} = \frac{m v_f - m v_i}{F_{rocket}} =$$

$$\frac{(3900 \text{ Kg})(1.0 \times 10^4 \text{ m/s}) - (3900 \text{ Kg})(2.7 \times 10^4 \text{ m/s})}{1.9 \times 10^5 \text{ N}} = \frac{-6.63 \times 10^7 \text{ Kg m/s}}{1.9 \times 10^5 \text{ Kg m/s}^2}$$

$\Delta t = -349$ seconds   Don't worry about the negative, its just there because I took $\Delta p = P_f - P_i$ it would be positive if $\Delta p = P_i - P_f$

11)

$m_{sub} = 2.3 \times 10^6$ Kg $\quad V_{sub\,initial} = 0$ m/s $\quad V_{sub\,final} = ?$

$m_{torpedo} = 250$ Kg $\quad V_{torpedo\,initial} = 0$ m/s $\quad V_{torpedo\,final} = 100.6$ m/s

Conservation of energy means

$$(m_{sub} V_{sub\,initial}) + (m_{torp} V_{torp\,initial}) = (m_{sub} V_{sub\,final}) + (m_{torp} V_{torp\,final})$$

plugging in known values

$$(2.3 \times 10^6 \text{ Kg})(0 \text{ m/s}) + (250 \text{ Kg})(0 \text{ m/s}) = (2.3 \times 10^6 \text{ Kg})(V_{sub\,final}) + (250 \text{ Kg})(100.6 \text{ m/s})$$

$$0 \text{ Kg m/s} + 0 \text{ Kg m/s} = (2.3 \times 10^6 \text{ Kg})(V_{sub\,final}) + 25150 \text{ Kg m/s}$$

$$0 \text{ Kg m/s} = (2.3 \times 10^6 \text{ Kg})(V_{sub\,final}) + 25150 \text{ Kg m/s}$$

$$-25150 \text{ Kg m/s} = (2.3 \times 10^6 \text{ Kg})(V_{sub\,final})$$

$$V_{sub\,final} = \frac{-25150 \text{ Kg m/s}}{2.3 \times 10^6 \text{ Kg}} = -.011 \text{ m/s}$$

(negative because it is moving in the opposite direction of the torpedo)